

Today's agenda:

## **Magnetic Fields.**

You must understand the similarities and differences between electric fields and field lines, and magnetic fields and field lines.

## **Magnetic Force on Moving Charged Particles.**

You must be able to calculate the magnetic force on moving charged particles.

## **Magnetic Flux and Gauss' Law for Magnetism.**

You must be able to calculate magnetic flux and recognize the consequences of Gauss' Law for Magnetism.

## **Motion of a Charged Particle in a Uniform Magnetic Field.**

You must be able to calculate the trajectory and energy of a charged particle moving in a uniform magnetic field.

# Magnetism

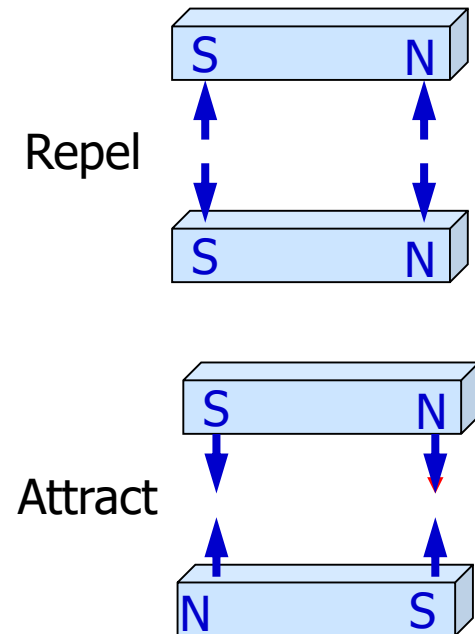
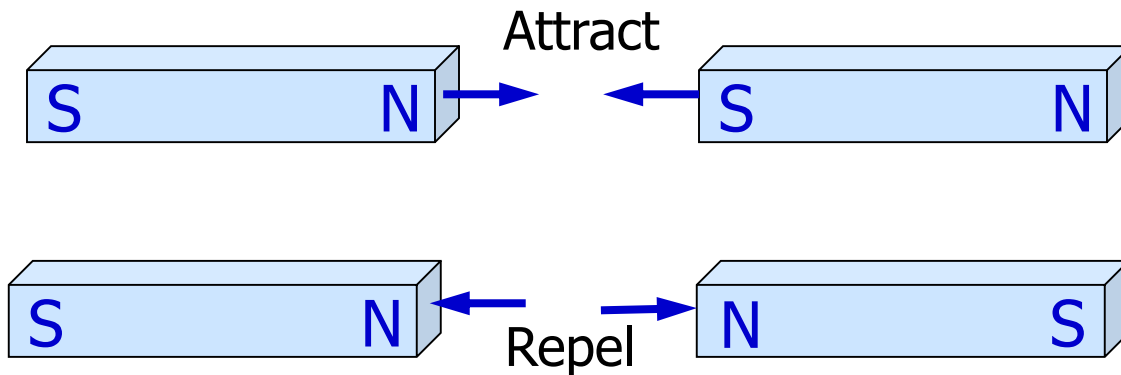
## Recall: (lecture 1)

- electric (Coulomb) force between electric charges (+ and -), like charges repel, opposite charges attract

## Analogously:

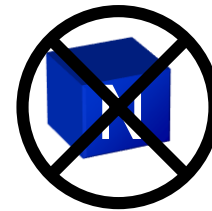
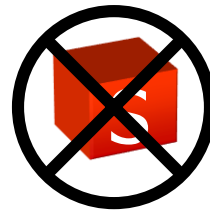
- two kinds of magnetic poles (north and south), like poles repel, opposites attract

Example: bar magnets:

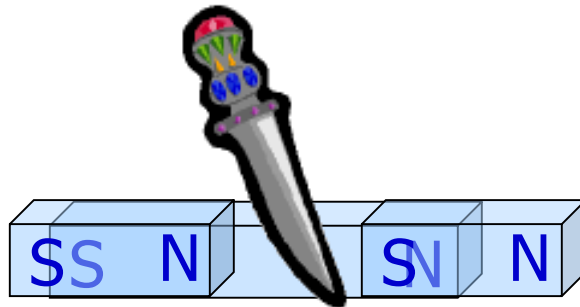


## Important difference:

- isolated + or – electric charges exist
- isolated N and S poles (magnetic monopoles) are **not** observed (topic of active physics research\*)



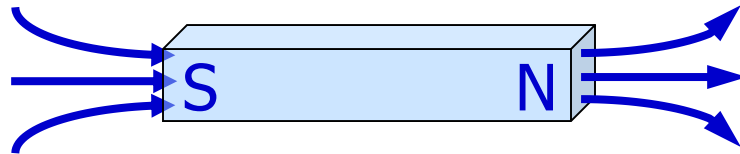
- every magnet has BOTH a N and a S pole
- basic magnetic structure is a dipole



\*See, for example, [here](#), [here](#), [here](#), and [here](#).

# Magnetic Fields

- magnetic dipoles create **magnetic fields**
- magnetic field lines point away from north and towards south



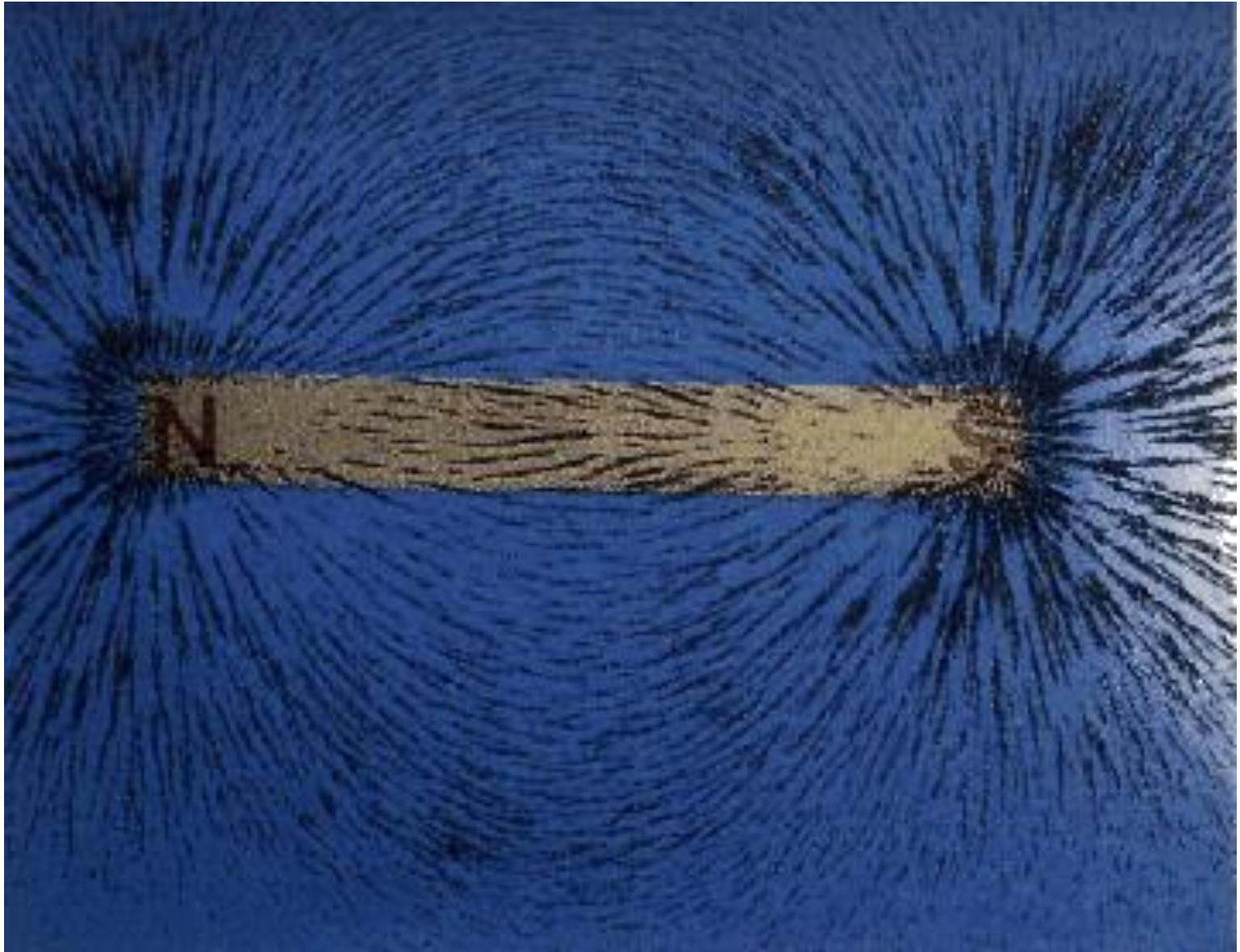
- symbol for magnetic field:  $\vec{B}$
- SI unit\* for magnetic field: T (Tesla)

$$1 \text{ T} = \frac{1 \text{ kg}}{\text{C} \cdot \text{s}}$$

These units come from the magnetic force equation, which appears three slides from now.

