Today's agenda:

Potential Changes Around a Circuit.
You must be able to calculate potential changes around a closed loop.

Electromotive force (EMF), Terminal Voltage, and Internal Resistance.
You must be able to incorporate all of the above quantities in your circuit calculations.
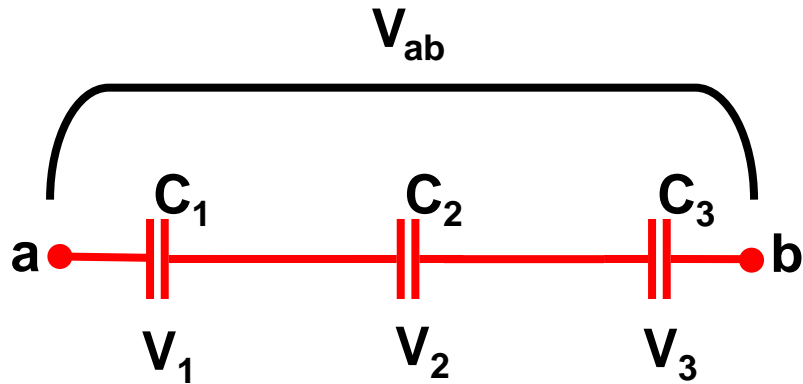
Electric Power.
You must be able to calculate the electric power dissipated in circuit components.

Examples.

Recall (lecture 7):
voltage drops across circuit components connected in series add

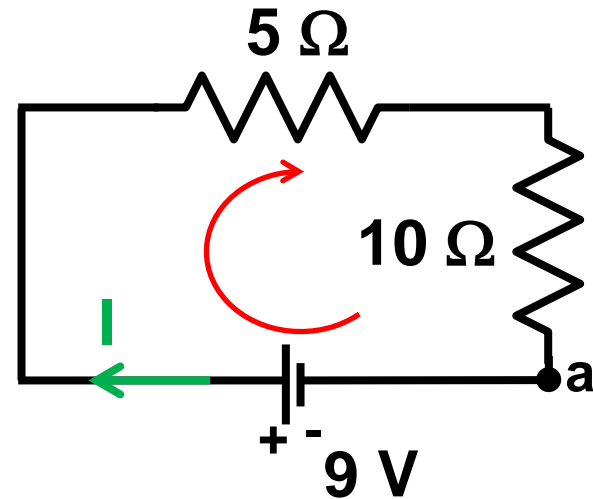$$V_{ab} = V_1 + V_2 + V_3$$

Generalization: Loop rule

- **around any closed loop:** $\sum_i V_i = 0$

  strictly: $\sum_i \Delta V_i = 0$

- reflects energy conservation (around any closed path, $U_f = U_i$)

Recipe for Kirchhoff's loop rule

1. Assign (guess) direction of current I
   positive I: current flows in this direction
   negative I: current flows in opposite direction

2. Pick travel direction
   (clockwise or counterclockwise)

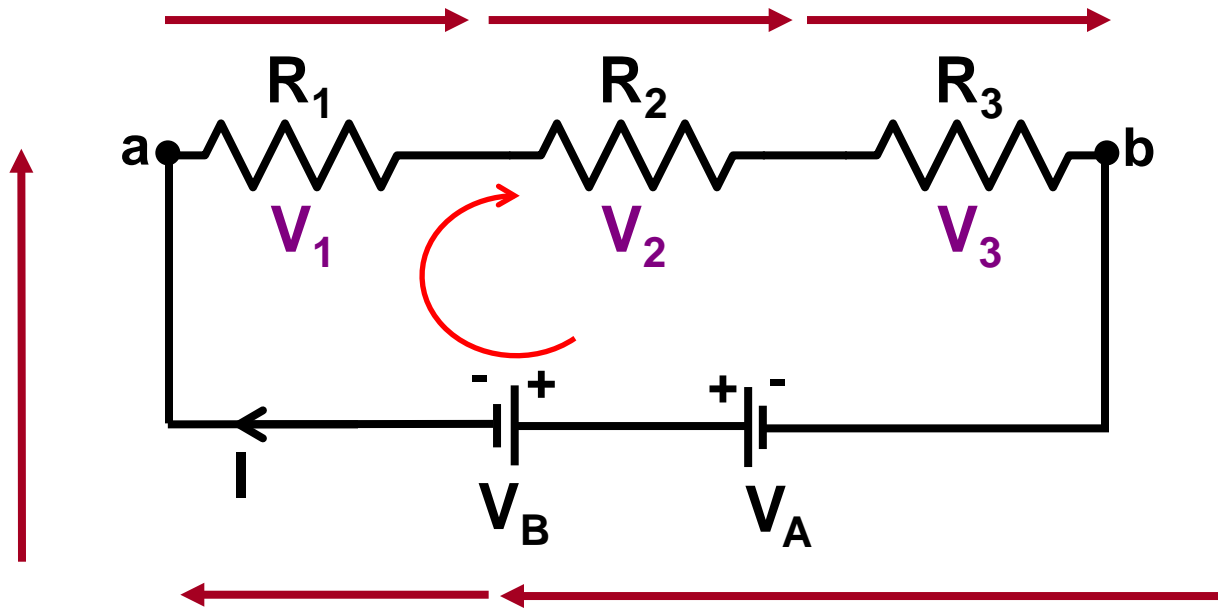3. Add up voltage differences V_i, set sum to zero
   sign rules:
   battery voltage counts positive if you travel from – to +
   resistor voltage counts negative of you travel with current

