Announcements

- final exam average (excluding regrades): 74.6%
- scores ranged from 43 to 200

- regrade requests are **due by Thursday, Feb 23** in recitation

On a separate sheet of paper, explain the reason for your request. This should be based on the work shown on paper, not what was in your head. Attach to the exam and hand it to your recitation instructor by next Thursday.

Today’s agenda:

Electric Current.

You must know the definition of current, and be able to use it in solving problems.

Current Density.

You must understand the difference between current and current density, and be able to use current density in solving problems.

Resistivity and Ohm’s Law

You must know the definition of resistivity and understand Ohm’s Law.

Resistivity vs. Resistance.

You must understand the relationship between resistance and resistivity, and be able to use resistance in solving circuit problems.

Temperature Dependence of Resistivity.

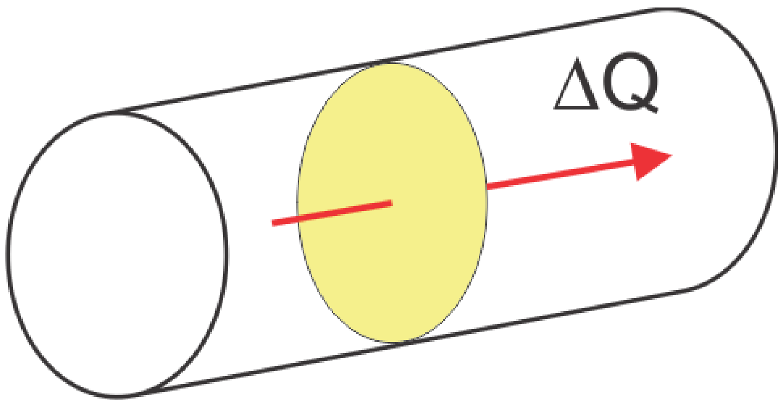
You must be able to use the temperature coefficient of resistivity to solve problems involving changing temperatures.

Definition of Electric Current

average current:

amount of charge ΔQ that passes through area during time Δt

$$I_{av} = \frac{\Delta Q}{\Delta t}$$



instantaneous current:

$$I = \frac{dQ}{dt}$$

unit of current: ampere (A): $A = \frac{C}{s}.$

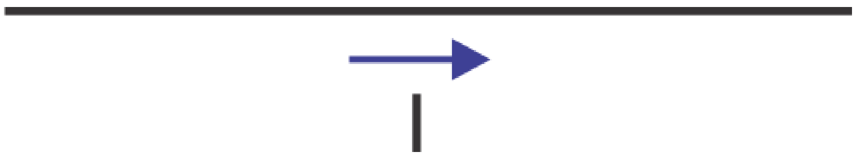
typical currents:

- 100 W light bulb: roughly 1A
- car starter motor: roughly 200A
- TV, computer, phone: nA to mA

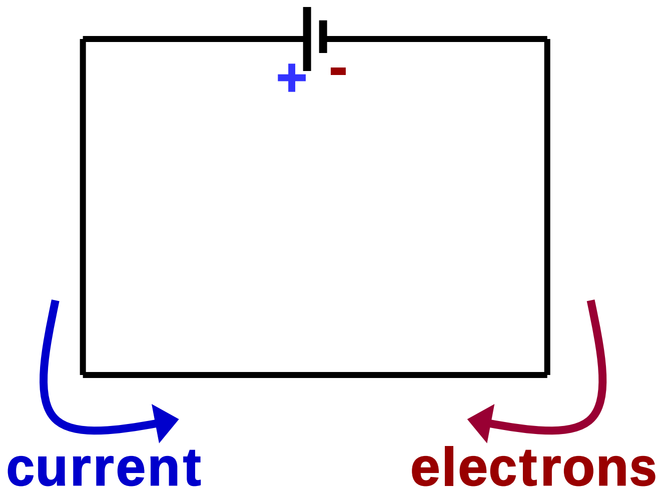
“m” for milli = 10^{-3}

current is a scalar (not a vector)

- has a **sign** associated with it
- **conventional** current is flow of **positive** charge



positive charge flows right or negative charge flows left in most conductors, charge carriers are **negative** electrons



an electron flowing from – to + gives rise to the same "conventional current" as a proton flowing from + to -

If your calculation produces a negative value for the current, that means the conventional current actually flows opposite to the direction indicated by the arrow.

