Physics 24 Exam 1

February 18, 2014

Exam Total

200 / 200

Printed Name: **Key**

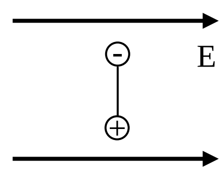
Rec. Sec. Letter: **N/A**

Five multiple choice questions, 8 points each. Choose the **best** or **most nearly correct** answer.

__B__ 1. The total electric flux passing through a Gaussian surface is found to be positive. Which of the following statements must be true?

[A] The total charge outside the surface is positive.
[B] The total charge inside the surface is positive.
[C] The total charge outside the surface is negative.
[D] The total charge inside the surface is negative.

__D__ 2. The figure shows an electric dipole with its dipole moment oriented perpendicular to a uniform electric field. For this orientation the magnitude of the torque on the dipole is ________ and the potential energy of the dipole is ________.



[A] 0, minimum [B] 0, maximum
[C] minimum, 0 [D] maximum, 0.

__D__ 3. A proton is released from rest in a uniform electric field. The proton then moves under the influence of the electric field. Which of the following is true for the proton?

[A] The proton's potential energy increases and it moves toward higher electric potential.
[B] The proton's potential energy decreases and it moves toward higher electric potential.
[C] The proton's potential energy increases and it moves toward lower electric potential.
[D] The proton's potential energy decreases and it moves toward lower electric potential.

__A__ 4. A parallel plate capacitor with capacitance C_0 is connected to a battery of potential V_0 and acquires a charge Q_0. With the battery still connected, the separation between the plates is **decreased** by a factor of 2. What are the new charge on the plates and potential difference between them after this change is made?

[A] $2Q_0$, V_0 [B] $Q_0/2$, V_0 [C] Q_0, $2V_0$ [D] Q_0, $V_0/2$

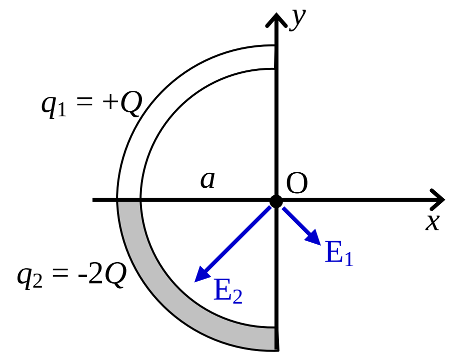
__ABCD__ 5. The Hoverdog® in the picture is negatively charged, and the rings positively charged. If the Hoverdog® is released from rest, it will

[A] Oscillate back and forth between the rings.
[B] Launch to the right at a high speed.
[C] Lick the nearest human's face.
[D] Ask Dr. Parris, he's the Hoverdog® expert.

Note: no Hoverdogs® were harmed in the preparation of this exam.

__40__ /40 for page 1

6. (40 points total) A positive charge Q is uniformly distributed over one half of a semi-circle of radius a. Negative charge of twice that magnitude is uniformly distributed over the bottom half of the semi-circle, as shown in the figure.



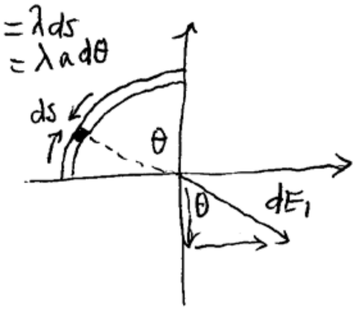
(a) (5 points) Draw on the figure at right the individual electric fields due to the positive and the negatively charged halves of the semi-circle.

(b) (20 points) Calculate the electric field at the origin due to the positively charged half of the semicircle, using the coordinate system indicated. Express your answer **in unit vector notation**, in terms of the parameters Q, a, and Coulomb's constant k.