   start at point a: $+9V - I\,5\Omega - I\,10\Omega = 0$

   $I = 9V / 15\Omega = 0.6A$

5 Ω

10 Ω

•a

+ -
9 V

Ohm's law V=I R

Example: Loop rule in circuit of 3 resistors and 2 batteries

1. Assign current direction
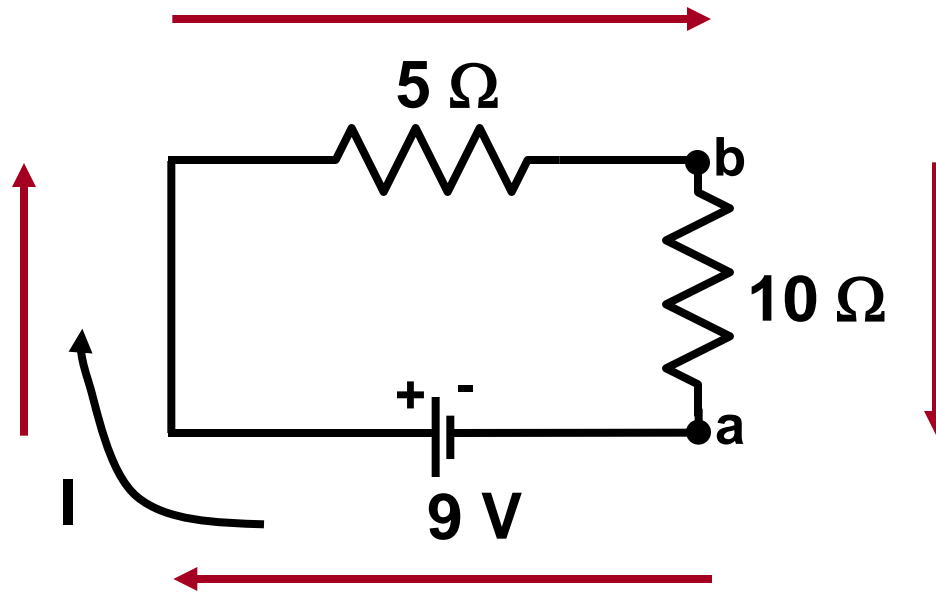
2. Pick travel direction: clockwise

3. Add voltages

$$- V_1 - V_2 - V_3 + V_A - V_B = 0$$

$$- IR_1 - IR_2 - IR_3 + V_A - V_B = 0$$

Recommendation: choose your path around the circuit in the same direction as your guessed current.

Example: calculate I, V_{ab}, and V_{ba} for the circuit shown.
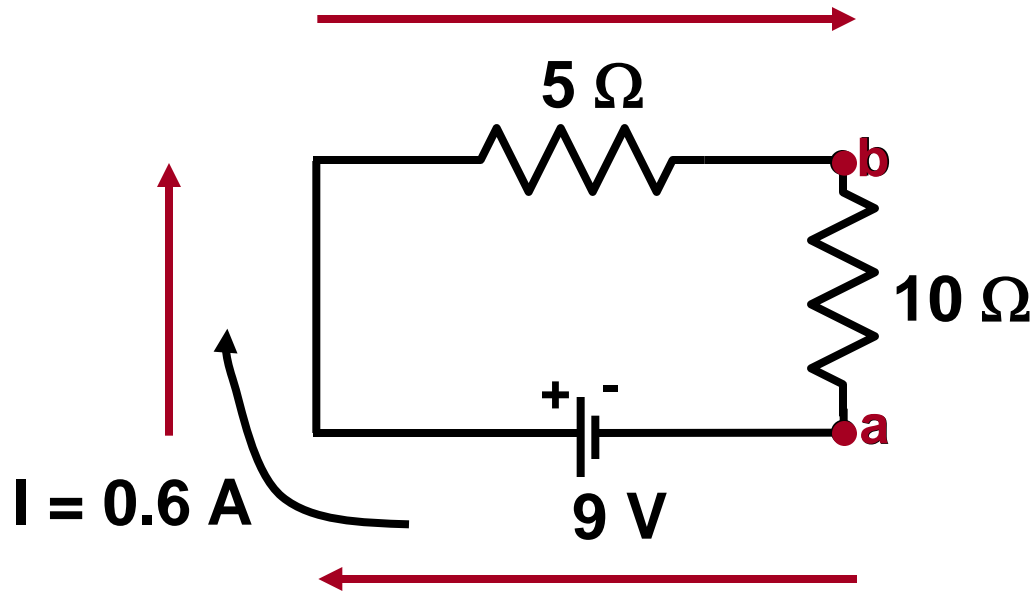
To be worked at the blackboard in lecture.

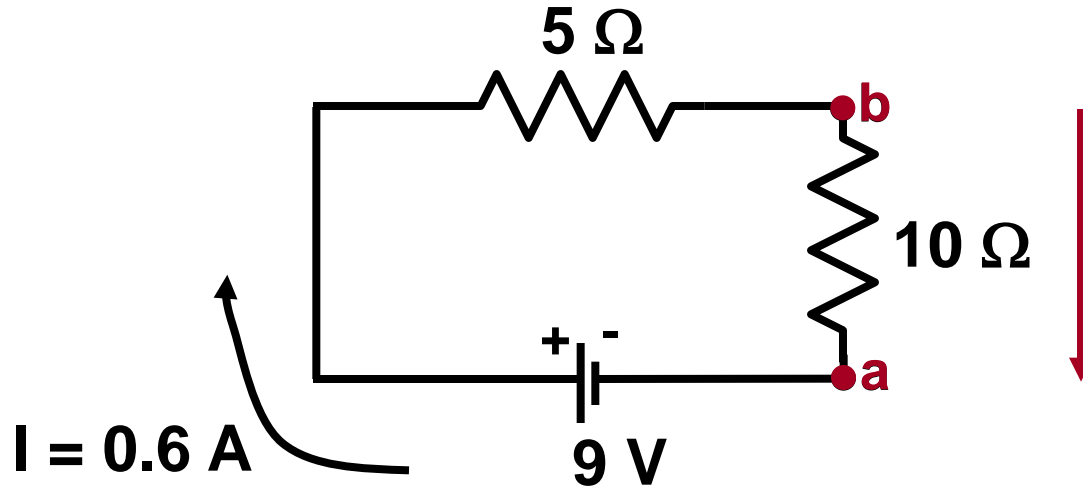$$+9 - 5\,I - 10\,I = 0$$

$$15\,I = +9$$

$$I = +9/15 = 0.6\ A$$

Example: calculate I, V_{ab}, and V_{ba} for the circuit shown.