Example: 3.8×10^{21} electrons pass through an area in a wire in 4 minutes. What was the average current?

$$I_{av} = \frac{\Delta Q}{\Delta t} = \frac{Ne}{\Delta t}$$

$$I_{av} = \frac{(3.8 \times 10^{21})(1.6 \times 10^{-19})}{(4 \times 60)} A$$

$$I_{av} = 2.53A$$

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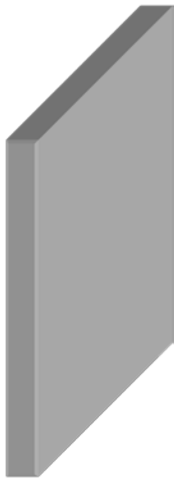
Current Density

current density J is **current per area** or, equivalently,

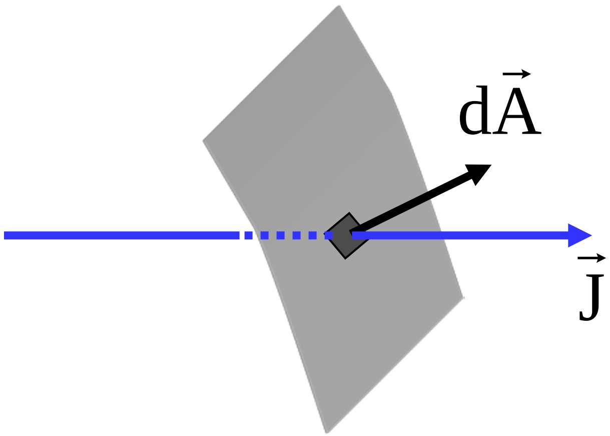
charge per area and time

unit of J: A/m^2



directions are important ...

- current density is a **vector**
  (direction is direction of velocity of positive charge carriers)

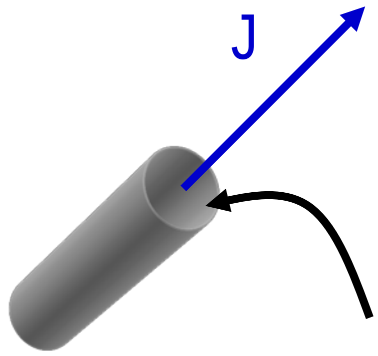


- current density $\vec{J}$ flowing through infinitesimal area $d\vec{A}$ produces infinitesimal current $dI = \vec{J} \cdot d\vec{A}$

- total current passing through A is

$$I = \int_{\text{surface}} \vec{J} \cdot d\vec{A}$$

No OSE's on this page.
Simpler, less-general
OSE on next page.

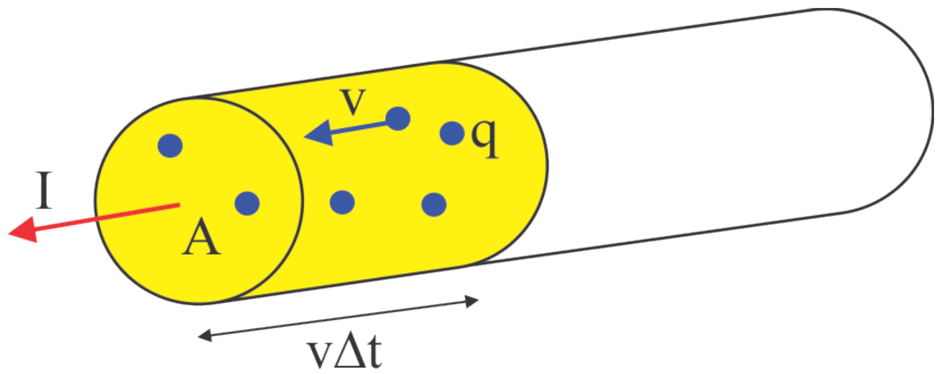


if $\vec{J}$ is uniform and parallel to $d\vec{A}$:

$$I = \int_{\text{surface}} \vec{J} \cdot d\vec{A} = J \int_{\text{surface}} dA = JA \quad \Rightarrow \boxed{J = \frac{I}{A}}$$

Microscopic view of electric current

- carrier density n (number of charge carriers per volume)
- carriers move with speed v



number of charges that pass through surface A in time Δt:

$$\frac{\text{number}}{\text{volume}} \times \text{volume} = n \ \ v\Delta t \ A$$

amount of charge passing through A in time Δt: $\Delta Q = q \ \ nv\Delta t \ \ A$

divide by Δt to get the current... $I = \frac{\Delta Q}{\Delta t} = nqv \ A$

...and by A to get the current density: $J = nqv$.

