Exam 1: Tuesday, Feb 14, 5:00-6:00 PM

Test rooms:

Instructor	Sections	Room
Dr. Hale	F, H	104 Physics
Dr. Kurter	B, N	125 BCH
Dr. Madison	K, M	199 Toomey
Dr. Parris	J, L	St Pat's Ballroom*
Mr. Upshaw	A, C, E, G	G-3 Schrenk
Dr. Waddill	D	120 BCH
Special Accommodations		Testing Center

*exam 1 only

If at 5:00 on test day you are lost, go to 104 Physics and check the exam room schedule, then go to the appropriate room and take the exam there.

Today's agenda:

Capacitors and Capacitance.

You must be able to apply the equation C=Q/V.

Capacitors: parallel plate, cylindrical, spherical.

You must be able to calculate the capacitance of capacitors having these geometries, and you must be able to use the equation C=Q/V to calculate parameters of capacitors.

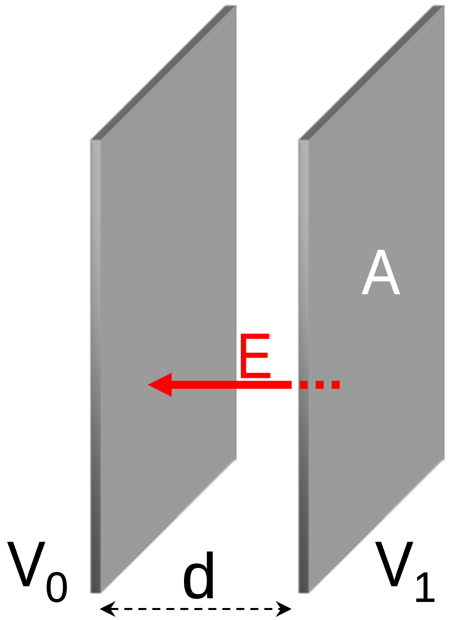
Circuits containing capacitors in series and parallel.

You must understand the differences between, and be able to calculate the "equivalent capacitance" of, capacitors connected in series and parallel.

Capacitors: the basics

What is a capacitor?

- device for **storing charge**
- simplest example: two parallel conducting plates separated by air

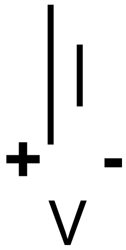


assortment of capacitors

Capacitors in circuits

symbol for capacitor (think parallel plates)

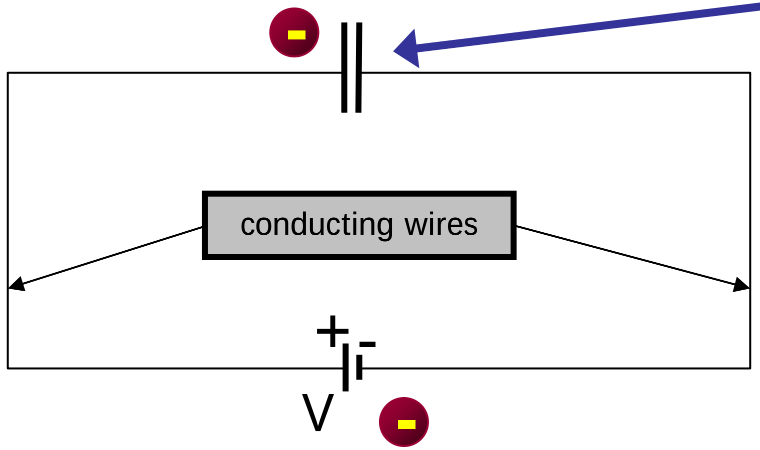
symbol for battery, or external potential



battery voltage V is actually **potential difference** between the terminals

- when capacitor is connected to battery, charges flow onto the plates

Capacitor plates build up charges +Q and -Q



- when battery is disconnected, charge remains on plates

Capacitance

How much charge can a capacitor store?

Better question: How much charge can a capacitor store per voltage?

Capacitance: $$C = \frac{Q}{V}$$

V is really $|\Delta V|$, the potential difference across the capacitor

capacitance C is a **device property**, it is always positive

unit of C: farad (F)
1 F is a large unit, most capacitors have values of C ranging from picofarads to microfarads (pF to μF).

micro $\Rightarrow 10^{-6}$, nano $\Rightarrow 10^{-9}$, pico $\Rightarrow 10^{-12}$ (Know for exam!)

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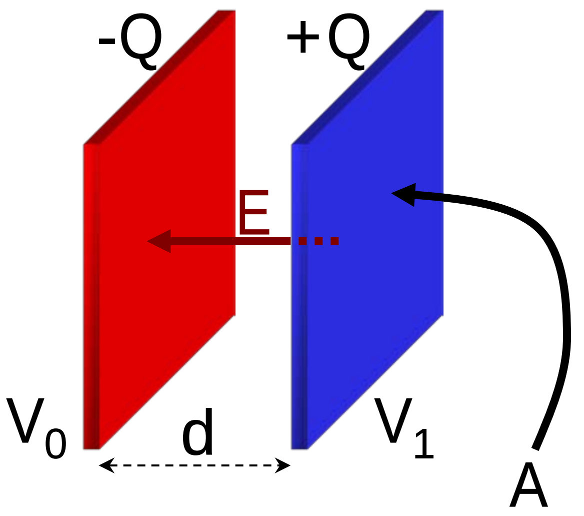
Capacitance of parallel plate capacitor

electric field between two parallel charged plates:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \ .$$

Q is magnitude of charge on either plate.



potential difference:

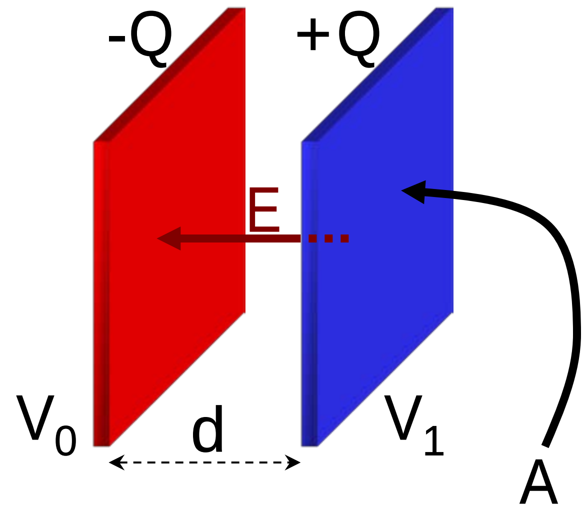
$$\Delta V = V_1 - V_0 = -\int_0^d \vec{E} \cdot d\vec{\ell} = E\int_0^d dx = Ed$$

capacitance:

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{Q}{\left(\frac{Q}{\varepsilon_0 A}\right)d} = \frac{\varepsilon_0 A}{d}$$

Parallel plate capacitance depends "only" on geometry.

$$C = \frac{\varepsilon_0 A}{d}$$



This expression is approximate, and must be modified if the plates are small, or separated by a medium other than a vacuum (lecture 9).