5 Ω

b

10 Ω

+ -

a

I = 0.6 A

9 V

$$V_a + 9 - 5\,(0.6) = V_b$$

$$V_{ab} = V_a - V_b = -9 + 5\,(0.6) = -6\ V$$

5 Ω

b

10 Ω

+ -

a

I = 0.6 A

9 V

$$V_b - 10\,(0.6) = V_a$$

$$V_{ba} = V_b - V_a = +\,10\,(0.6) = +6\ V$$

5 Ω

b

10 Ω

+ -

a

I = 0.6 A

9 V
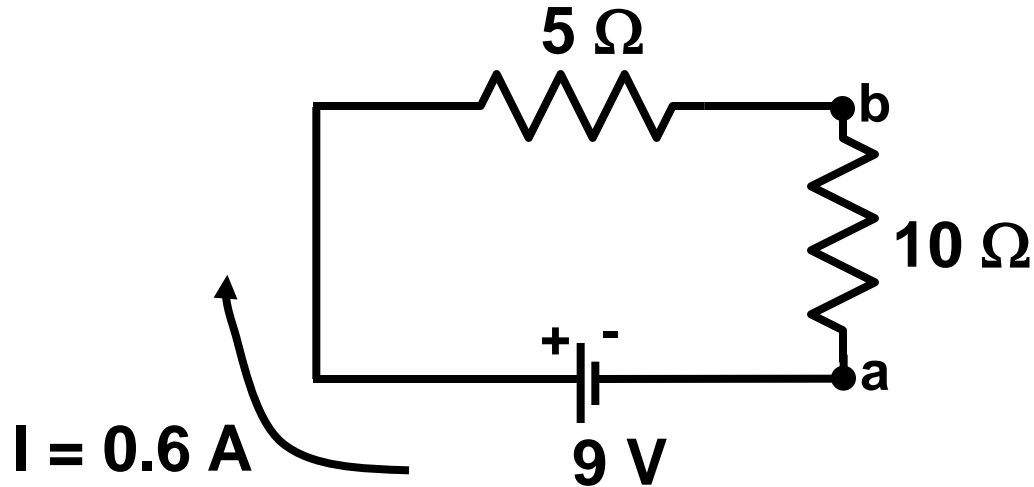
Note: V_{ba} = +6 V = - V_{ab} , as expected

Graph the potential rises and drops in this circuit.
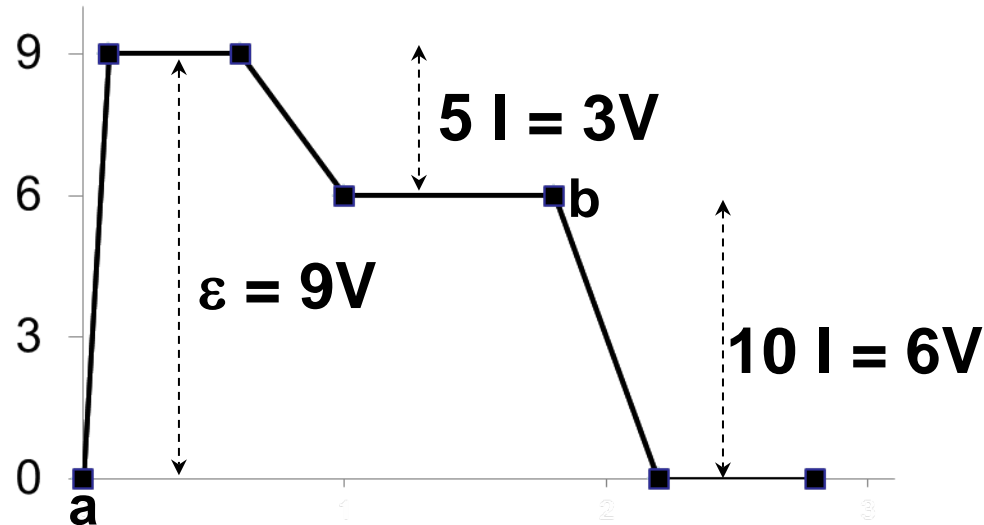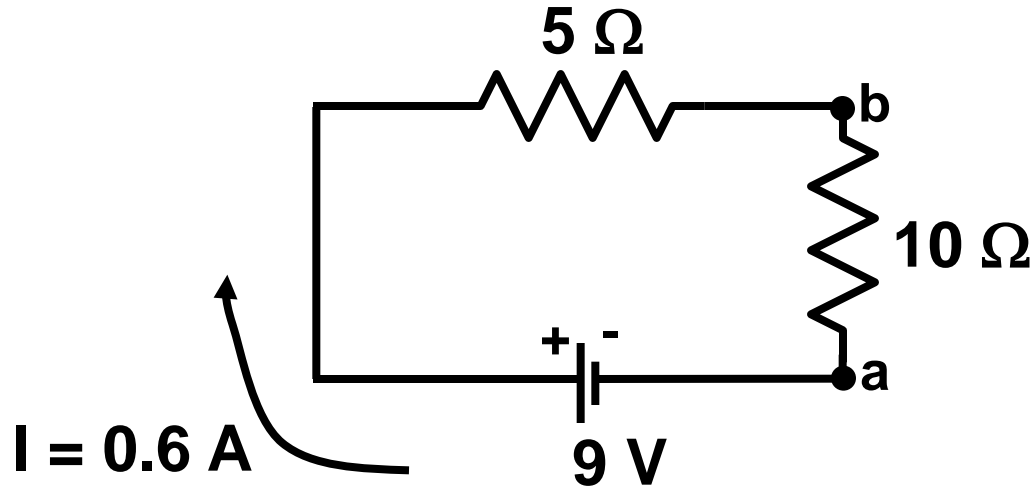
5 Ω

b

10 Ω

+ -

a

I = 0.6 A

9 V

9

6

3

0

a

5 I = 3V

b

ε = 9V

10 I = 6V

Example: calculate **I**, V_{ab}, and V_{ba} for the circuit shown.