To account for the vector nature of the current density,

$$\vec{J} = nq\vec{v}$$

Not quite "official" yet.

and if the charge carriers are electrons, q=-e so that

$$\vec{J}_e = -n\,e\,\vec{v}.$$

Not quite "official" yet.

The – sign demonstrates that the velocity of the electrons is antiparallel to the conventional current direction.

Currents in Materials

Metals are conductors because they have “free” electrons, which are not bound to metal atoms.

In a cubic meter of a typical conductor there roughly 10^{28} free electrons, moving with typical speeds of 1,000,000 m/s...

...but the electrons move in random directions, and there is no net flow of charge, until you apply an electric field.

The velocity that should be used in the equation for current density

$$\vec{J} = n\, q\, \vec{v}.$$

is **not** the charge carrier's **instantaneous velocity**

Instead, use the **net or drift** velocity $\vec{v}_d$ (left over after the random motions is averaged out)

$$\boxed{\vec{J} = n\, q\, \vec{v}_d.}$$

if $\vec{J}$ is parallel to $\vec{A}$:

$$I = nqv_d A \qquad\qquad v_d = \frac{I}{nqA}$$

Example: the 12-gauge copper wire in a home has a cross-sectional area of $3.31\text{x}10^{-6}$ m^2 and carries a current of 10 A. The conduction electron density in copper is $8.49\text{x}10^{28}$ electrons/m^3. Calculate the drift speed of the electrons.

$$v_d = \frac{I}{nqA}$$

$$\left|v_d\right| = \frac{I}{neA}$$

$$\left|v_d\right| = \frac{10\,\text{C/s}}{(8.49\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})(3.31\times10^{-6}\ \text{m}^2)}$$

$$\left|v_d\right| = 2.22\times10^{-4}\ \text{m/s}$$

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Resistivity

Why does electric current flow?

- electric field creates force acting on charge carriers
- in many materials:
  current density (approximately) **proportional** to electric field

$$\vec{J} = \sigma \vec{E} = \frac{1}{\rho} \vec{E}$$

Ohm's law
(misnamed, not a law of nature)

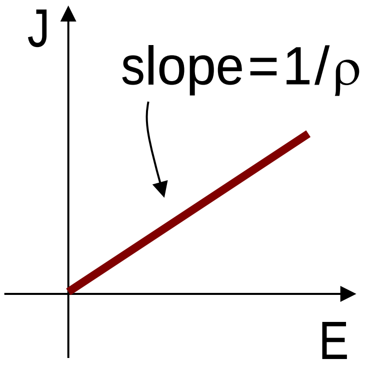
- σ is electrical conductivity
- ρ is **electrical resistivity**
- σ and ρ are material properties
- unit of ρ: $\frac{V/m}{A/m^2} = \frac{V}{A} m = \Omega m$

Ohm

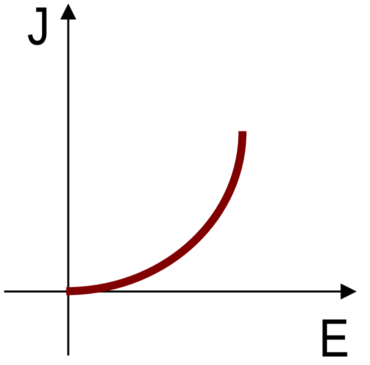
Caution!
ρ is not volume density!
σ is not surface density!