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

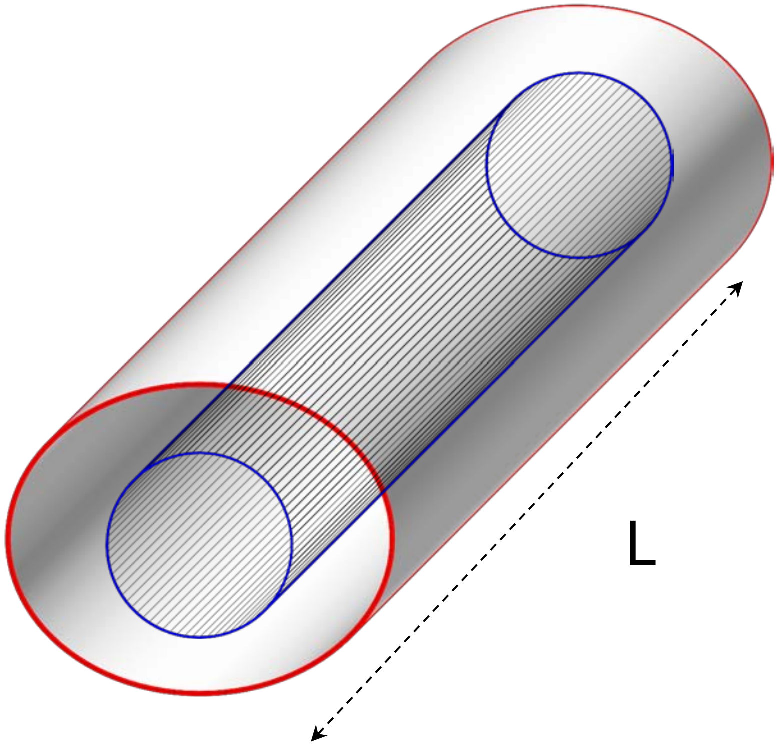
Greek letter Kappa. For today's lecture (and for exam 1), use κ=1.

κ is NOT the same as $k=9\text{x}10^9$!

Capacitance of coaxial cylinder

- capacitors do not have to consist of parallel plates, other geometries are possible

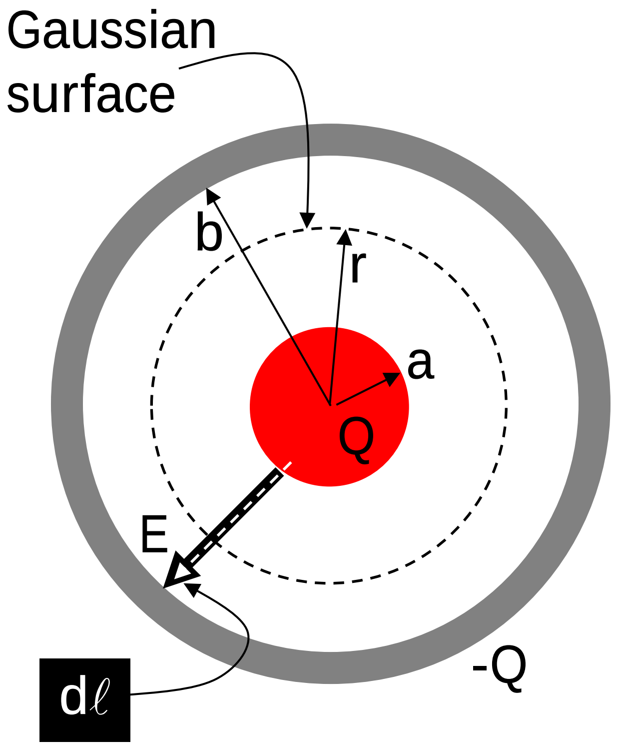
- capacitor made of two coaxial cylinders:



$$\Delta V = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{\ell} = -\int_a^b E_r\, dr$$

from Gauss law: $E = \dfrac{2k\lambda}{r}$
(see lectures 4 and 6)

$$\Delta V = -2k\,\lambda \int_a^b \frac{dr}{r} = -2k\,\lambda \ln\left(\frac{b}{a}\right)$$

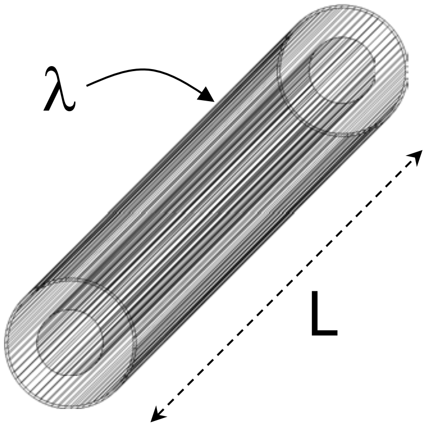


$$C = \frac{Q}{|\Delta V|} = \frac{\lambda L}{|\Delta V|} = \frac{\lambda L}{2k\,\lambda \ln\left(\frac{b}{a}\right)}$$

$$C = \frac{L}{2k \ln\left(\frac{b}{a}\right)} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

capacitance per unit length: $\dfrac{C}{L} = \dfrac{2\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)}$

Example application:



coaxial cables, capacitance per length is a critical part of the specifications.