To be worked at the blackboard in lecture.

$$+ 9 - 6 - 5\,I - 10\,I = 0$$

$$15\,I = 3$$

$$I = 0.2\ A$$

5 Ω

b

10 Ω

- +   + -

a

I = 0.2 A   6 V   9 V
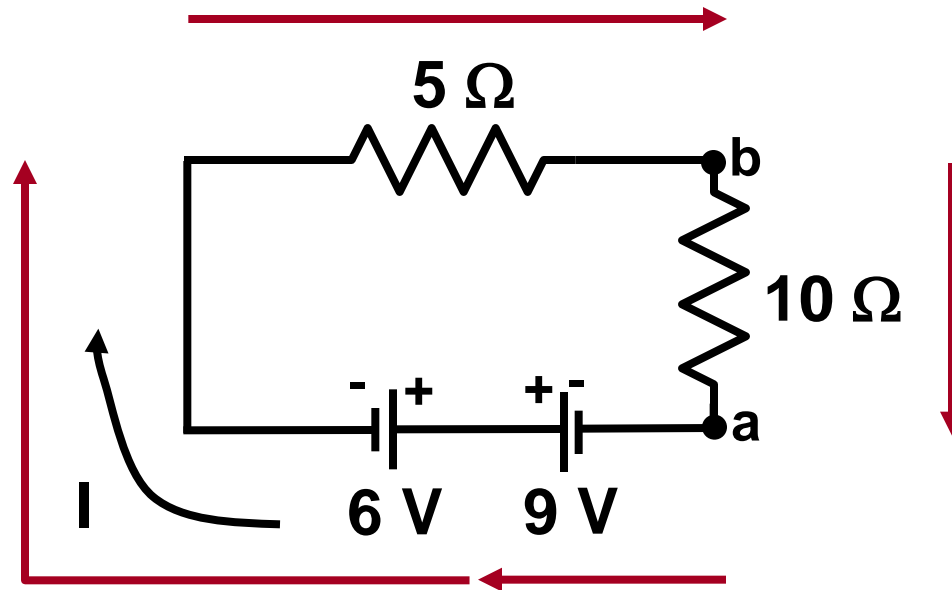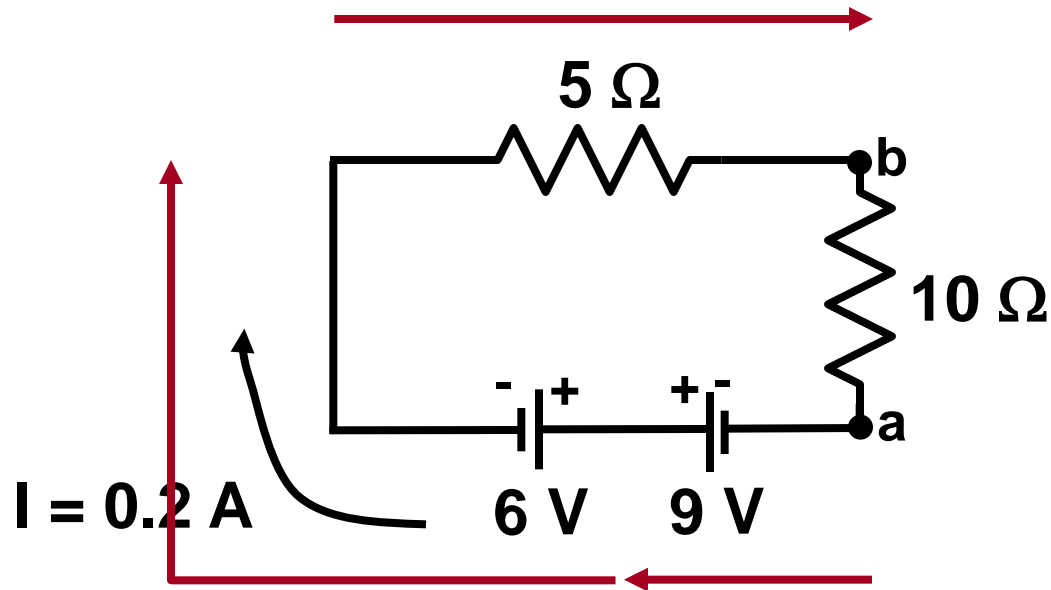
$$V_a + 9 - 6 - 5\,(0.2) = V_b$$

$$V_{ab} = V_a - V_b = -9 + 6 + 5\,(0.2) = -2\ V$$

The smart way: $V_{ba} = - V_{ab} = +2$ V

5 Ω

10 Ω

b

a

- +   + -

I = 0.2 A   6 V   9 V
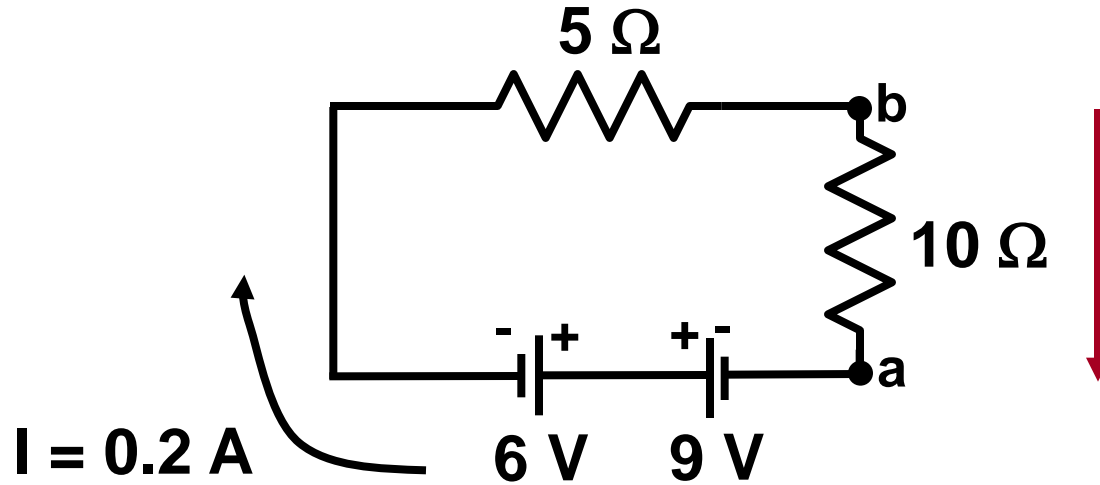
$$V_b - 10\,(0.2) = V_a$$