\*Old unit, still sometimes used: 1 Gauss =  $10^{-4}$  Tesla.

magnetic field of a bar magnet, visualized using iron filings

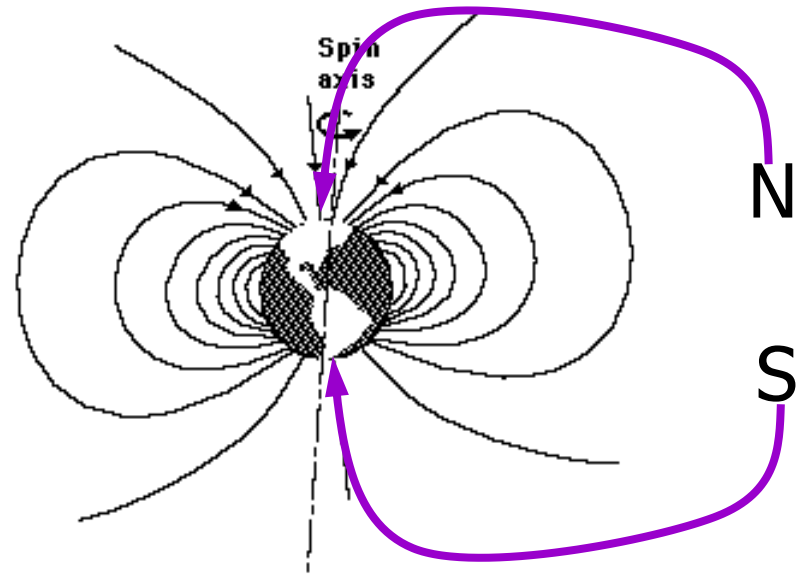


# Earth's Magnetic Field

- earth has magnetic field, with poles near the geographic poles

## Magnetic compass:

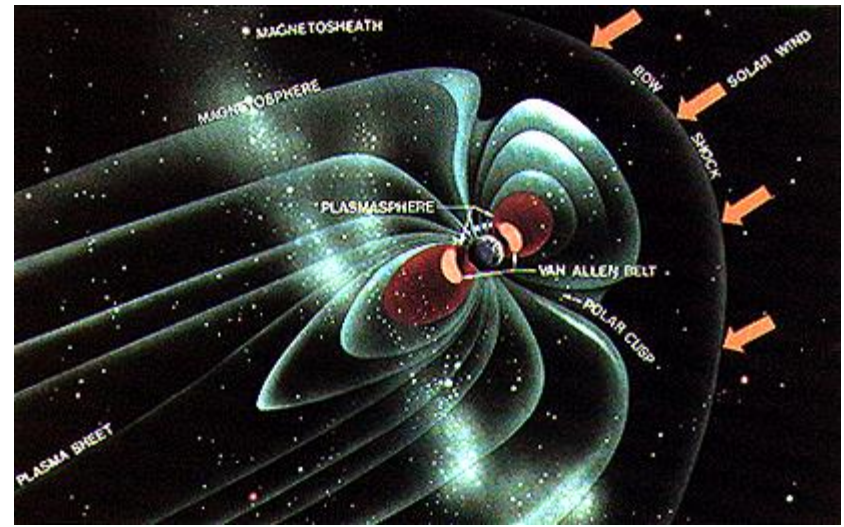
- magnetic poles of compass needle (small bar magnet) are attracted to and repelled by earth's magnetic poles
- magnetic north pole of needle is attracted to geographic N pole
- magnetic south pole of needle is attracted to geographic S pole



<http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/magearth.html>

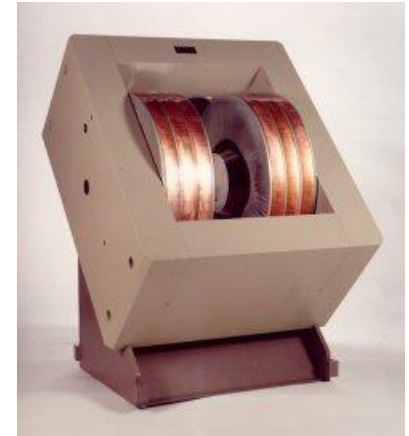
Magnetic north pole of earth is (close to) geographic south pole and vice versa

The earth's magnetic field has a magnitude of roughly 0.5 G, or 0.00005 T. A powerful permanent magnet, like the kind you might find in headphones, might produce a magnetic field of 1000 G, or 0.1 T.



[http://liftoff.msfc.nasa.gov/academy/space/mag\\_field.html](http://liftoff.msfc.nasa.gov/academy/space/mag_field.html)

The electromagnet that used to be in the basement of the Physics Bldg produced a field of 26000 G = 26 kG = 2.6 T.



Superconducting magnets can produce a field of over 10 T. Never get near an operating superconducting magnet while wearing a watch or belt buckle with iron in it!

# Magnetic Fields

## Two questions:

- How can one create magnetic fields?  
(permanent magnets and ???)
- What are the effects of magnetic fields?

We start with the second question!



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## Magnetic Flux and Gauss' Law for Magnetism.

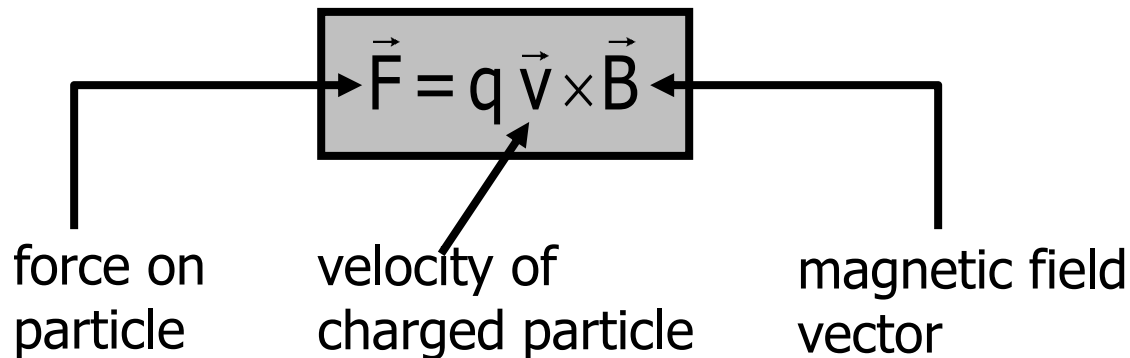
You must be able to calculate magnetic flux and recognize the consequences of Gauss' Law for Magnetism.

## Motion of a Charged Particle in a Uniform Magnetic Field.

You must be able to calculate the trajectory and energy of a charged particle moving in a uniform magnetic field.

## Magnetic force on a charge

- magnetic field  $\vec{B}$  creates force acting on **moving** charged particle



What is the magnetic force if the charged particle is at rest?

Question: motion with respect to what? You, the earth, the sun?  
Highly nontrivial, leads to Einstein's theory of relativity.

Recall (Physics 1135):

Vector cross product: Magnitude

$$\vec{A} \times \vec{B} = \vec{C}$$

$$C = AB \sin\theta = A_{\perp B}B = AB_{\perp A}$$

# Vector cross product: Direction

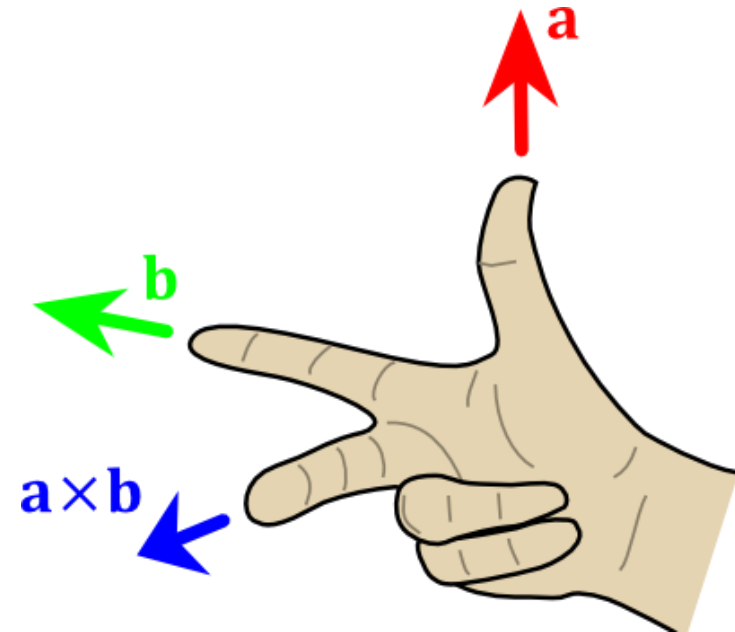
$$\vec{A} \times \vec{B} = \vec{C}$$

$\vec{C}$  is perpendicular  
to both  $\vec{A}$  and  $\vec{B}$

**Direction: right hand rule**

$$\vec{A} \times \vec{B} = \vec{C}$$

thumb  $\times$  index finger = middle finger



# Many alternative versions of the right hand rule exist!

- textbook presents several versions
- other classes may use different right hand rules
- Wikipedia, Youtube