6ft High-quality Coaxial Audio/Video RCA CL2 Rated Cable - RG6/U 75ohm (for S/PDIF, Digital Coax, Sub

This Digital Coax Cable is made from premium quality RG-6/U with double copper braid shielding to preve digital audio signals and other high-bandwidth content, but it can also be used for composite video and o

The CL2 rating on this cable indicates that the jacket has been treated so that it complies with fire safety ı

Features:

- Gold plated RCA male connectors
- Rubber-covered, molded connector housings
- 97% pure oxygen-free copper conductor
- Double shielded with copper braiding
- 22 pF per foot capacitance
- 75 ohm impedance

Isolated Sphere Capacitance

isolated sphere can be thought of as concentric spheres with the outer sphere at an infinite distance and zero potential.

We already know the potential outside a conducting sphere:

$$V = \frac{Q}{4\pi\varepsilon_0 r} \ .$$

The potential at the surface of a charged sphere of radius R is

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

so the capacitance at the surface of an isolated sphere is

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 R.$$

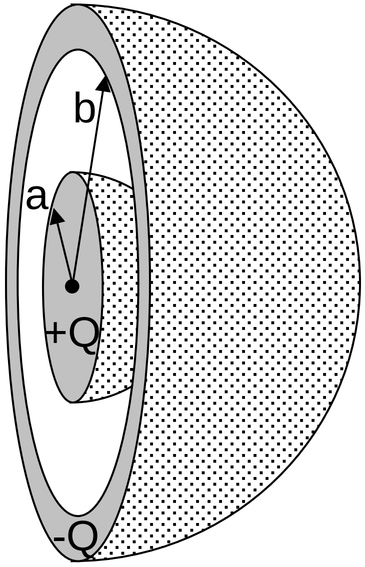
Capacitance of Concentric Spheres

If you have to calculate the capacitance of a concentric spherical capacitor of charge Q...

In between the spheres (Gauss' Law)

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$|\Delta V| = \frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$



$$C = \frac{Q}{|\Delta V|} = \frac{4\pi\varepsilon_0}{\left[\frac{1}{a} - \frac{1}{b}\right]}$$

You need to do this derivation ***if*** you have a problem on spherical capacitors!

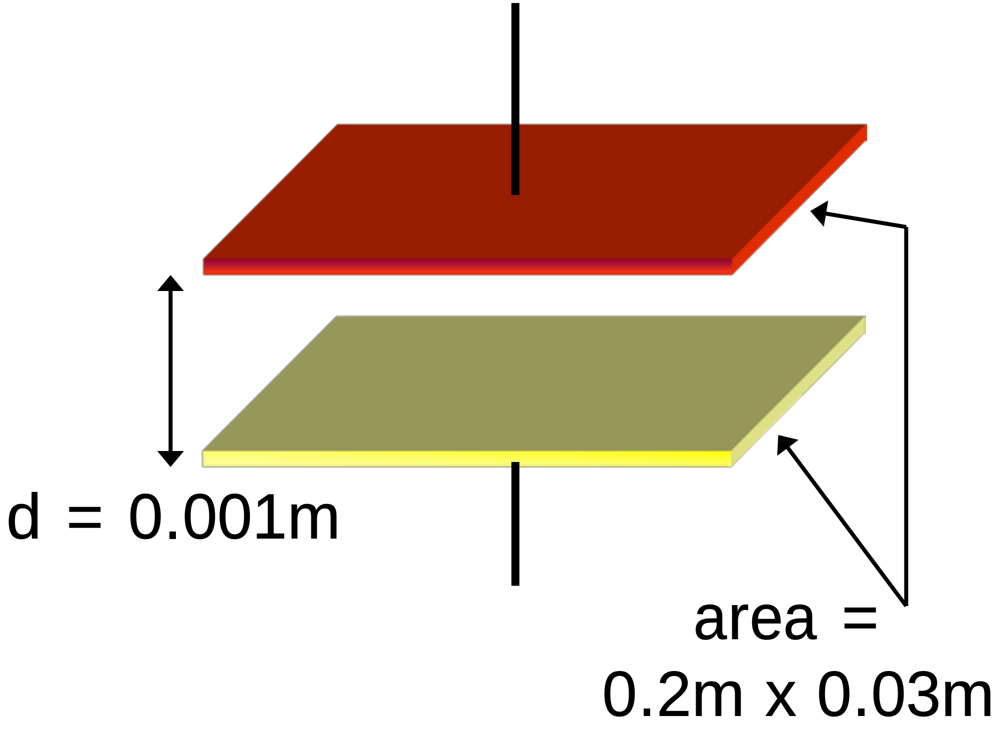
Example: calculate the capacitance of a capacitor whose plates are 20 cm x 3 cm and are separated by a 1.0 mm air gap.

$$C = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\left(8.85\times10^{-12}\right)\left(0.2\times0.03\right)}{0.001}\,\mathrm{F}$$

$$C = 53\times10^{-12}\ \mathrm{F}$$

$$\boxed{C = 53\ \mathrm{pF}}$$



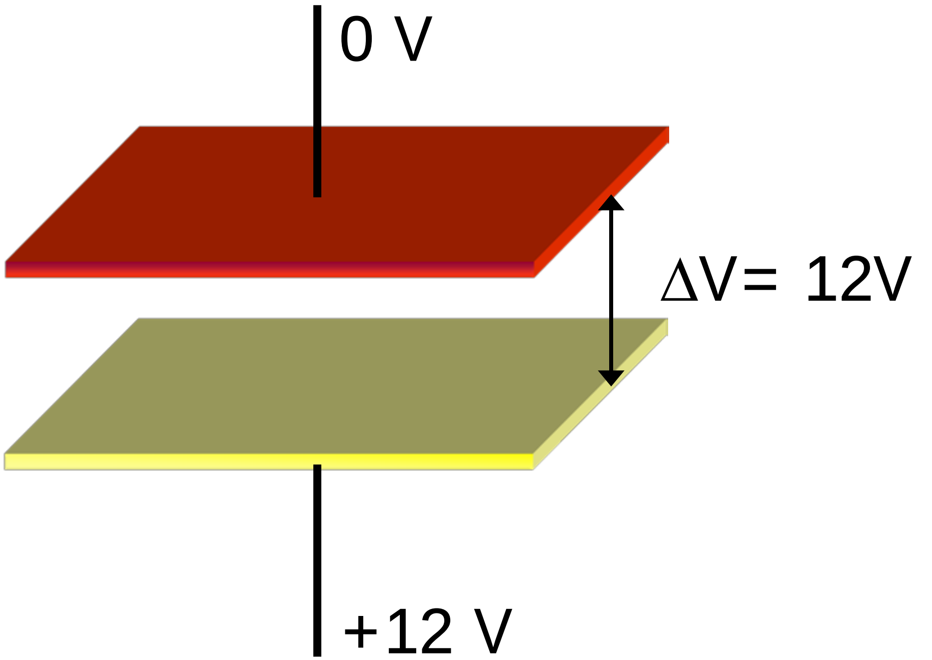
If you keep everything in SI (mks) units, the result is “automatically” in SI units.