$$V_{ba} = V_b - V_a = + 10\,(0.2) = + 2\text{ V}$$

What if you guess the wrong current direction?
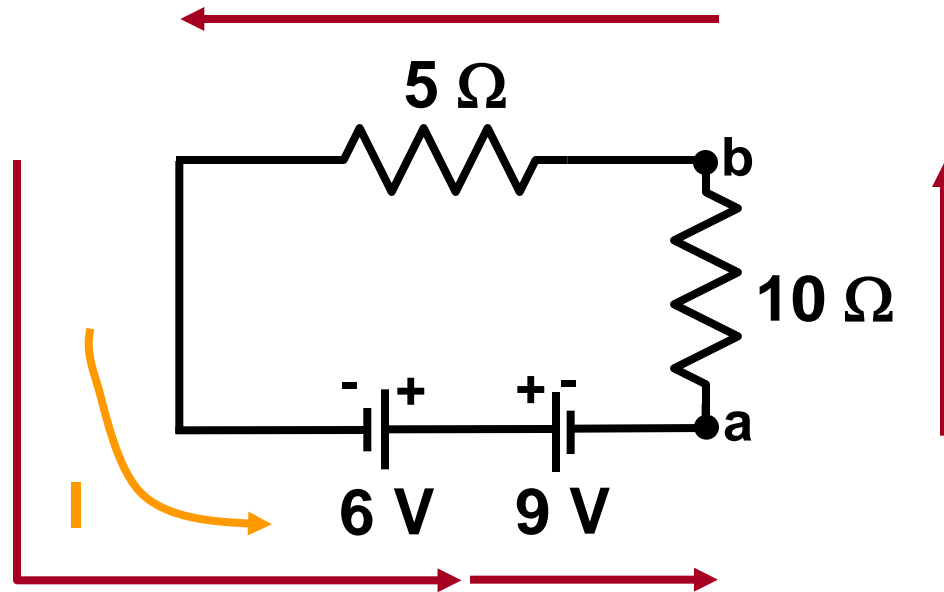
$$- 10\ I - 5\ I + 6 - 9 = 0$$

$$15\ I = -3$$

I = − 0.2 A   oops, guessed wrong direction, no big deal!

DC Currents

- in Physics 2135, (almost) all currents are "direct current."

- d.c. currents flow in one direction, from + to -.

- we will not encounter ac (alternating currents) much

- analysis of ac current is more complex

- household current is ac, assuming dc will be "close enough" to give you "a feel" for the physics.

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Electric Power.
You must be able to calculate the electric power dissipated in circuit components, and incorporate electric power in work-energy problems.

Examples.

Ideal versus real voltage sources

- **ideal** battery (or other voltage source):
  voltage does **not** depend on the current flowing

- **real** battery: voltage does depend on current,
  typically voltage **decreases** with increasing current (load)

How to model a real battery?

- real battery consists of ideal battery + internal resistance

Voltage of ideal battery is called electromotive force (emf) ε

internal resistance r
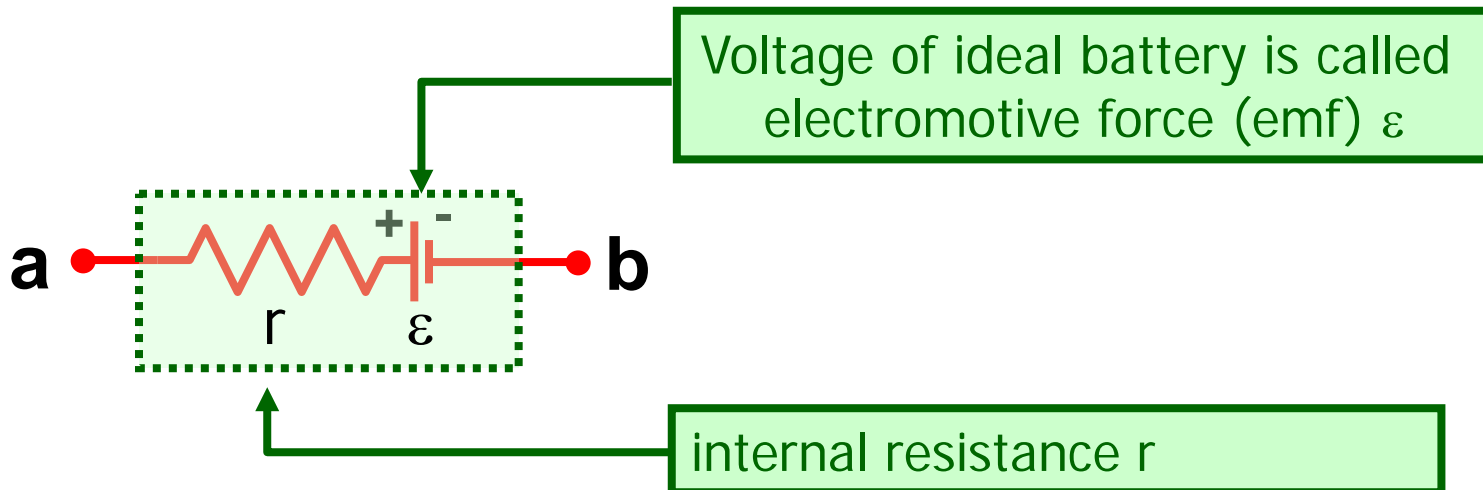
EMF and terminal voltage

The electromotive force (emf) of a voltage source is the potential difference it produces when no current is flowing.

emf is not a force!

Can the emf be measured?

- hook up a voltmeter:

- as soon as you connect the voltmeter, current flows

- you can only measure **terminal voltage** V_{ab}, but not emf ε

ε (emf)

a   +   -   b

I

An ideal voltmeter would be able to measure ε.
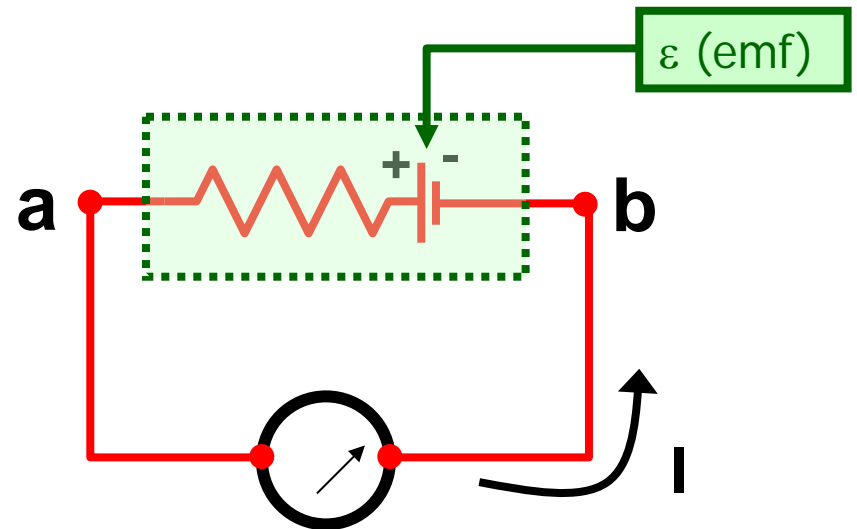
The "battery" is everything inside the green box.