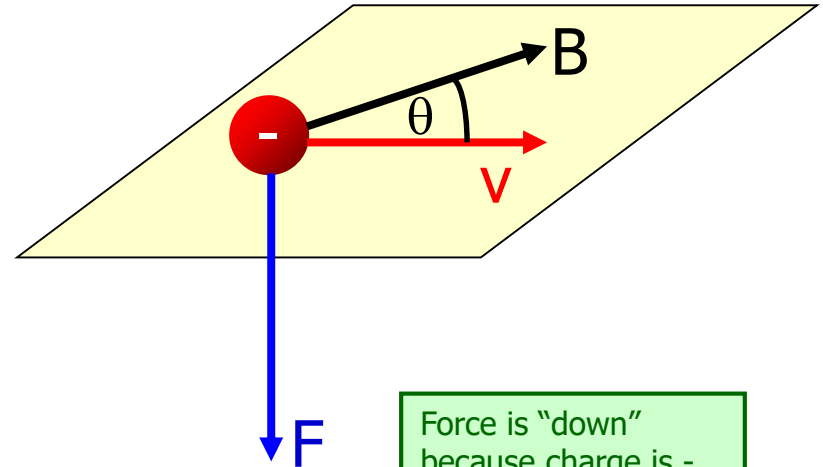
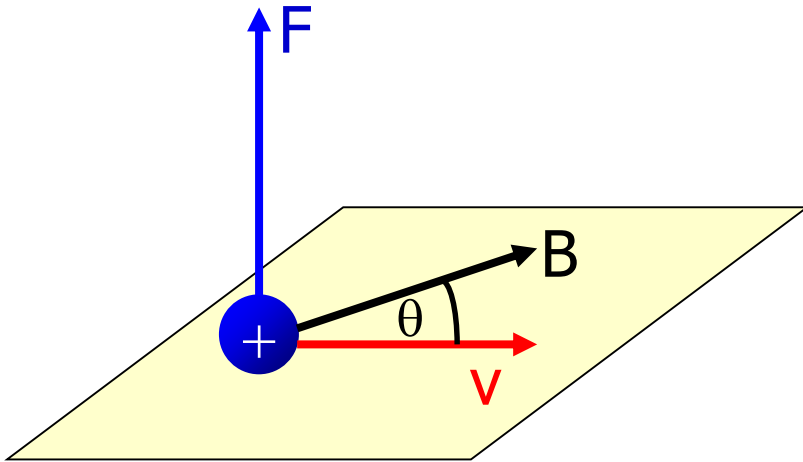
**All versions are equivalent!**

## Recommendation:

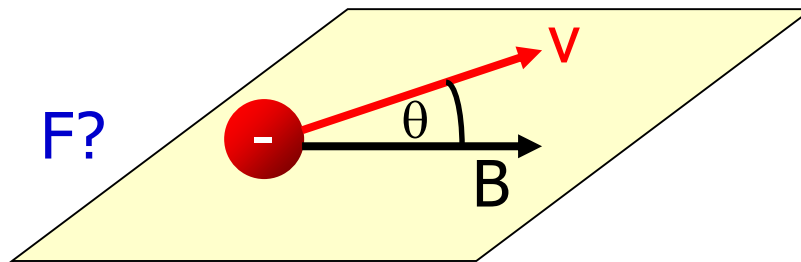
Memorize one version and stick to it!

still more variations: <http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/magfor.html>

# Examples: Direction of the magnetic force



Force is "down" because charge is -.



## Cross product in terms of vector components

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{F} = q \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{pmatrix}$$

<http://www.youtube.com/watch?v=21LWuY8i6Hw>

All of the right-hand rules are just techniques for determining the direction of vectors in the cross product without having to do any actual math.

Example: a proton is moving with a velocity  $\vec{v} = v_0 \hat{j}$  in a region of uniform magnetic field. The resulting force is  $\vec{F} = F_0 \hat{i}$ . What is the magnetic field (magnitude and direction)?

To be worked at the blackboard.

$$\left[ \begin{array}{c} a \\ \times \\ \left| \begin{array}{cc} e & f \\ h & i \end{array} \right| \end{array} \right] - \left[ \begin{array}{c} b \\ \times \\ \left| \begin{array}{cc} d & f \\ g & i \end{array} \right| \end{array} \right] + \left[ \begin{array}{c} \left| \begin{array}{cc} d & e \\ g & h \end{array} \right| \\ \times \\ c \end{array} \right]$$

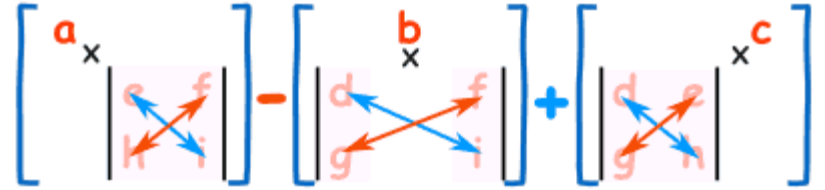
Blue is +, red is -. Image from <http://www.mathsisfun.com/algebra/matrix-determinant.htm>



Example: a proton is moving with a velocity  $\vec{v} = v_0 \hat{j}$  in a region of uniform magnetic field. The resulting force is  $\vec{F} = F_0 \hat{i}$ . What is the magnetic field (magnitude and direction)?

$$\vec{v} = v_0 \hat{j}$$

$$\vec{F} = F_0 \hat{i}$$



$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{F} = F_0 \hat{i} = q \left[ \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{pmatrix} \right] = (+e) \left[ \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & v_0 & 0 \\ B_x & B_y & B_z \end{pmatrix} \right]$$

$$F_0 \hat{i} = (+e) \left[ \hat{i}(v_0 \cdot B_z - B_y \cdot 0) - \hat{j}(0 \cdot B_z - B_x \cdot 0) + \hat{k}(0 \cdot B_y - B_x \cdot v_0) \right]$$

Example: a proton is moving with a velocity  $\vec{v} = v_0 \hat{j}$  in a region of uniform magnetic field. The resulting force is  $\vec{F} = F_0 \hat{i}$ . What is the magnetic field (magnitude and direction)?

$$F_0 \hat{i} = ev_0 B_z \hat{i} - 0 \hat{j} - eB_x v_0 \hat{k} = (ev_0 B_z) \hat{i} + (-0) \hat{j} + (-eB_x v_0) \hat{k}$$

$$F_0 = ev_0 B_z \quad \text{and} \quad 0 = -eB_x v_0$$

$$B_z = \frac{F_0}{ev_0} \quad \text{and} \quad B_x = 0$$

What is  $B_y$ ???? It could be anything. Not enough information is provided to find  $B_y$ . We can't find the magnitude and direction of the magnetic field using only the information given!

Example: a proton is moving with a velocity  $\vec{v} = v_0 \hat{j}$  in a region of uniform magnetic field. The resulting force is  $\vec{F} = F_0 \hat{i}$ . What is the **minimum magnitude** magnetic field that can be present?

$$B_z = \frac{F_0}{ev_0} \quad \text{and} \quad B_x = 0$$

$B_y = 0$  for a minimum magnitude magnetic field, so...

$$B_{\min} = \frac{F_0}{ev_0}$$

## Vector notation conventions:



$\odot$  is a vector pointing out of the paper/board/screen (looks like an arrow coming straight for your eye).



$\otimes$  is a vector pointing into the paper/board/screen (looks like the feathers of an arrow going away from eye).

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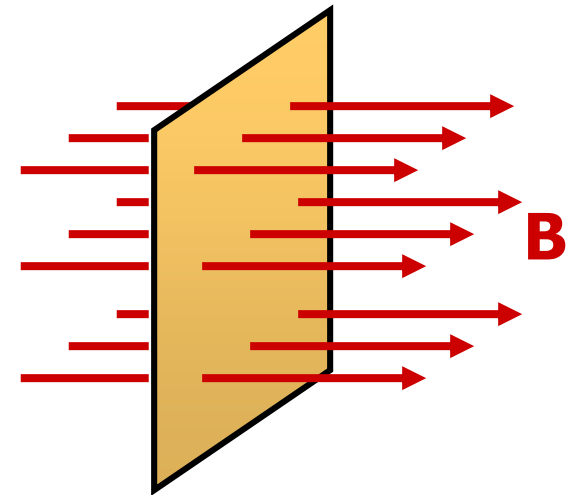
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# Magnetic Flux and Gauss' Law for Magnetism

## Define magnetic flux:

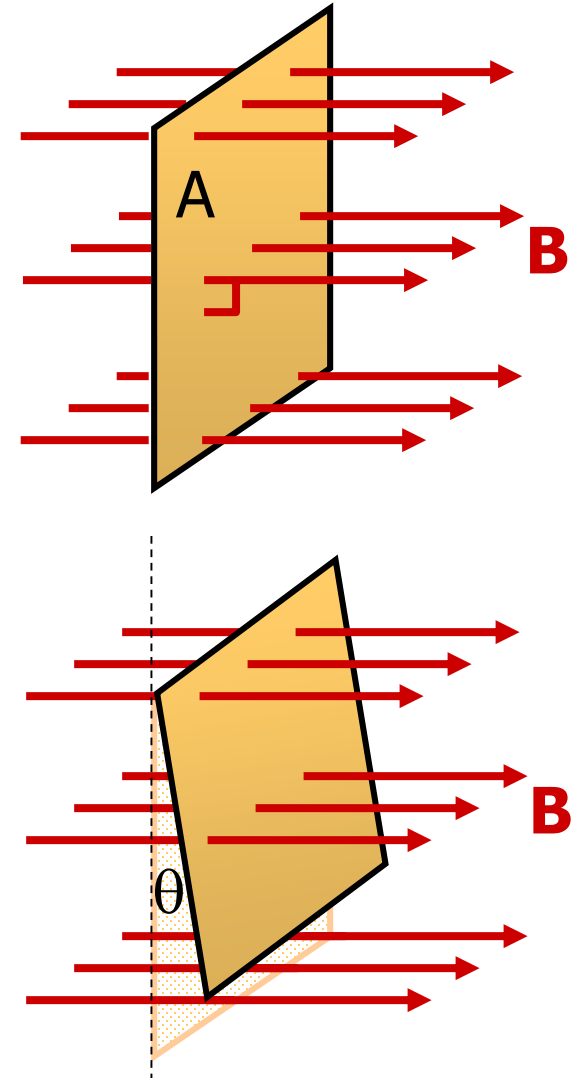
- in complete analogy to electric flux
- count number of magnetic field lines passing through a surface



magnetic flux passing through a surface is (proportional to) number of magnetic field lines that pass through it

if  $B$  is uniform and normal to surface  
 $\Phi_B = BA$ .

if the surface is tilted, fewer lines cut the surface.