Ohmic vs non-Ohmic materials

- materials that follow Ohm's Law are called **"ohmic"** materials
- resistivity ρ is constant
- linear J vs. E graph



- materials that do not follow Ohm's Law are called **"non-Ohmic"** materials
- nonlinear J vs. E graph



Resistivity

- resistivities vary enormously
- roughly 10^{-8} $\Omega\cdot$m for copper
- roughly 10^{15} $\Omega\cdot$m for hard rubber
- incredible range of 23 orders of magnitude

Example: the 12-gauge copper wire in a home has a cross-sectional area of 3.31×10^{-6} m^2 and carries a current of 10 A. Calculate the magnitude of the electric field in the wire.

ρ of copper

$$E = \rho J = \rho \frac{I}{A}$$

$$E = \frac{(1.72\times10^{-8}\ \Omega\cdot m)(10\,A)}{(3.31\times10^{-6}\ m^2)}$$

$$E = 5.20\times10^{-2}\ V/m$$

Homework hint you can look up the resistivity of a material in a table in your text.

Homework hint (not needed in this particular example): in this chapter it is safe to use ΔV=Ed.

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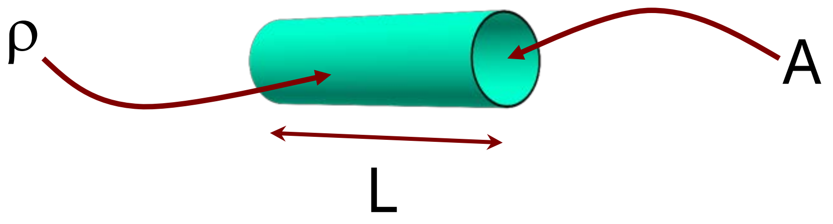
Resistance

current in a wire:

- length L, cross section A
- material of resistivity ρ



start from $E = \rho J$

$$V = EL = \rho JL = \rho \frac{I}{A} L = IR$$

$$\boxed{R = \frac{\rho L}{A}}$$

resistance of the wire,

unit $\frac{V}{A} = \Omega$ (Ohm)

Ohm' law (device version)

$$\boxed{V = IR}$$

Resistance

- resistance of wire (or other device) measures how easily charge flows through it

$$R = \frac{\rho L}{A}$$

- the longer a wire, the harder it is to push electrons through it
- the greater the cross-sectional area, the “easier” it is to push electrons through it
- the greater the resistivity, the “harder” it is for the electrons to move in the material

Distinguish:

Resistivity = material’s property

Resistance = device property

Example (will not be worked in class): Suppose you want to connect your stereo to remote speakers.
(a) If each wire must be 20 m long, what diameter copper wire should you use to make the resistance 0.10 Ω per wire.

$$R = \rho L / A$$

$$A = \rho L / R$$

$$A = \pi (d/2)^2$$

geometry!

$$\pi (d/2)^2 = \rho L / R$$

$$(d/2)^2 = \rho L / \pi R$$

$$d/2 = (\rho L / \pi R)^{1/2}$$

don't skip steps!

$$\boxed{d = 2 (\rho L / \pi R)^{1/2}}$$

$$d = 2 \left[(1.68 \times 10^{-8}) (20) / \pi (0.1) \right]^{1/2} \text{ m}$$

$$\boxed{d = 0.0021 \text{ m} = 2.1 \text{ mm}}$$

(b) If the current to each speaker is 4.0 A, what is the voltage drop across each wire?

$$V = I R$$

$$V = (4.0) (0.10) \text{ V}$$

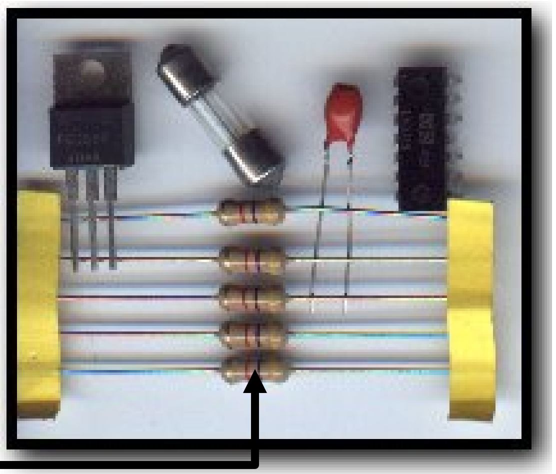
$$V = 0.4 \text{ V}$$

Resistors in circuits

- symbol we use for a “resistor:”