Example: what is the charge on each plate if the capacitor is connected to a 12 volt* battery?

$$Q = CV$$

$$Q = \left(53\times10^{-12}\right)\left(12\right)\text{C}$$

$$\boxed{Q = 6.4\times10^{-10}\ \text{C}}$$



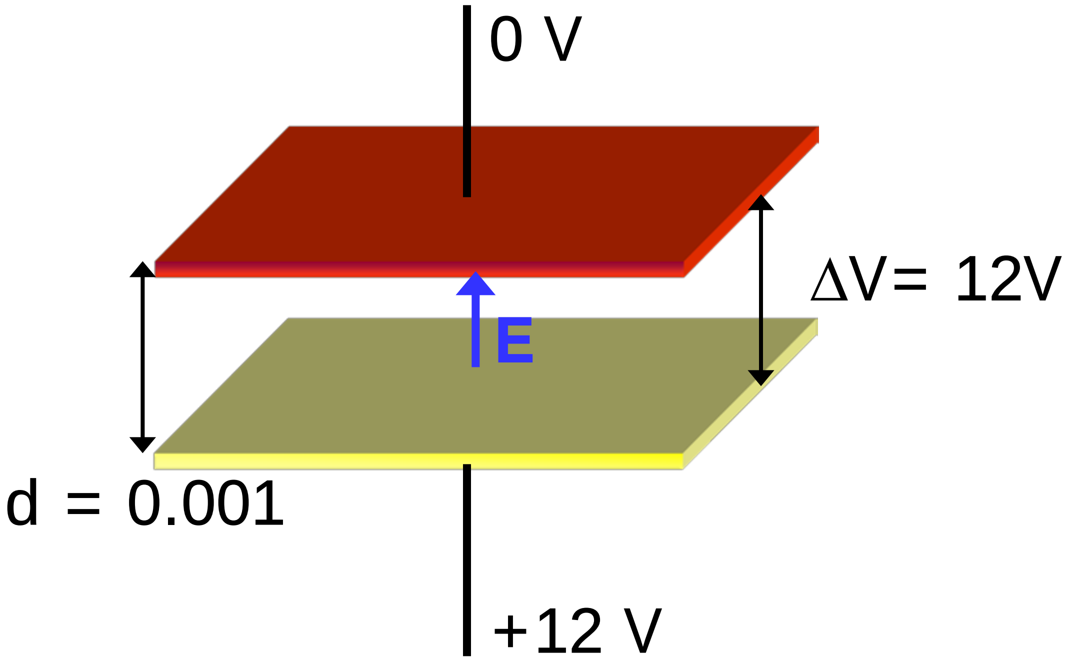
*Remember, it's the potential difference that matters.

Example: what is the electric field between the plates?

$$E = \frac{\Delta V}{d}$$

$$E = \frac{12\text{V}}{0.001\text{ m}}$$

$$\vec{E} = 12000\ \frac{\text{V}}{\text{m}}\ ,\text{"up."}$$



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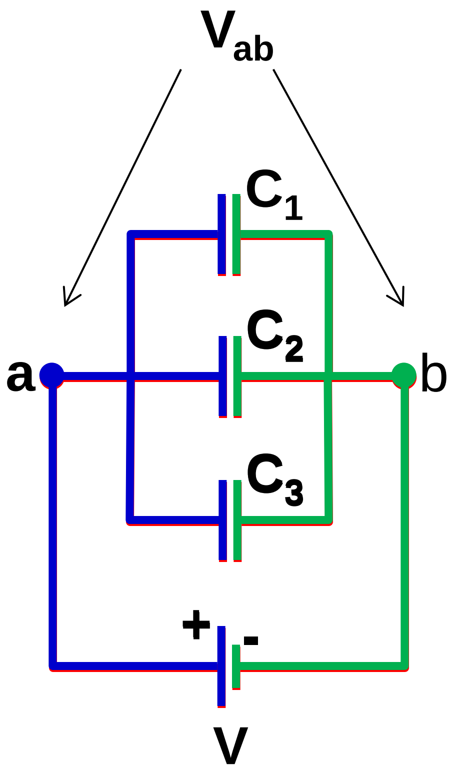
Circuits Containing Capacitors in Parallel

Capacitors connected in parallel:



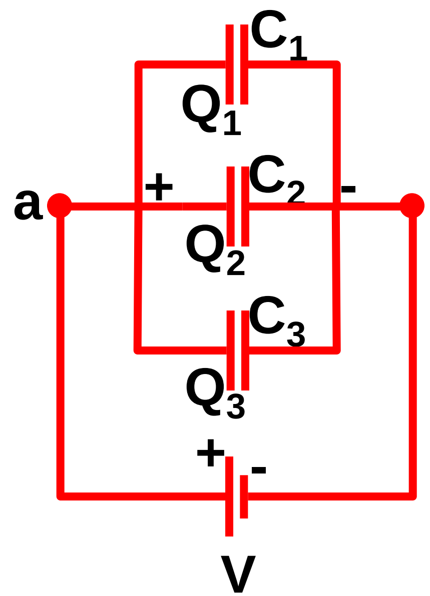
all three capacitors must have the same potential difference (voltage drop) $V_{ab} = V$

General concept: When circuit components are connected in parallel, then the voltage drops across these components are all the same.