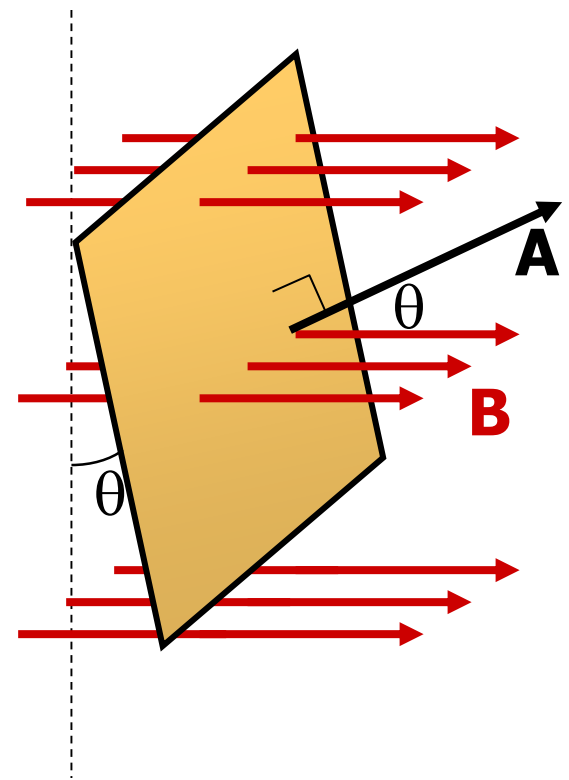
If these slides look familiar, refer back to lecture 4!

$\vec{A}$  is vector having a magnitude equal to surface area, in direction normal to surface.

The “amount of surface” perpendicular to magnetic field is  $A \cos \theta$ .

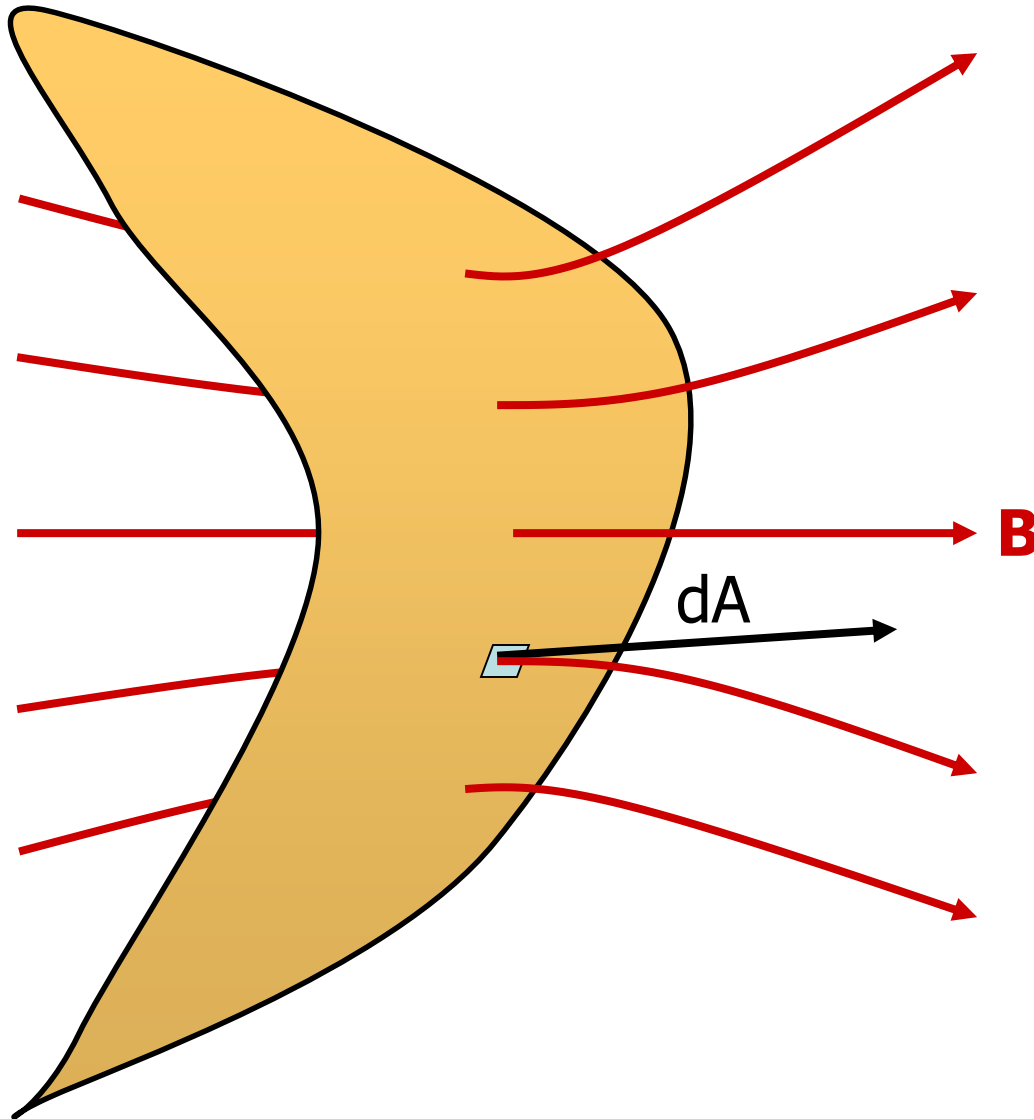
Because  $\vec{A}$  is perpendicular to surface, amount of  $A$  parallel to magnetic field is  $A \cos \theta$ .

$$A_{\parallel} = A \cos \theta \quad \text{so} \quad \Phi_B = BA_{\parallel} = BA \cos \theta.$$
$$\Phi_B = \vec{B} \cdot \vec{A}$$





If magnetic field is not uniform, or surface is not flat...



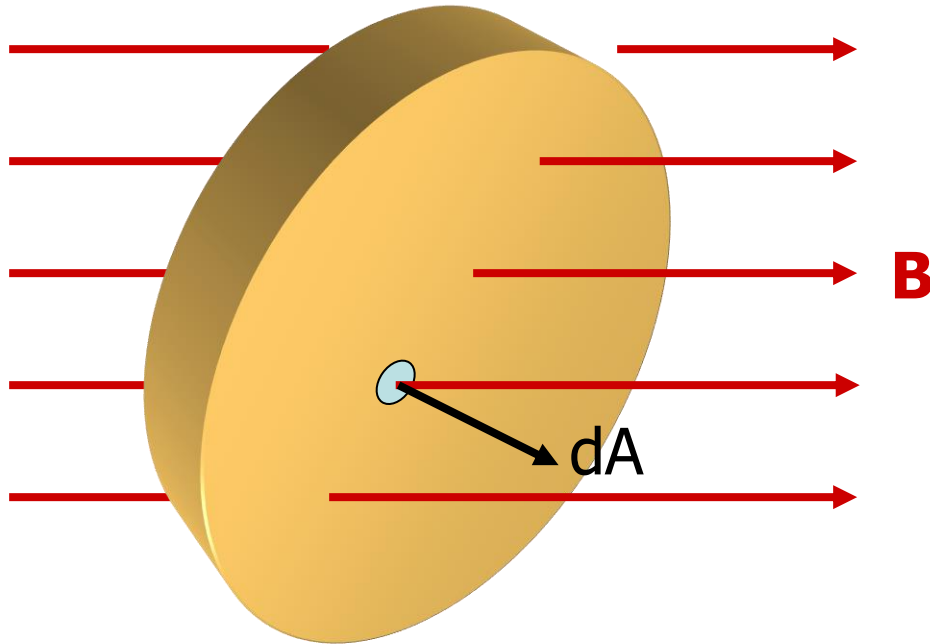
divide surface into infinitesimal surface elements and add flux through each...

$$\Phi_B = \lim_{\Delta A_i \rightarrow 0} \sum_i \vec{B}_i \cdot \Delta \vec{A}_i$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

**definition of magnetic flux**

If the surface is closed (completely encloses a volume)...



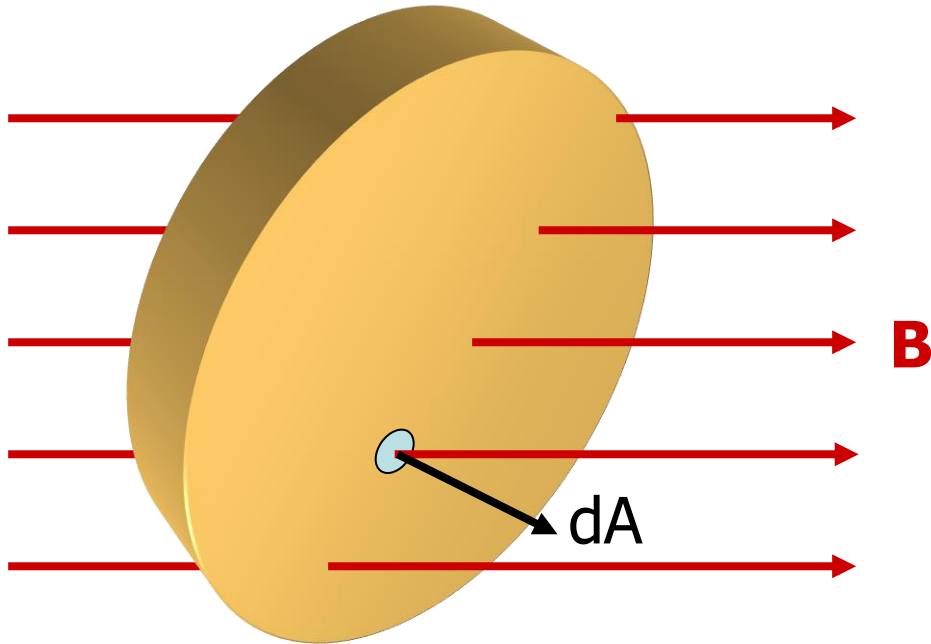
...we count lines going out as positive and lines going in as negative...