To model a battery, simply include an extra resistor to represent the internal resistance, and label the voltage source* as an emf instead of V (units are still volts):
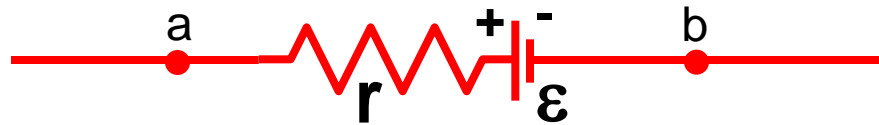

If the internal resistance is negligible, simply don't include it!

If you are asked to calculate the terminal voltage, it is just $V_{ab} = V_a - V_b$, calculated using the techniques I am showing you today.

(Terminal voltage is usually expressed as a positive number, so it is better to take the absolute value of V_{ab}.)

*Remember, all sources of emf—not just batteries—have an internal resistance.

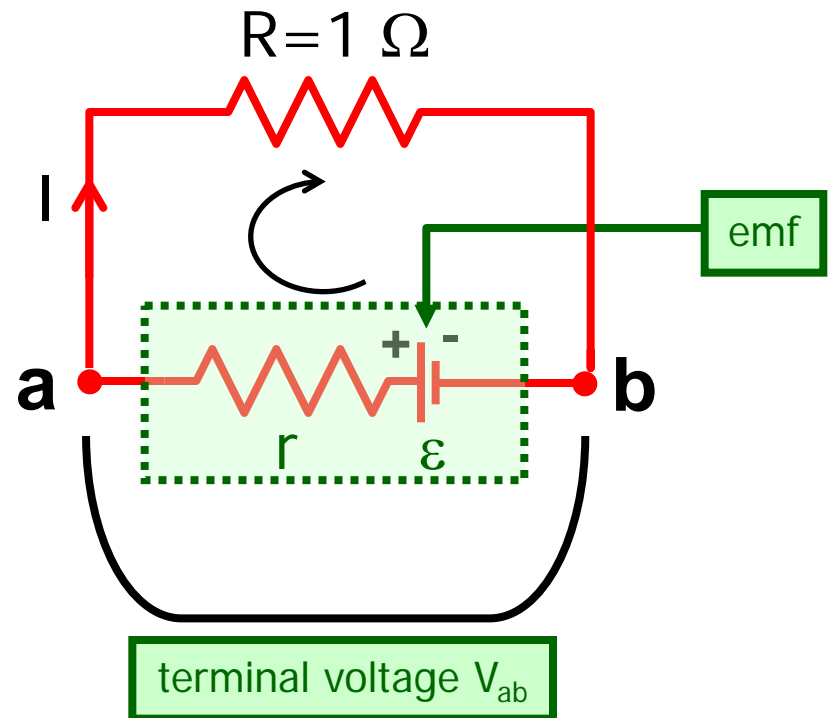Example: a battery is known to have an emf of 9 volts. If a 1 ohm resistor is connected to the battery, the terminal voltage is measured to be 3 volts. What is the internal resistance of the battery?

Loop rule:

$$\varepsilon - Ir - IR = 0$$

terminal voltage

$$V_{ab} = V_a - V_b = IR$$

R=1 Ω

I

emf

a   b

r ε

terminal voltage V_{ab}

$$\varepsilon - Ir - IR = 0$$

$$V_{ab} = IR$$

$$Ir = \varepsilon - IR$$

$$I = \frac{V_{ab}}{R}$$
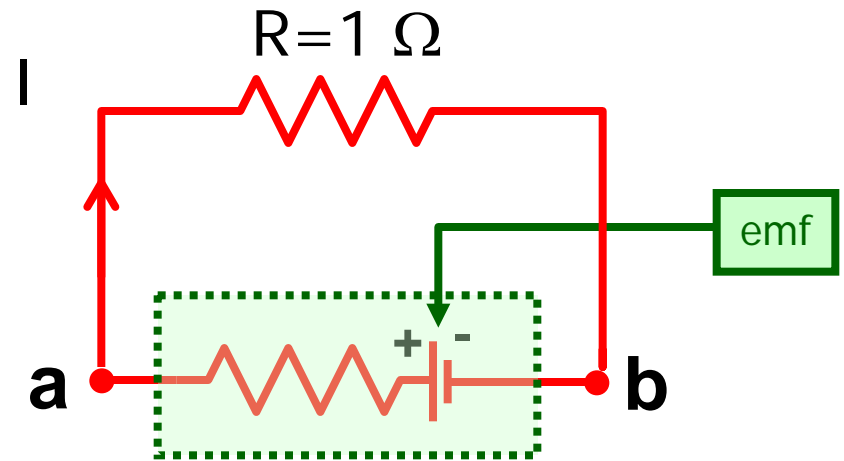
$$r = \frac{\varepsilon - IR}{I}$$

$$r = \frac{\varepsilon}{I} - R$$

$$r = \frac{\varepsilon R}{V_{ab}} - R$$

R=1 Ω

I

emf

a

b

$$r = R\left(\frac{\varepsilon}{V_{ab}} - 1\right)$$

$$r = 1\left(\frac{9}{3} - 1\right) = (3 - 1) = 2\Omega$$

A rather unrealistically large value for the internal resistance of a 9V battery.

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Examples.