$\Rightarrow Q_1 = C_1 V$

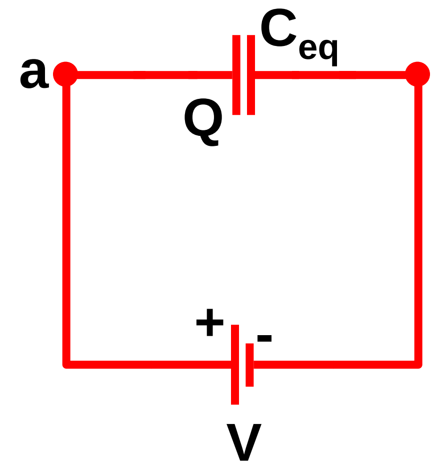
$\& \quad Q_2 = C_2 V$

$\& \quad Q_3 = C_3 V$



Imagine replacing the parallel combination of capacitors by a single **equivalent capacitor**

"equivalent" means "stores the same total charge if the voltage is the same."



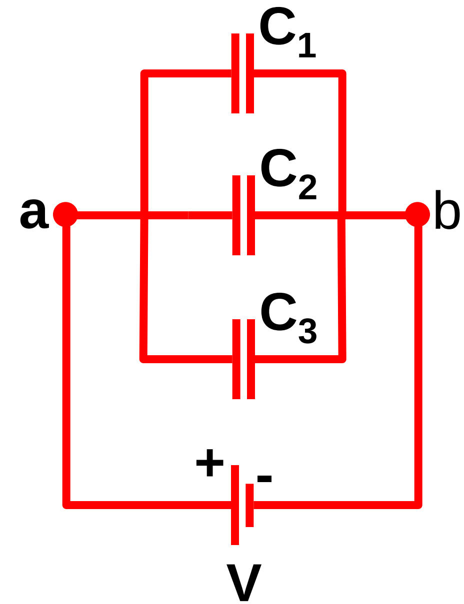
$$Q_{total} = C_{eq} V = Q_1 + Q_2 + Q_3$$

Important!

Summarizing the equations on the last slide:

$$Q_1 = C_1 V \qquad Q_2 = C_2 V \qquad Q_3 = C_3 V$$

$$Q_1 + Q_2 + Q_3 = C_{eq} V$$



Using $Q_1 = C_1V$, etc., gives

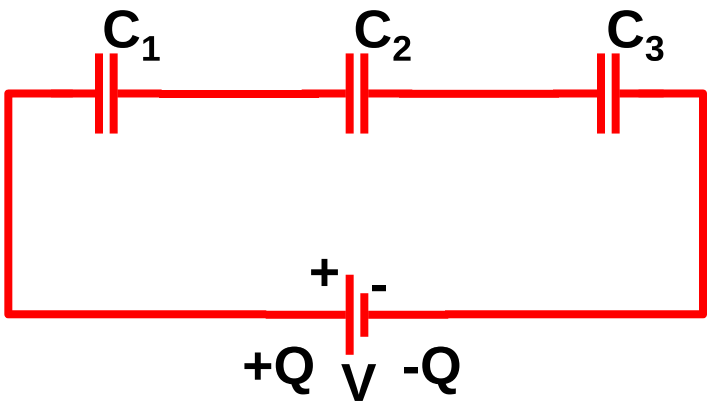
$$C_1V + C_2V + C_3V = C_{eq} V$$

$$C_1 + C_2 + C_3 = C_{eq}$$ (after dividing both sides by V)

Generalizing: $$C_{eq} = \sum_i C_i$$ (capacitances in parallel add up)

Circuits Containing Capacitors in Series

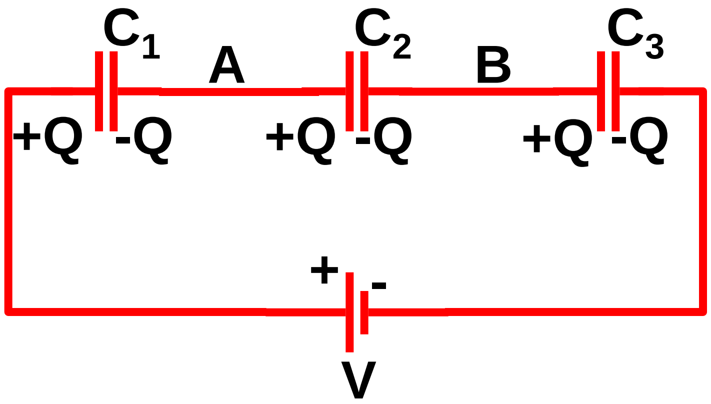
Capacitors connected in series:



charge +Q flows from the battery to the left plate of C_1

charge -Q flows from the battery to the right plate of C_3
(+Q and –Q: the same in magnitude but of opposite sign)

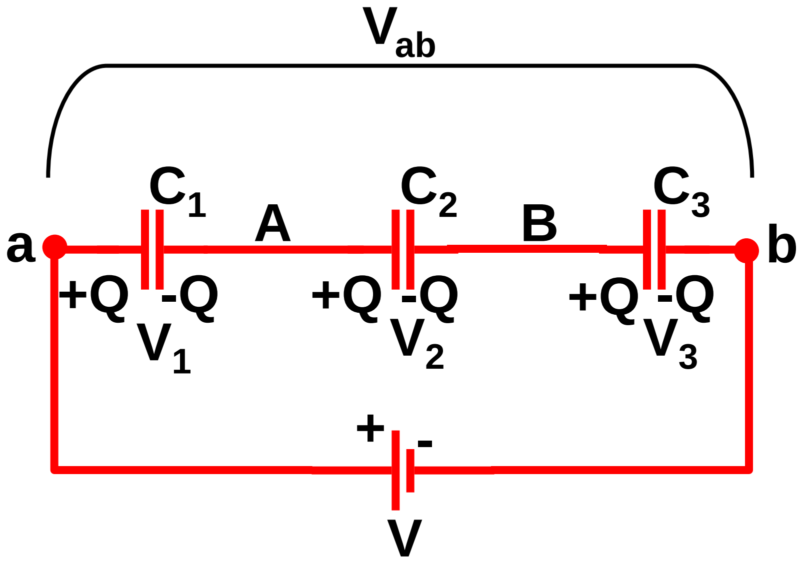
Charges +Q and –Q attract equal and opposite charges to the other plates of their respective capacitors:



These equal and opposite charges came from the originally neutral circuit regions A and B.