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

a surface integral, therefore a double integral  $\iint$

## Recall:

- field lines begin and end at charges (monopoles)
- there are **no magnetic monopoles** in nature
- all field lines entering surface have to leave it again



Therefore

$$\Phi_M = \oint \vec{B} \cdot d\vec{A} = 0$$

## Gauss' Law for Magnetism!

This law may require modification if the existence of magnetic monopoles is confirmed.

Gauss' Law for magnetism is not very useful in this course. The concept of magnetic flux is extremely useful, and will be used later!

You have now learned Gauss's Law for both electricity and magnetism.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

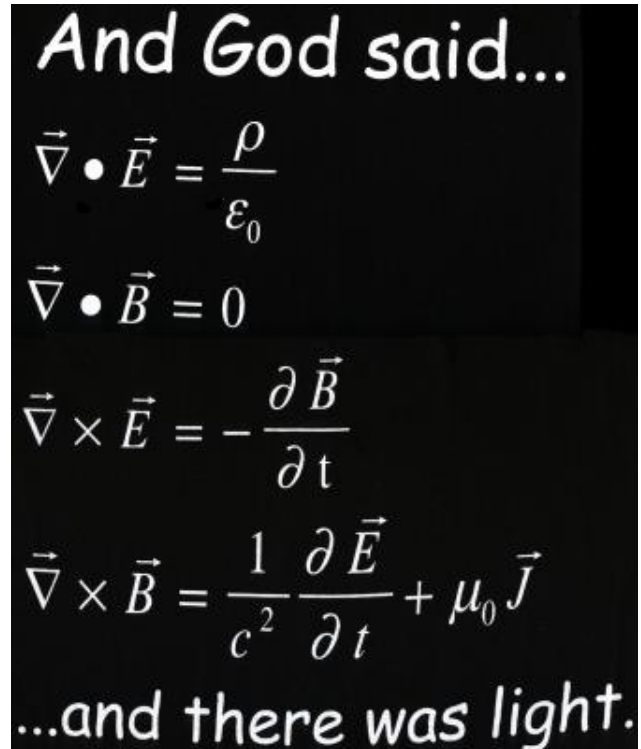
These equations can also be written in differential form:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

Congratulations! You are 1/2 of the way to being qualified to wear...

# The Missouri S&T Society of Physics Student T-Shirt!



This will not be tested on the exam.

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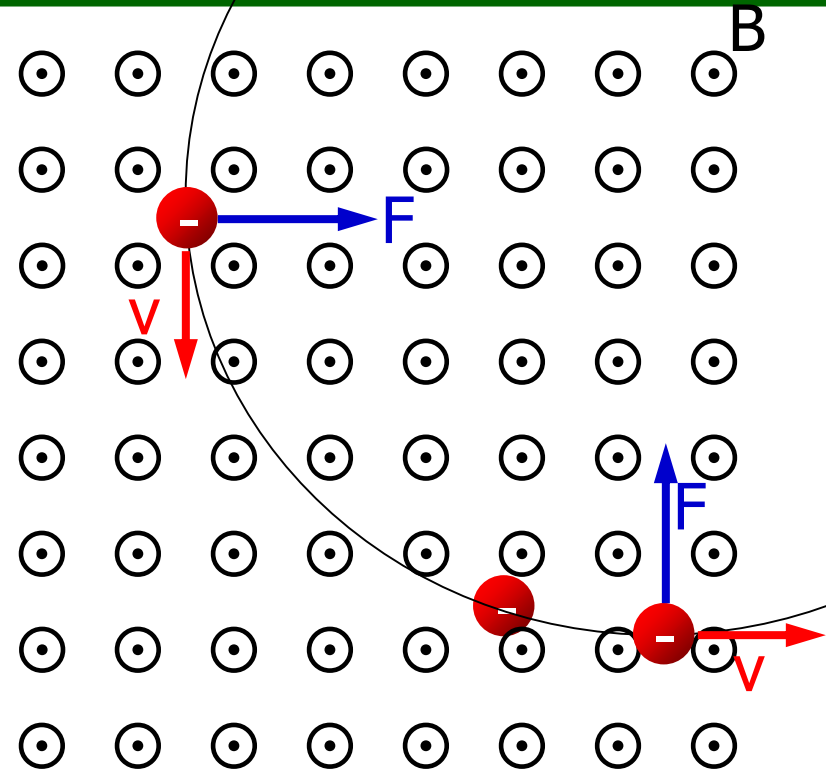
You must be able to calculate the trajectory and energy of a charged particle moving in a uniform magnetic field.

## Motion of a charged particle in a uniform magnetic field

Example: an electron travels at  $2 \times 10^7$  m/s in a plane perpendicular to a 0.01 T uniform magnetic field. Describe its path.

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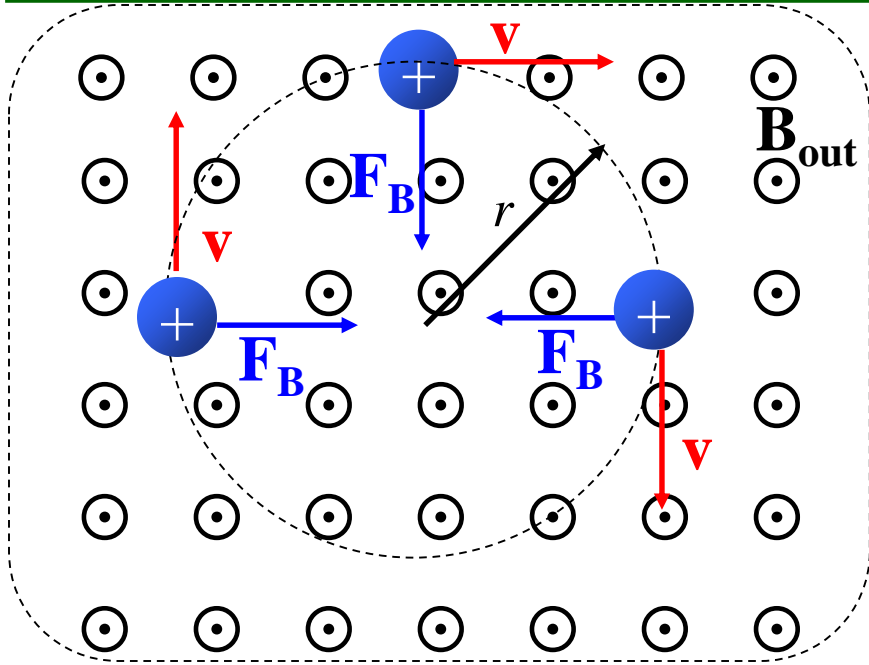
- force on electron is always perpendicular to the velocity
- force does not change speed of electron
- force only changes direction of electron



Electron will move in a **circular path with a constant speed and acceleration**  $= v^2/r$ , where  $r$  is the radius of the circle.



Example: a proton of speed  $v$  travels in a plane perpendicular to a uniform magnetic field. Find the radius of its orbit.



- force is in radial direction
- magnitude  $F = |q|vB$
- for circular motion,  $a = v^2/r$

$$F = |q|vB = \frac{mv^2}{r}$$

$$v = \frac{|q|rB}{m} \quad r = \frac{mv}{|q|B}$$

period  $T$  of orbit:

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{|q|B}$$

The rotational frequency  $f$  is called the cyclotron frequency

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m}$$

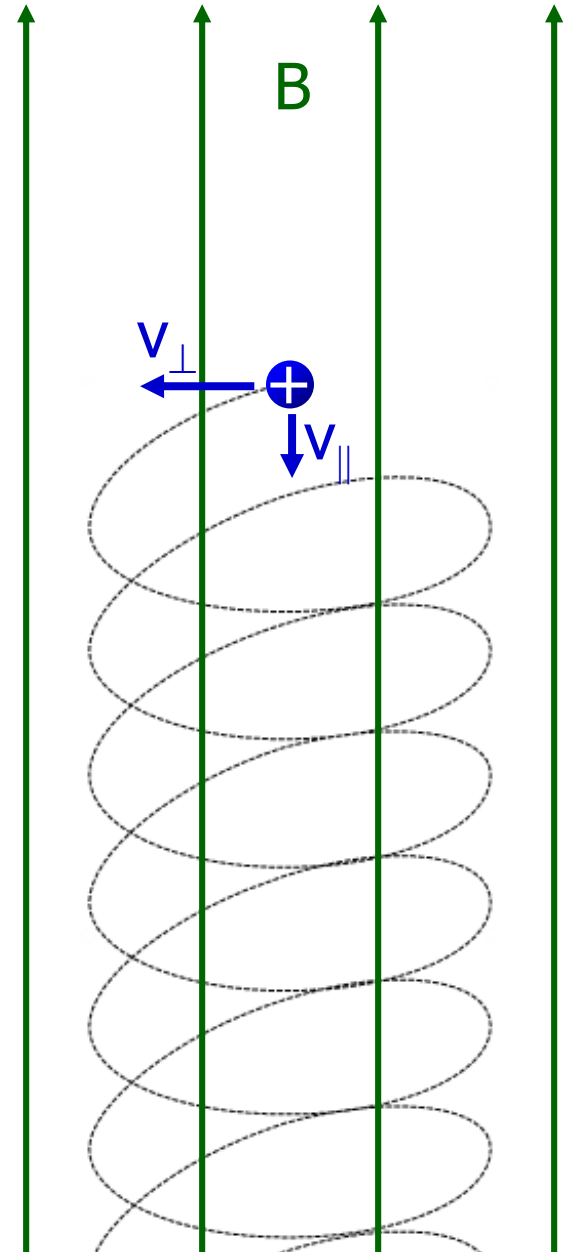
# Helical motion in a uniform magnetic field

If  $\vec{v}$  and  $\vec{B}$  are perpendicular, a charged particle travels in a circular path.  $v$  remains constant but the direction of  $\vec{v}$  constantly changes.