Electric Power

- power in terms of work done by a force: $P_F = \dfrac{dW_F}{dt}$

- work done by electric force moving charge q through potential difference ΔV

$$W_{i \to f} = U_i - U_f = q(V_i - V_f).$$

- for infinitesimal charge dq

$$dW_{i \to f} = dq\ (V_i - V_f).$$

- **instantaneous power:**

$$P = \frac{dW_{i \to f}}{dt} = \frac{dq\ (V_i - V_f)}{dt} = \frac{dq\Delta V}{dt}.$$

$$P = \frac{dW}{dt} = \frac{dq}{dt} V = IV.$$

$$\boxed{P = IV}$$

- P<0 means loss of electric energy
- P>0 means gain of electric energy

- **For resistors**: using Ohm's "law" V=IR, we can write

$$\boxed{P = I^2 R = V^2 / R.}$$

Example: an electric heater draws 15.0 A on a 120 V line. How much power does it use and how much does it cost per 30 day month if it operates 3.0 h per day and the electric company charges 10.5 cents per kWh. For simplicity assume dc current.

Household current is ac rather than dc. Our calculation will be a reasonable approximation to reality.

An electric heater draws 15.0 A on a 120 V line. How much power does it use.

$$P = IV$$

$$P = (15\ A)(120\ V) = 1800\ W = 1.8\ kW$$

How much does it cost per 30 day month if it operates 3.0 h per day and the electric company charges 10.5 cents per kWh.

$$\cos t = (1.8\ kW)(30\ days)\left(\frac{3\ h}{day}\right)\left(\frac{\$0.105}{kWh}\right)$$

$$\cos t = \$17.00$$

$$(1 \text{ kW})(1 \text{ h}) = (1000 \text{ W})(3600 \text{ s})$$

$$= \left(1000 \ \frac{\text{J}}{\text{s}}\right)(3600 \text{ s})$$

$$= 3.6 \times 10^6 \text{ J}$$

So a kWh is a "funny" unit of energy. K (kilo) and h (hours) are lowercase, and W (James Watt) is uppercase.

$$P_{\text{average}} = \frac{W_{\text{done by force}}}{\text{time}} = \frac{\text{Energy Transformed}}{\text{time}}$$
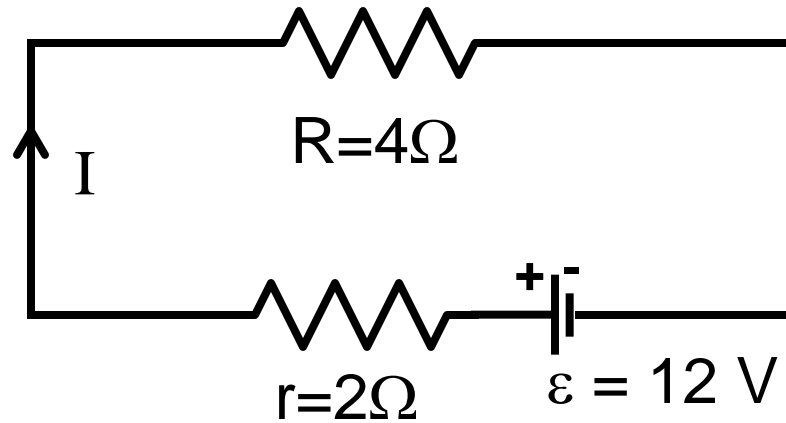
$$\text{Energy Transformed} = \left(P_{\text{average}}\right)\left(\text{time}\right)$$

$$\text{Energy Transformed} = \left(1800\frac{J}{s}\right)\left(30 \text{ days}\right)\left(\frac{3 \text{ h used}}{\text{day}}\right)\left(\frac{3600 \text{ s}}{h}\right)$$

$$\text{Energy Transformed} = 583,200,000 \text{ Joules used}$$

Example: A 12 V battery with 2 Ω internal resistance is connected to a 4 Ω resistor. Calculate (a) the rate at which chemical energy is converted to electrical energy in the battery, (b) the power dissipated internally in the battery, and (c) the power output of the battery.
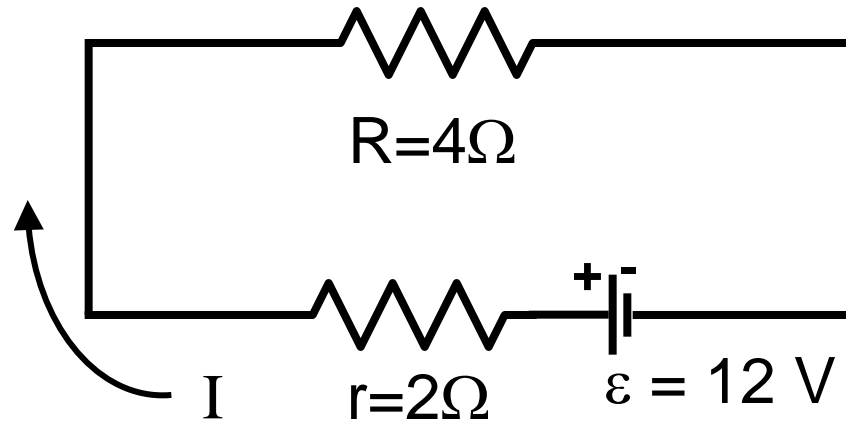
R=4Ω

+ -

I   r=2Ω ε = 12 V

Calculate (a) the rate at which chemical energy is converted to electrical energy in the battery.

$R=4\Omega$

I

$+$ $-$

$r=2\Omega$ $\varepsilon = 12$ V

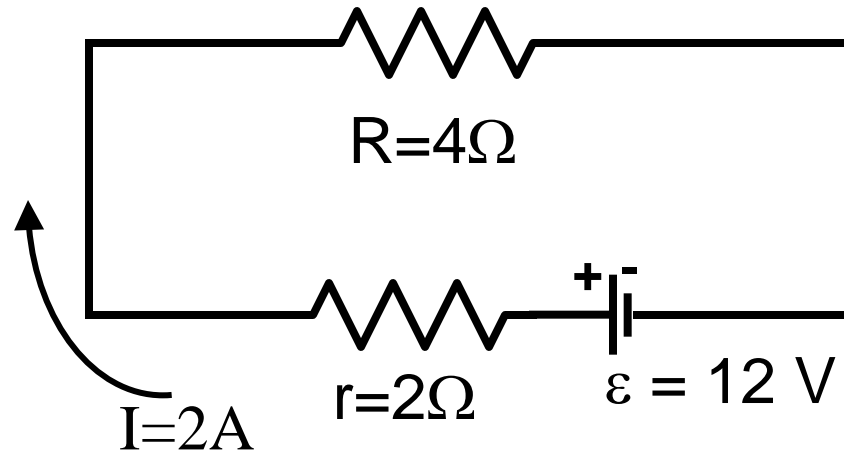$$\sum V = 0 \text{ around any closed circuit loop}$$