Because region A must be neutral, there must be a charge +Q on the left plate of C_2.

Because region B must be neutral, there must be a charge -Q on the right plate of C_2.



The charges on C_1, C_2, and C_3 are the same, and are

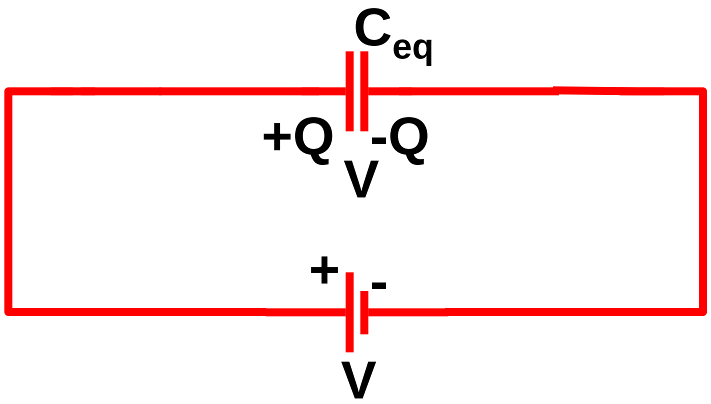
$$Q = C_1 V_1 \qquad Q = C_2 V_2 \qquad Q = C_3 V_3$$

The voltage drops across C_1, C_2, and C_3 add up

$$V_{\mathbf{ab}} = V_1 + V_2 + V_3.$$

General concept: When circuit components are connected in series, then the voltage drops across these components add up to the total voltage drop.

replace the three capacitors by a single **equivalent capacitor**



“equivalent” means it has the same charge Q and the same voltage drop V as the three capacitors

$$Q = C_{eq} V$$

Collecting equations:

$Q = C_1 V_1 \qquad Q = C_2 V_2 \qquad Q = C_3 V_3$

Important!

$V_{\mathbf{ab}} = V = V_1 + V_2 + V_3.$

$Q = C_{eq} V$

Substituting for V_1, V_2, and V_3:

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

Substituting for V:

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

Dividing both sides by Q:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

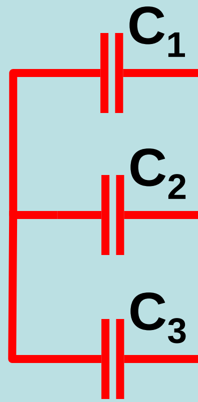
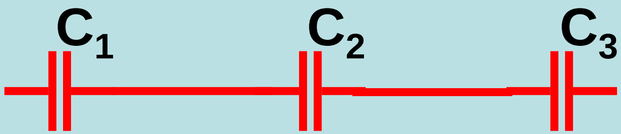
Generalizing:

OSE:

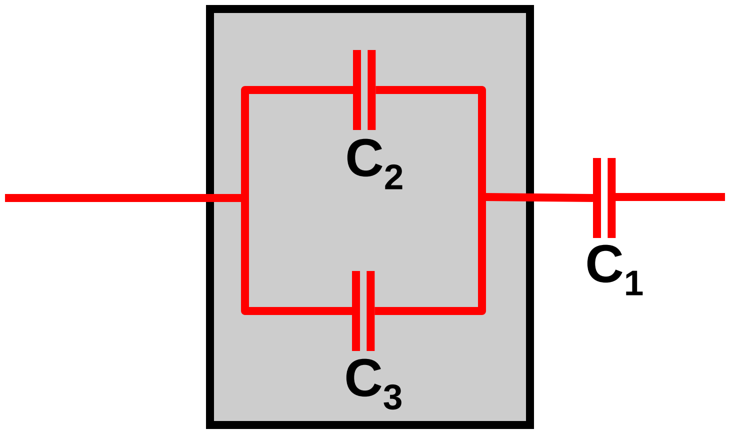
$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$

(capacitors in series)

Summary (know for exam!):

	Parallel 	**Series** 
equivalent capacitance	$C_{eq} = \sum_i C_i$	$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$
charge	Q's add	V's add
voltage	same V	same Q

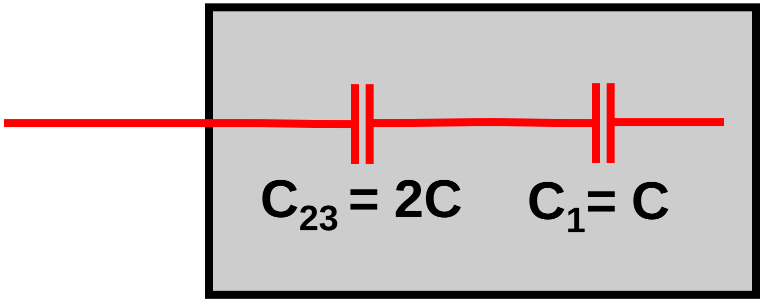
Example: determine the capacitance of a single capacitor that will have the same effect as the combination shown. Use $C_1 = C_2 = C_3 = C$.



Start by combining parallel combination of C_2 and C_3

$$C_{23} = C_2 + C_3 = C + C = 2C$$

Now I see a series combination.

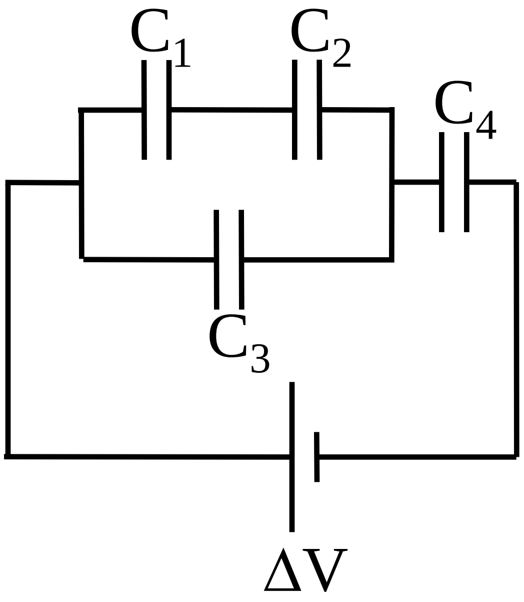


$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{23}}$$

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} = \frac{2}{2C} + \frac{1}{2C} = \frac{3}{2C}$$