If  $\vec{v}$  has a component parallel to  $\vec{B}$ , then  $v_{\parallel}$  remains constant, and the charged particle moves in a helical path.

There won't be any test problems on helical motion.

\*or antiparallel

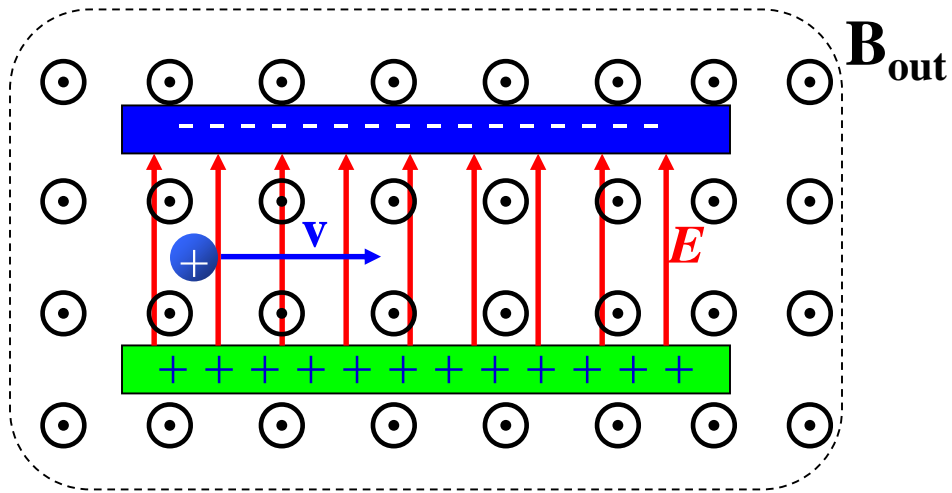


## Lorentz Force Law

If both electric and magnetic fields are present,

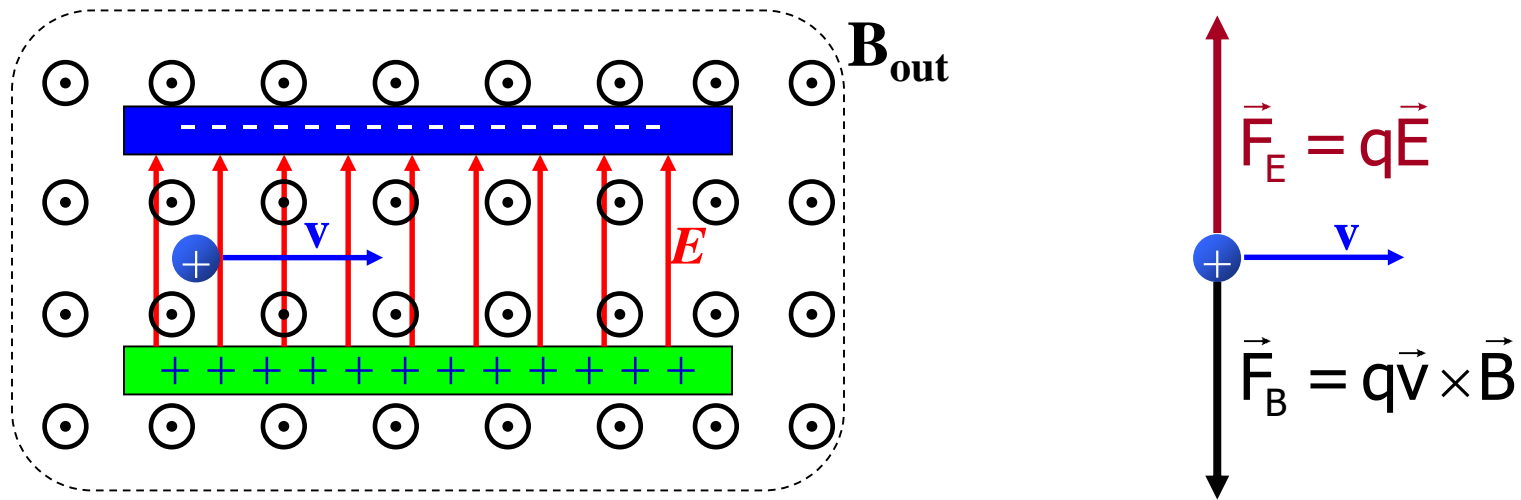
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}).$$

# Application: Velocity Selector



The magnetic field is uniform inside the dashed-line region. What speed will allow the proton to pass undeflected through region of uniform electric field?

# Velocity Selector

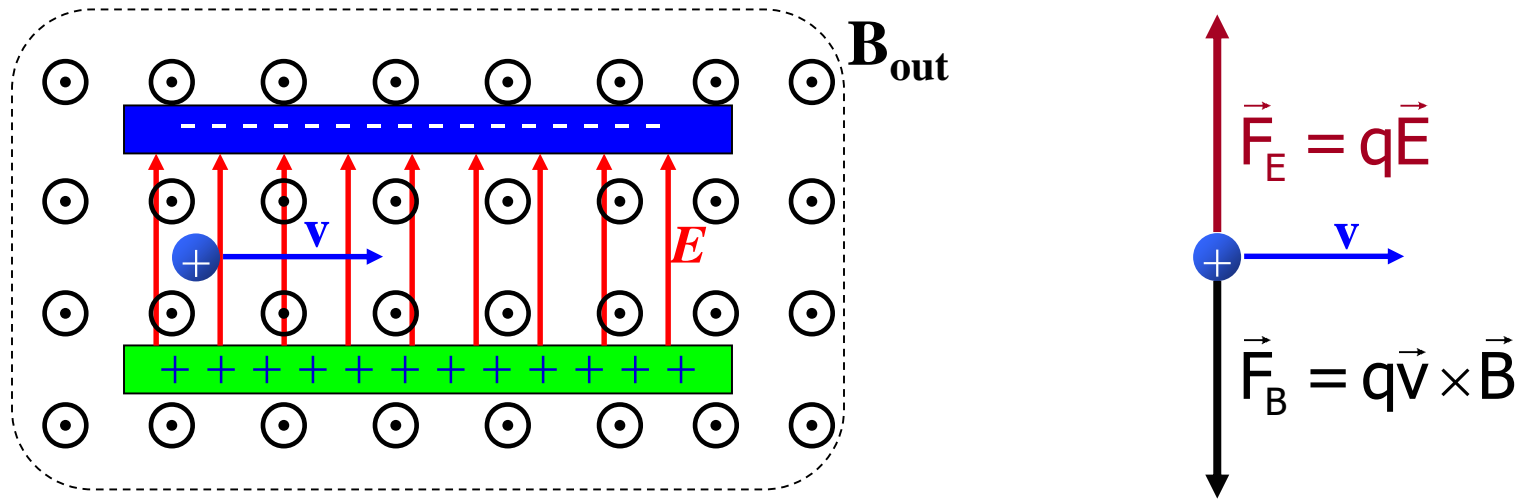


When the electric and magnetic forces balance then the charge will pass straight through.

$$|\vec{F}_E| = |\vec{F}_B|$$

$$|q|E = |q|vB \quad \text{or} \quad v = \frac{E}{B}$$

# Velocity Selector



This only works if the electric and magnetic fields are perpendicular and oriented so that  $q\vec{v} \times \vec{B}$  is antiparallel to  $\vec{E}$ .

Also, we simplified the calculation by making  $\vec{v}$  perpendicular to  $\vec{B}$ .

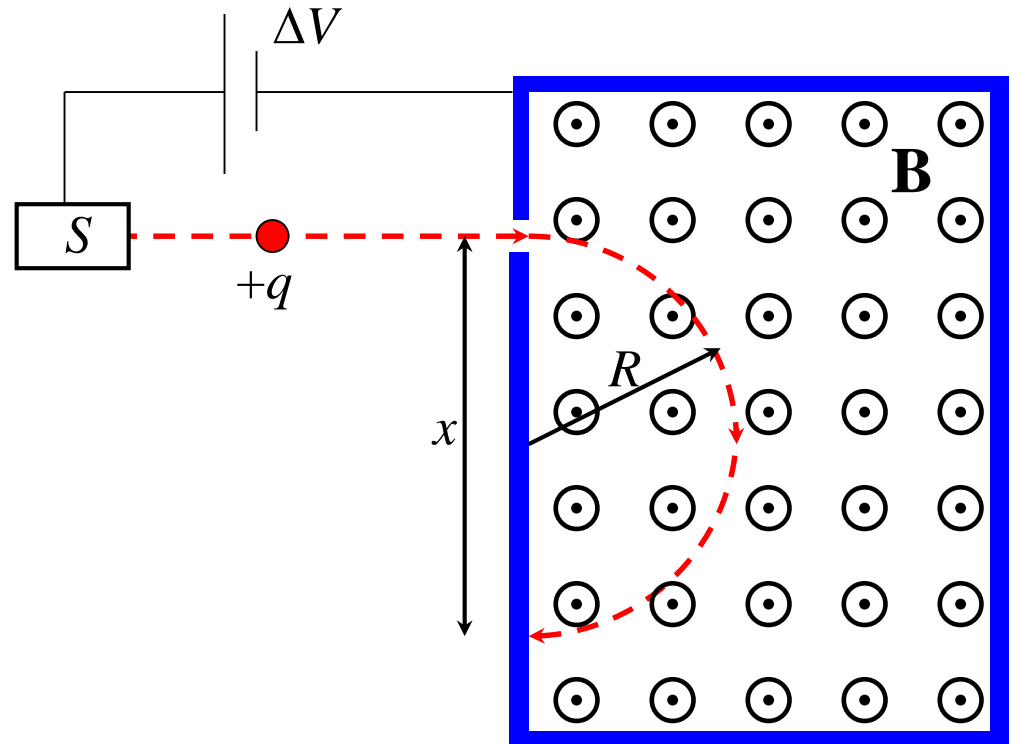
# Application: Mass Spectrometer

Mass spectrometers separate charges of different mass.