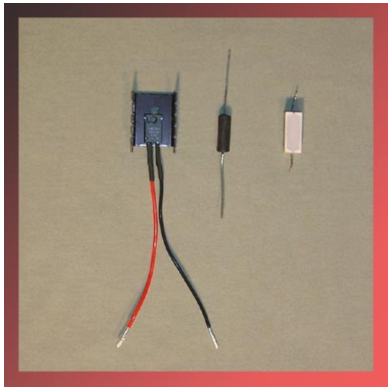
- in principle, **every circuit component** has some resistance
- all wires have resistance
- for efficiency, we want wires to have low resistance
- in **idealized** problems, consider wire resistance to be **zero**
- lamps, batteries, and other devices in circuits also have resistance

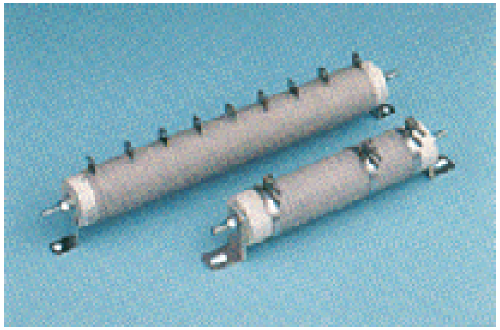
Resistors are often intentionally used in circuits. The picture shows a strip of five resistors (you tear off the paper and solder the resistors into circuits).



The little bands of color on the resistors have meaning. Here are a couple of handy web links:

1. http://www.dannyg.com/examples/res2/resistor.htm
2. http://www.digikey.com/en/resources/conversion-calculators/conversion-calculator-resistor-color-code-4-band





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Temperature Dependence of Resistivity

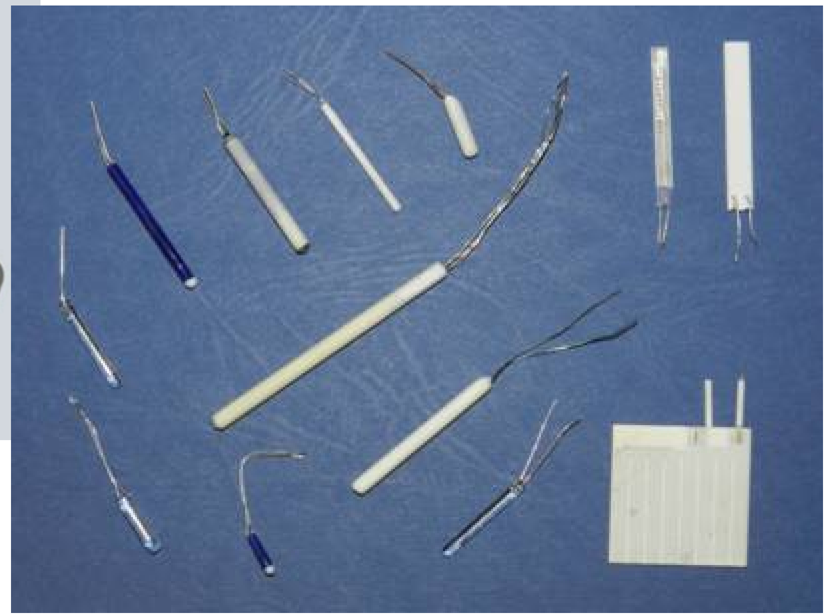
Many materials have resistivities that depend on temperature. We can **model*** this temperature dependence by an equation of the form

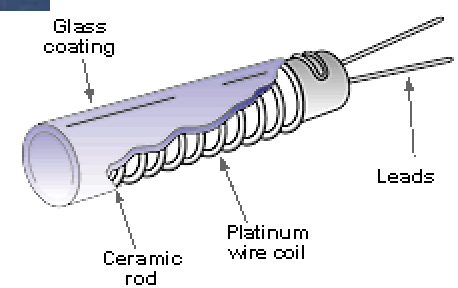
$$\rho = \rho_0 \left[1 + \alpha \left(T - T_0 \right) \right],$$

where ρ_0 is the resistivity at temperature T_0, and α is the temperature coefficient of resistivity.