Start at negative terminal of battery...

$$\varepsilon - I\,R_{2\Omega} - I\,R_{4\Omega} = 0$$

$$I = \varepsilon \,/\, (R_{2\Omega} + R_{4\Omega}) = 12 \text{ V} / 6 \text{ } \Omega = 2 \text{ A}$$

Energy is converted at the rate $P_{converted} = I\varepsilon = (2 \text{ A})(12 \text{ V}) = 24W$.

$$P_{dissipated} = I^2r = (2 \text{ A})^2 (2 \text{ } \Omega) = 8 \text{ W.}$$

$$P_{output} = P_{converted} - P_{dissipated} = 24 \text{ W} - 8 \text{ W} = 16W.$$

$$R=4\Omega$$

$$+ \quad -$$

$$r=2\Omega \qquad \varepsilon = 12 \text{ V}$$

$$I=2A$$
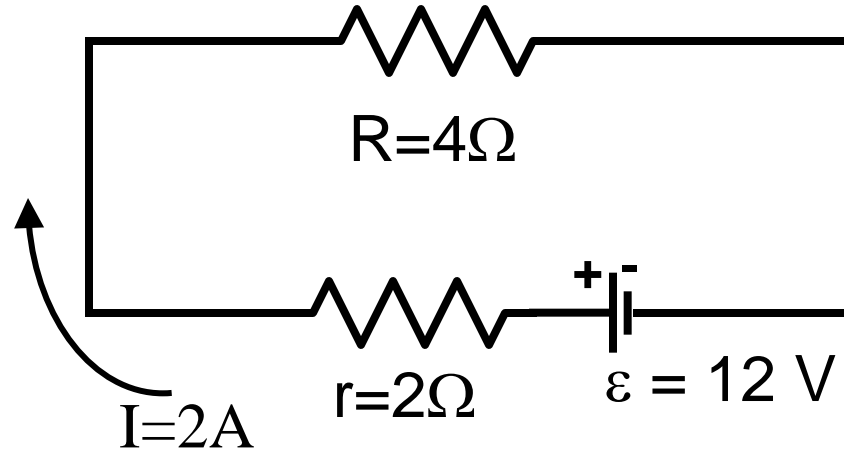
The output power is delivered to (and dissipated by) the resistor:

$$P_{output} = P_{resistor} = I^2 R = (2 \text{ A})^2 (4 \ \Omega) = 16W.$$

Example: a 3 volt and 6 volt battery are connected in series, along with a 6 ohm resistor. The batteries* are connected the "wrong" way (+ to + or - to -). What is the power dissipated in the resistor? In the 3 volt battery?

6 Ω

- **+**   **+** **-**

I   **3 V**   **6 V**   ●a
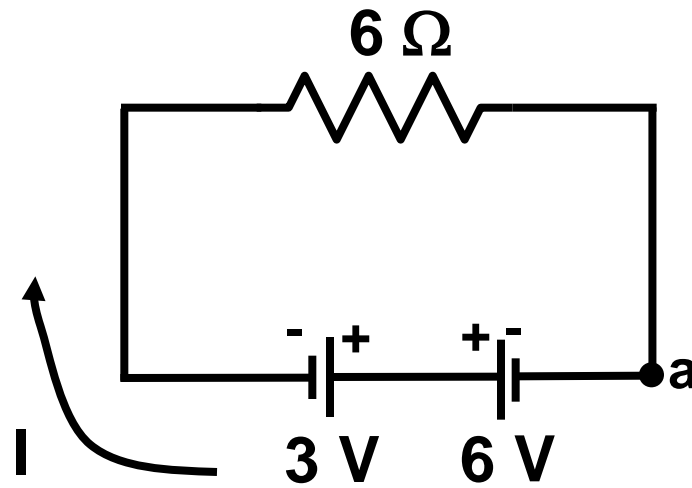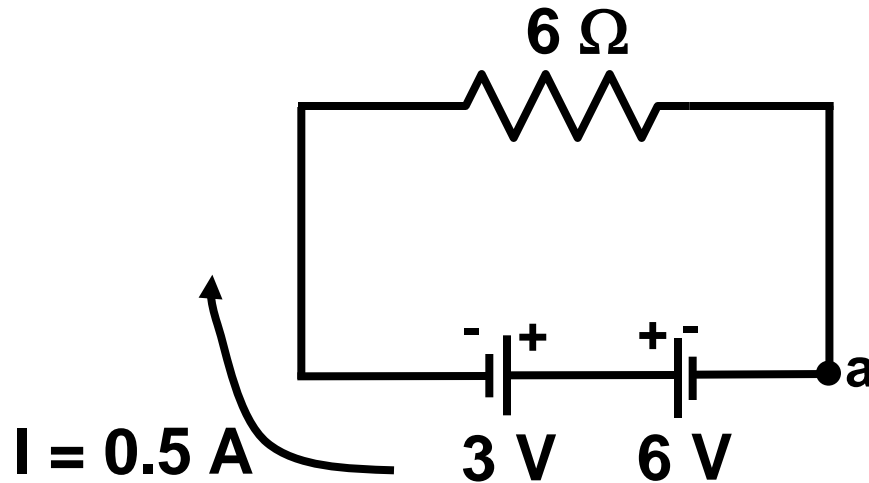
Starting at point a...

$$+ \, 6 - 3 - 6 \, I = 0$$

$$I = (6 - 3) \, / \, 6 = 0.5 \text{ A}$$

*Assume zero internal resistance unless the problem suggests otherwise.

What is the power dissipated in the resistor?

$$P_R = I^2R = (0.5)^2\,(6) = 1.5\ \text{W}$$

What is the power dissipated in the 3 volt battery?

$$P_{3V} = IV = (0.5)\,(3) = 1.5\ \text{W}$$

Note: $P_{6V} = IV = (0.5)\,(6) = 3\ \text{W} = P_R + P_{3V}$