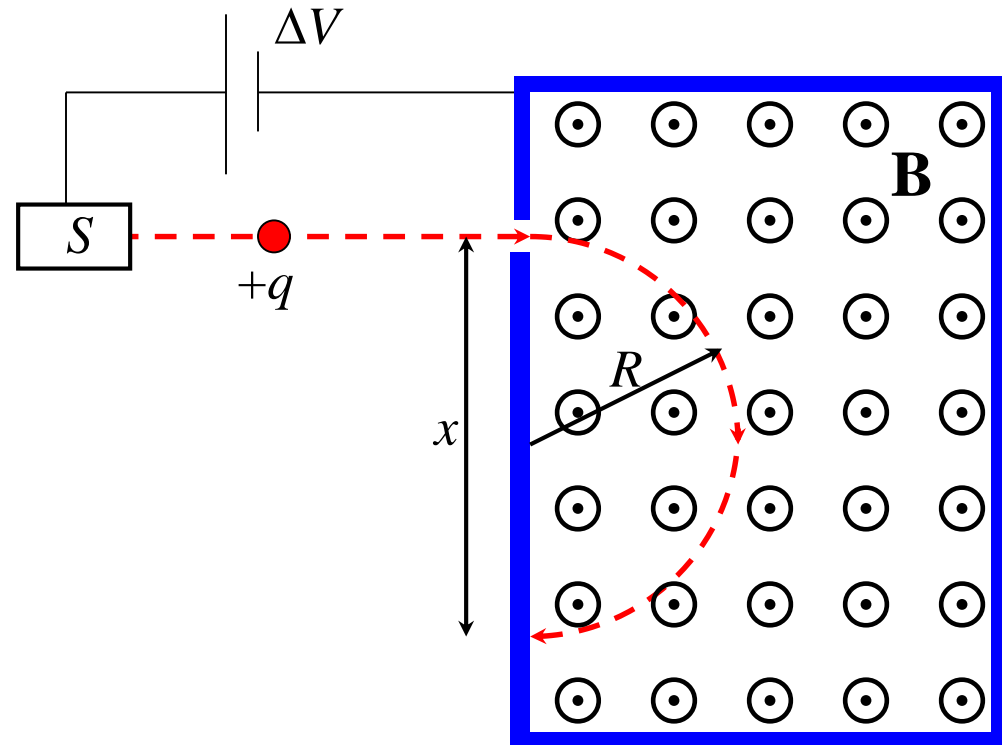
When ions of fixed energy enter a region of constant magnetic field, they follow a circular path.

The radius of the path depends on the mass/charge ratio and speed of the ion, and the magnitude of the magnetic field.

$$x = 2r \quad \text{and} \quad r = \frac{mv}{qB}$$



Example: ions from source  $S$  enter a region of uniform magnetic field  $B$  that is perpendicular to the ion path. The ions follow a semicircle and strike the detector plate at  $x = 1.7558$  m from the point where they entered the field. If the ions have a charge of  $1.6022 \times 10^{-19}$  C, the magnetic field has a magnitude of  $B = 80.0$  mT, and the accelerating potential is  $\Delta V = 1000.0$  V, what is the mass of the ion?

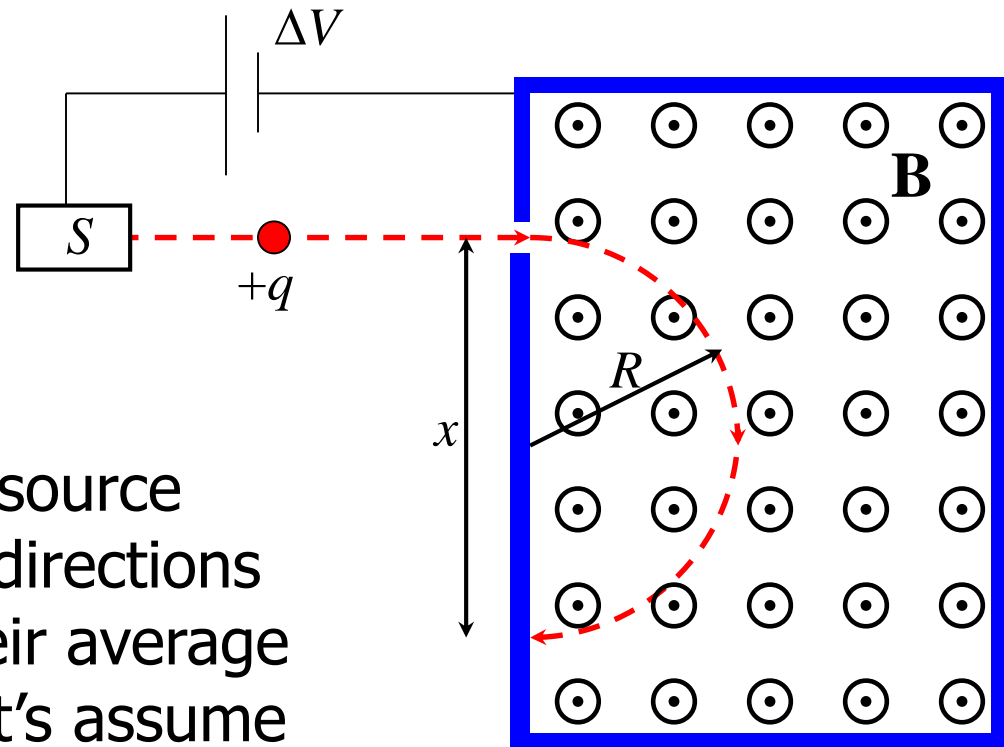




Step 1: find the final speed of the ions if they have a charge of  $1.6022 \times 10^{-19} \text{ C}$  and the accelerating potential is  $\Delta V = 1000.0 \text{ V}$ ?

Let's focus on the part of the diagram where the ions from the source are being accelerated.

If the ions are "boiling" off a source filament, their initial velocity directions will be almost random, so their average speed will be almost zero. Let's assume the ions start at rest (corresponding to zero average speed).



Step 1: find the final speed of the ions if they have a charge of  $1.6022 \times 10^{-19} \text{ C}$  and the accelerating potential is  $\Delta V = 1000.0 \text{ V}$ ?

$$E_f - E_i = [W_{\text{other}}]_{i \rightarrow f}$$

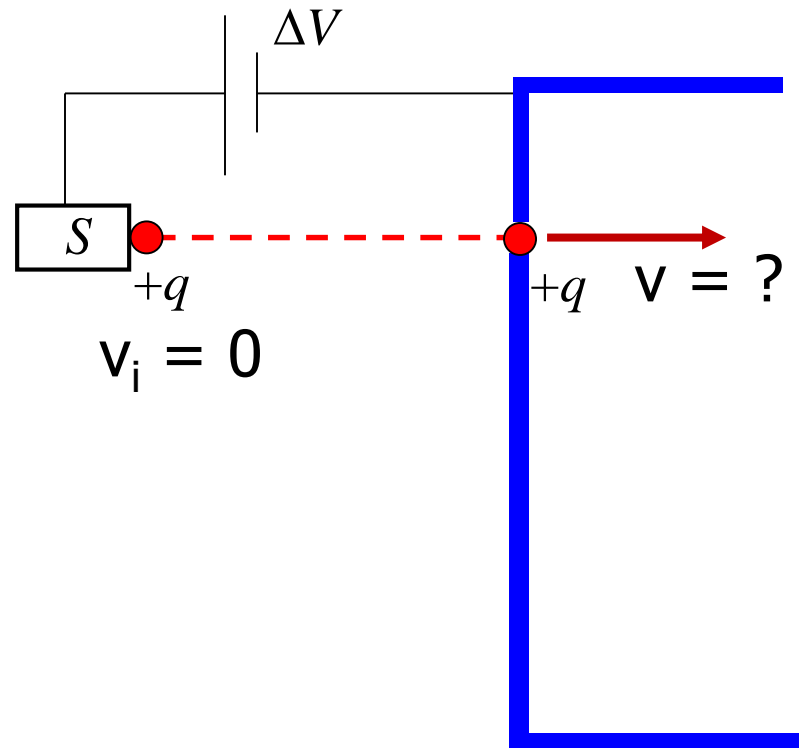
$$K_f + U_f - (\cancel{K_i} + \cancel{U_i}) = [\cancel{W}_{\text{other}}]_{i \rightarrow f}$$