***T_0 is a reference temperature, often taken to be 0 °C or 20 °C. This approximation can be used if the temperature range is "not too great;" i.e. 100 °C or so.**

Resistance thermometers made of carbon (inexpensive) and platinum (expensive) are widely used to measure very low temperatures.





Example: a carbon resistance thermometer in the shape of a cylinder 1 cm long and 4 mm in diameter is attached to a sample. The thermometer has a resistance of 0.030 Ω. What is the temperature of the sample?

This is the starting equation:

$$\rho = \rho_0 \left[1 + \alpha \left(T - T_0 \right) \right]$$

We can look up the resistivity of carbon at 20 °C.

We use the thermometer dimensions to calculate the resistivity when the resistance is 0.03 Ω, and use the above equation directly.

Or we can rewrite the equation in terms or R. Let’s first do the calculation using resistivity.

Example: a carbon resistance thermometer in the shape of a cylinder 1 cm long and 4 mm in diameter is attached to a sample. The thermometer has a resistance of 0.030 Ω. What is the temperature of the sample?

The resistivity of carbon at 20 °C is

$$\rho_0 = 3.519 \times 10^{-5}\ \Omega \cdot \mathrm{m}$$

$$R = \frac{\rho L}{A}$$

$$\rho(R) = \frac{RA}{L}$$

$$\rho(R = 0.03) = \frac{(0.03)\left(\pi \cdot 0.002^2\right)}{(0.01)} = 3.7699 \times 10^{-5}\ \Omega \cdot \mathrm{m}$$

Example: a carbon resistance thermometer in the shape of a cylinder 1 cm long and 4 mm in diameter is attached to a sample. The thermometer has a resistance of 0.030 Ω. What is the temperature of the sample?

$$\rho = \rho_0\left[1+\alpha\left(T-T_0\right)\right] \qquad \alpha = -0.0005\ °C^{-1}$$

$$\alpha\left(T-T_0\right) = \frac{\rho}{\rho_0} - 1$$

$$T = T_0 + \frac{1}{\alpha}\left(\frac{\rho}{\rho_0} - 1\right)$$

$$T = 20 + \frac{1}{-0.0005}\left(\frac{3.7699\times10^{-5}}{3.519\times10^{-5}} - 1\right) = -122.6\ °C$$

Example: a carbon resistance thermometer in the shape of a cylinder 1 cm long and 4 mm in diameter is attached to a sample. The thermometer has a resistance of 0.030 Ω. What is the temperature of the sample?

Alternatively, we can use the resistivity of carbon at 20 °C to calculate the resistance at 20 °C.

$$T_0 = 20°C \qquad \rho_0 = 3.519\times10^{-5}\ \Omega\cdot m \qquad L = 0.01\ m \qquad r = 0.002\ m$$

$$R_0 = \frac{\rho_0 L}{\pi r^2} = 0.02800\ \Omega$$

This is the resistance at 20 °C.

Example: a carbon resistance thermometer in the shape of a cylinder 1 cm long and 4 mm in diameter is attached to a sample. The thermometer has a resistance of 0.030 Ω. What is the temperature of the sample?

$$\rho = \rho_0 \left[1 + \alpha \left(T - T_0\right)\right]$$

$$\frac{RA}{L} = \frac{R_0 A_0}{L_0} \left[1 + \alpha \left(T - T_0\right)\right]$$

If we assume $A/L = A_0/L_0$, then

$$R = R_0 \left[1 + \alpha \left(T - T_0\right)\right]$$

Example: a carbon resistance thermometer in the shape of a cylinder 1 cm long and 4 mm in diameter is attached to a sample. The thermometer has a resistance of 0.030 Ω. What is the temperature of the sample?

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right]$$

$$\alpha\left(T-T_0\right) = \frac{R}{R_0} - 1$$

$$T = T_0 + \frac{1}{\alpha}\left(\frac{R}{R_0} - 1\right)$$

$$T = 20 + \frac{1}{-0.0005}\left(\frac{.030}{.028} - 1\right) = -122.9\ ^\circ C$$

The result is very sensitive to significant figures in resistivity and α.