$$\boxed{C_{eq} = \frac{2}{3}C}$$

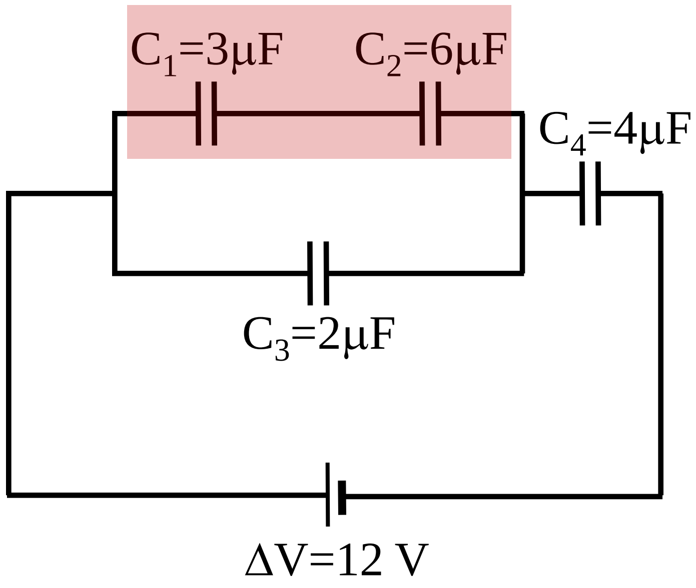
Example: for the capacitor circuit shown, C_1 = 3μF, C_2 = 6μF, C_3 = 2μF, and C_4 =4μF. (a) Find the equivalent capacitance. (b) if ΔV=12 V, find the potential difference across C_4.



I'll work this at the blackboard.

Homework Hint: each capacitor has associated with it a Q, C, and V. If you don't know what to do next, near each capacitor, write down Q= , C= , and V= . Next to the = sign record the known value or a "?" if you don't know the value. As soon as you know any two of Q, C, and V, you can determine the third. This technique often provides visual clues about what to do next.

(a) Find C_{eq}. (b) if ΔV=12 V, find V_4.



C_1 and C_3 are not in parallel. Make sure you understand why!

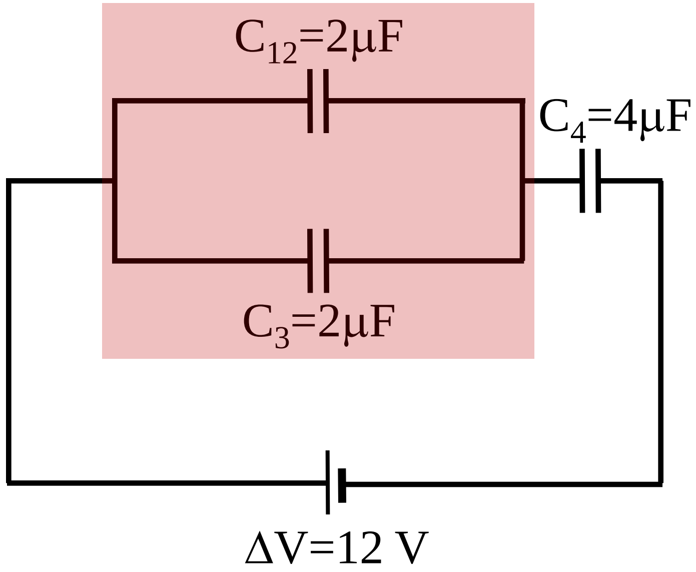
C_2 and C_4 are not in series. Make sure you understand why!

C_1 and C_2 are in series. Make sure you use the correct equation!

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

Don't forget to invert: $C_{12} = 2\ \mu F$.

(a) Find C_{eq}. (b) if ΔV=12 V, find V_4.

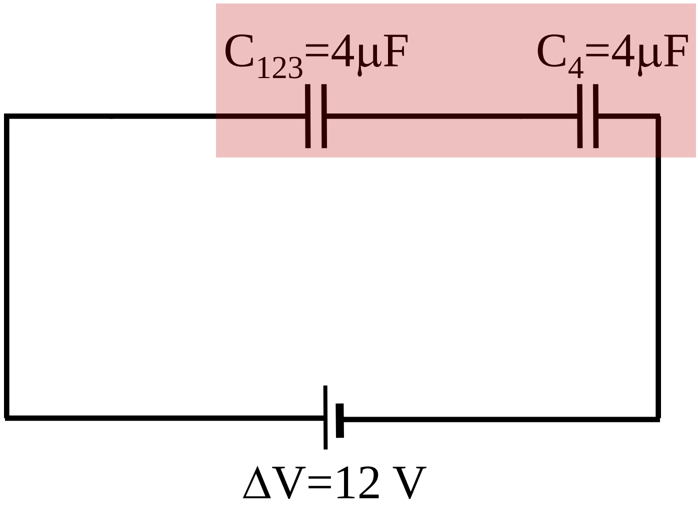


C_{12} and C_4 are not in series. Make sure you understand why!

C_{12} and C_3 are in parallel. Make sure you use the correct equation!

$$C_{123} = C_{12} + C_3 = 2 + 2 = 4\mu F$$

(a) Find C_{eq}. (b) if ΔV=12 V, find V_4.



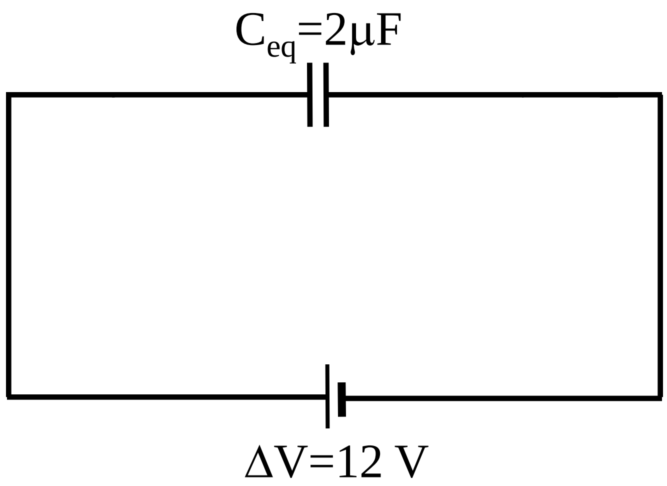
C_{123} and C_4 are in series. Make sure you understand why! Combined, they make give C_{eq}.