$$K_f = -U_f + U_i = -(U_f - U_i) = -\Delta U$$

$$\frac{1}{2}mv^2 = -\Delta U = -q\Delta V$$

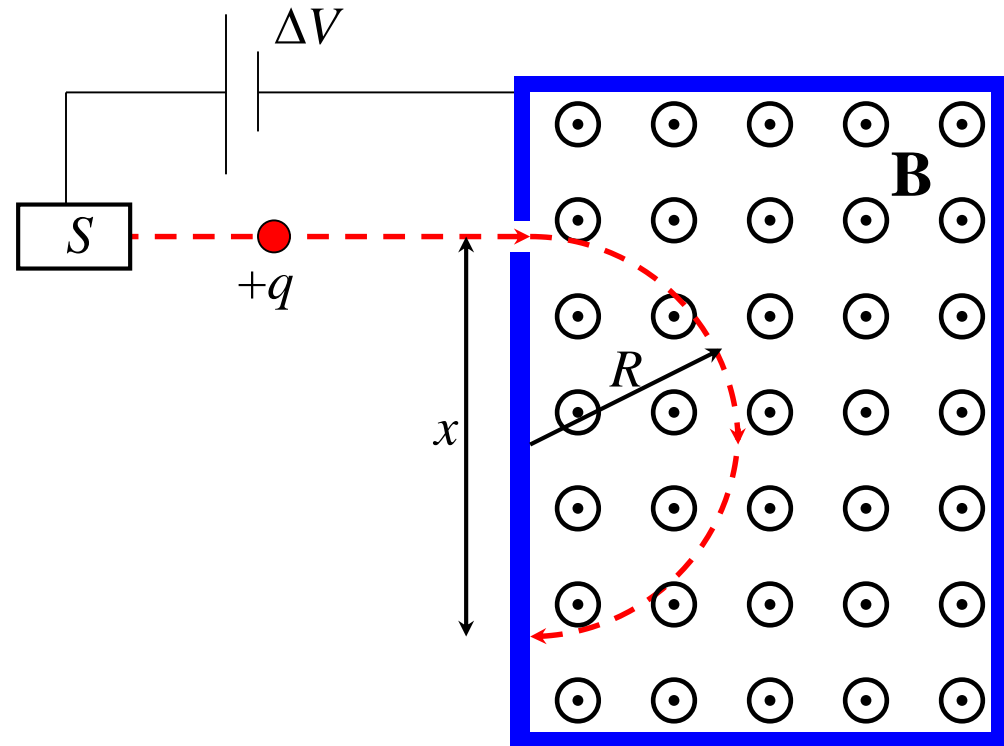
$$v = \sqrt{\frac{-2q\Delta V}{m}}$$

Notes: we don't know  $m$ , so let's keep this symbolic for now. Besides, we want to solve for  $m$  later. Also,  $\Delta V$  must be negative, which makes sense (a  $+$  charge would be accelerated away from a higher potential).



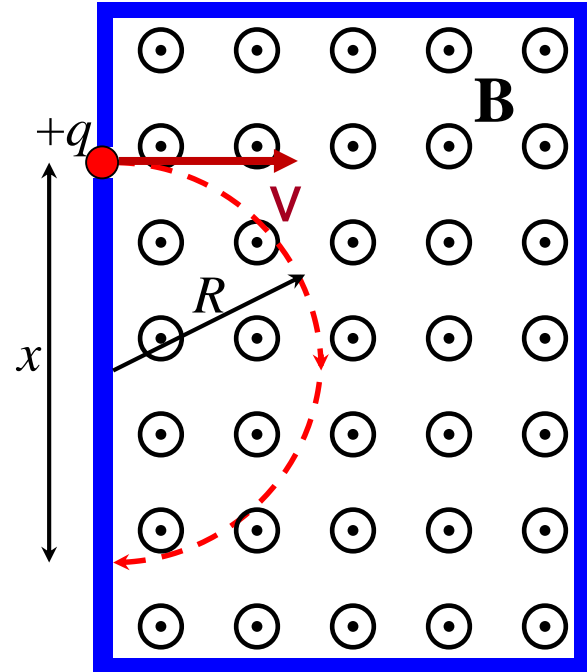
Don't get your  $v$ 's and  $V$ 's mixed up! Use a script  $v$  for speed!

Step 2: find the radius of the circular path followed by the ions in the region of uniform magnetic field of magnitude  $B$ .



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Let's focus on the magnetic field region of the diagram.



Step 2: find the radius of the circular path followed by the ions in the region of uniform magnetic field of magnitude  $B$ .

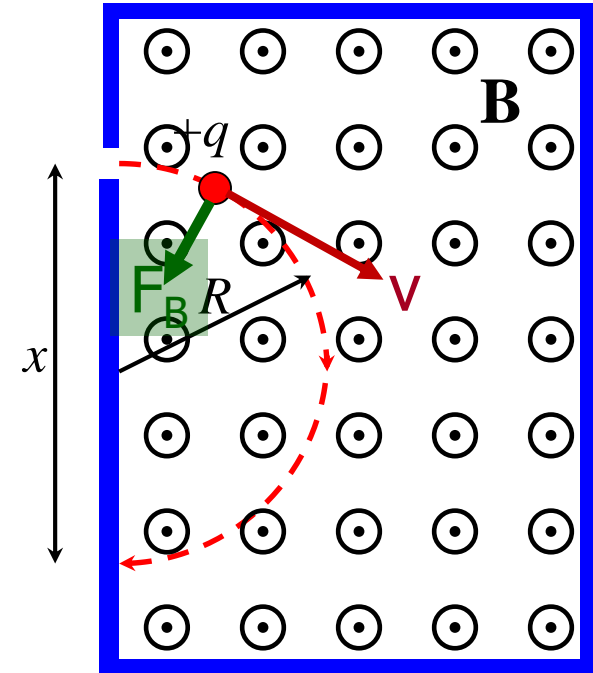
For motion in a circular path...

$$|\vec{F}_B| = |m\vec{a}| = \left| m \frac{v^2}{R} \right|$$

$$|\vec{F}_B| = |q\vec{v} \times \vec{B}| = |qvB \sin 90^\circ|$$

$$|q|vB = m \frac{v^2}{R}$$

$$R = \frac{mv}{|q|B}$$



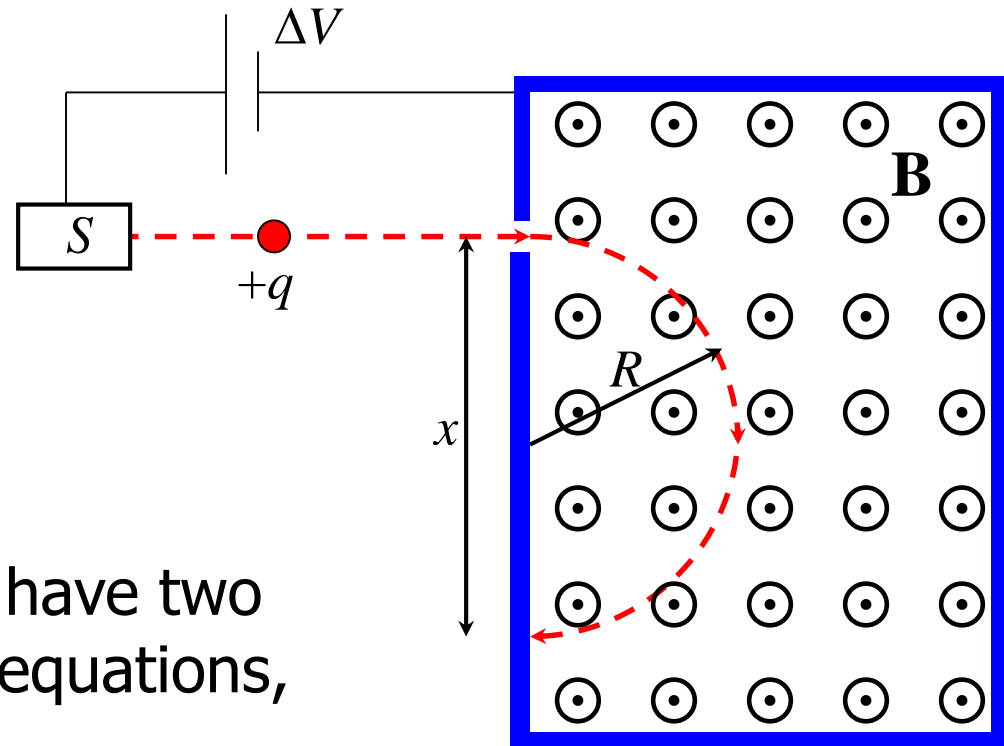
On an exam, we'll probably let you get away without using the absolute value signs, even though it makes our brains hurt when we do that (let you get away with something).

Step 3: solve for  $m$ , using  $q=1.6022 \times 10^{-19}$  C,  $\Delta V=1000.0$  V,  $B = 80.0$  mT, and  $R=x/2=1.7558/2=0.8779$  m.

Now we can put it all together.

$$v = \sqrt{\frac{-2q\Delta V}{m}} \quad R = \frac{mv}{|q|B}$$

We know  $q$ ,  $R$ ,  $B$ , and  $\Delta V$ . We have two unknowns,  $v$  and  $m$ , and two equations, so we can solve for  $m$ .



$$R^2 = \frac{m^2 v^2}{|q|^2 B^2} = \frac{m^2 \left( \frac{-2q\Delta V}{m} \right)}{|q|^2 B^2} = \frac{(-2q\Delta V)m}{|q|^2 B^2} = \frac{(-2\Delta V)m}{|q| B^2}$$

Step 3: solve for  $m$ , using  $q=1.6022 \times 10^{-19}$  C,  $\Delta V=-1000.0$  V,  $B = 80.0$  mT, and  $R=x/2=1.7558/2=0.8779$  m.

$$R^2 = \frac{(-2\Delta V)m}{|q|B^2}$$

$$m = \frac{|q|B^2R^2}{(-2\Delta V)}$$

$$m = \frac{(1.6022 \times 10^{-19})(80 \times 10^{-3})^2 (0.8779)^2}{-(2)(-1000)}$$

$$m = 3.9515 \times 10^{-25} \text{ kg} \quad \text{uranium-238!}$$

1 atomic mass unit (u) equals  $1.66 \times 10^{-27}$  kg, so  $m=238.04$  u.