Make sure you use the correct equation!

$$\frac{1}{C_{eq}} = \frac{1}{C_{123}} + \frac{1}{C_{24}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

Don't forget to invert: $C_{eq} = 2\ \mu F$.

(a) Find C_{eq}. (b) if ΔV=12 V, find V_4.

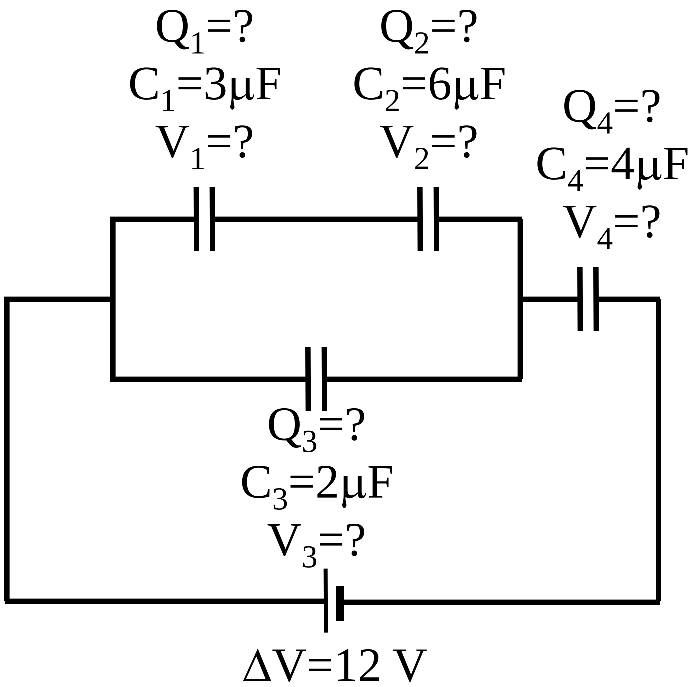


C_{eq} = 2 μF.

If you see a capacitor circuit on the test, read the problem first. Don't go rushing off to calculate C_{eq}. Sometimes you are asked to do other things.

Truth in advertising: there's a high probability you will need to calculate C_{eq} at some point in the problem.

(a) Find C_{eq}. (b) if ΔV=12 V, find V_4.



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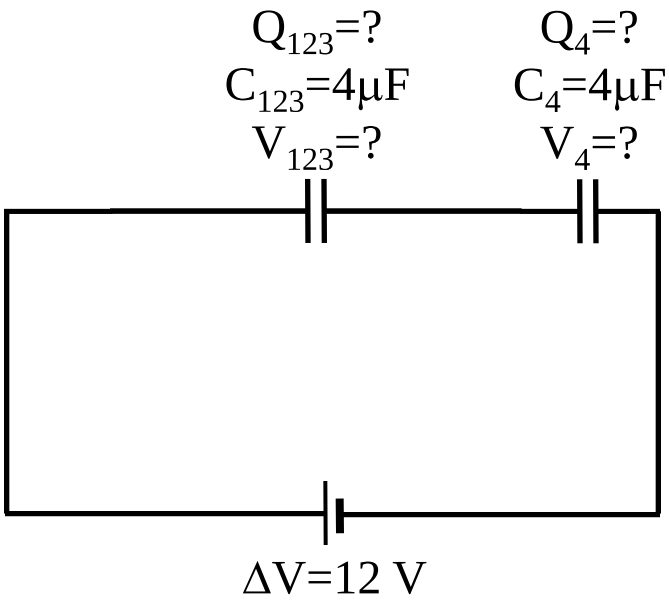
We know C_4 and want to find V_4. If we know Q_4 we can calculate V_4. Maybe that is a good way to proceed.

(a) Find C_{eq}. (b) if $\Delta V=12$ V, find V_4.



C_4 is in series with C_{123} and together they form C_{eq}.

Therefore $Q_4 = Q_{123} = Q_{eq}$.

$$Q_{eq} = C_{eq}\Delta V = (2)(12) = 24\mu C = Q_4$$

$$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C} \Rightarrow V_4 = \frac{Q_4}{C_4} = \frac{24}{4} = 6V$$

You really need to know this:

Capacitors in series...

all have the same charge

add the voltages to get the total voltage

Capacitors in parallel...

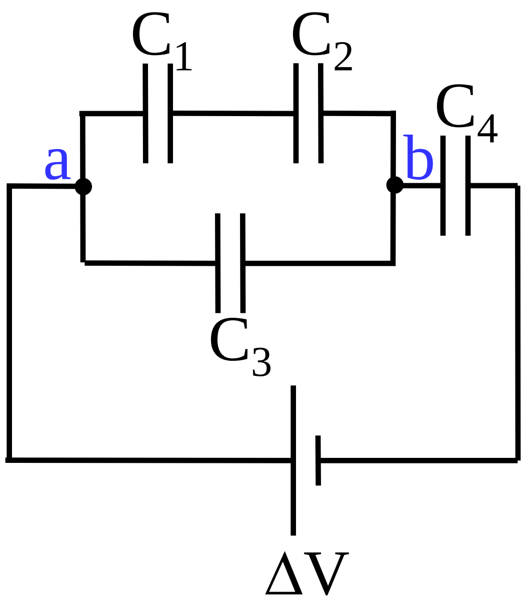
all have the same voltage

add the charges to get the total charge

(and it would be nice if you could explain why)

Homework Hint!

What does our text mean by V_{ab}?



Our text's convention is $V_{ab} = V_a - V_b$. This is explained on page 759. This is in contrast to Physics 1135 notation, where $V_{a\to b} = V_b - V_a$.

In the figure on this slide, if $V_{ab} = 100$ V then point a is at a potential 100 volts higher than point b, and $V_{a\to b} = -100$ V; there is a 100 volt drop on going from a to b.

A "toy" to play with...

http://phet.colorado.edu/en/simulation/capacitor-lab

(You might even learn something.)