Let $\vec{E}_1$ be field due to Q_1. Then $dE_1 = \frac{k\,dq}{a^2}$

$$dE_{1x} = +\frac{k\lambda a\,d\theta}{a^2}\sin\theta \qquad dE_{1y} = -\frac{k\lambda_1 a\,d\theta}{a^2}\cos\theta$$

$dq = \lambda ds$
$= \lambda a\,d\theta$



$$E_{1x} = \frac{k\lambda_1}{a}\int_0^{\pi/2}\sin\theta\,d\theta \qquad E_{1y} = -\frac{k\lambda_1}{a}\int_0^{\pi/2}\cos\theta\,d\theta$$

$$= -\frac{k\lambda_1}{a}\cos\theta\Big|_0^{\pi/2} \qquad = -\frac{k\lambda_1}{a}\sin\theta\Big|_0^{\pi/2}$$

$$= -\frac{k\lambda_1}{a}(0-1) \qquad = -\frac{k\lambda_1}{a}(1-0)$$

$$= \frac{k\lambda_1}{a} = \frac{k}{a}\frac{2Q}{\pi a} = \frac{2kQ}{\pi a^2} \qquad = -\frac{k\lambda_1}{a} = -\frac{k}{a}\frac{2Q}{\pi a} = -\frac{2kQ}{\pi a^2}$$

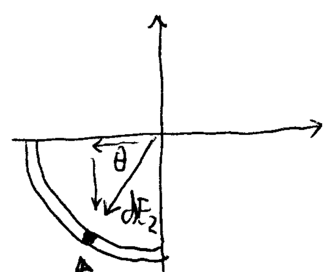
$$\lambda_1 = \frac{+Q}{\frac{1}{4}(2\pi a)} = \frac{2Q}{\pi a}$$

$$\boxed{\vec{E}_1 = \frac{2kQ}{\pi a^2}\hat{\imath} - \frac{2kQ}{\pi a^2}\hat{\jmath}}$$

(c) (15 points) Find the total electric field at the origin, due to the entire semi-circle of charge. Feel free to state and use any appropriate symmetry arguments to simplify your calculation. Express your answer **in unit vector notation**, in terms of the parameters Q, a, and Coulomb's constant k.

$$\lambda_2 = \frac{-2Q}{\frac{1}{4}(2\pi a)} = -\frac{4Q}{\pi a}$$

you may state that by symmetry $E_{2x} = E_{2y}$ and calculate only one of them



$|dq| = |\lambda|a\,d\theta$

following the logic in part a...

$$E_{2x} = -\frac{k|\lambda_2|}{a^2}\int_0^{\pi/2}\cos\theta\,d\theta \qquad E_{2y} = -\frac{k|\lambda_2|}{a}\int_0^{\pi/2}\sin\theta\,d\theta$$

$$= -\frac{k|\lambda_2|}{a^2}\sin\theta\Big|_0^{\pi/2} \qquad = +\frac{k|\lambda_2|}{a}\cos\theta\Big|_0^{\pi/2}$$

$$= -\frac{k|\lambda_2|}{a}(+1) = -\frac{k}{a}\frac{4Q}{\pi a} \qquad = +\frac{k|\lambda_2|}{a}(0-1) = -\frac{k}{a}\frac{4Q}{\pi a}$$

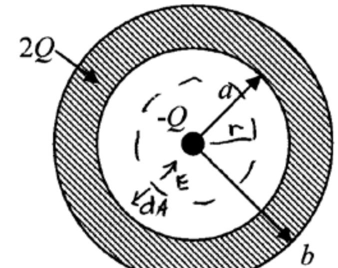
$$\vec{E}_2 = -\frac{4kQ}{\pi a^2}\hat{\imath} - \frac{4kQ}{\pi a^2}\hat{\jmath}$$

$$\boxed{\vec{E} = \vec{E}_1 + \vec{E}_2 = -\frac{2kQ}{\pi a^2}\hat{\imath} - \frac{6kQ}{\pi a^2}\hat{\jmath}}$$

40 /40 for page 2

7. (40 points total) A metal spherical shell of inner radius *a* and outer radius *b* holds a net positive charge of $+2Q$. Inside this spherical shell, held fixed at the center, is a negative point charge $-Q$.

(a) (10 points) Starting from a statement of Gauss's Law, find the magnitude of the electric field for $r<a$.



$\oint \vec{E}\cdot d\vec{A} = q_{encl}/\epsilon_0$ to find magnitude of E take $|q_{encl}|$

$E\,4\pi r^2 = |-Q|/\epsilon_0$

$$\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$$

(b) (5 points) What is the magnitude of the electric field for $a<r<b$?

$\boxed{0}$

(c) (10 points) Starting from a statement of Gauss's Law, find the magnitude of the electric field for $r>b$.

Draw a Gaussian sphere of radius $r>b$. Charge inside is $+2Q-Q=+Q$.
$\vec{E}$ is directed radially out, parallel to $d\vec{A}$.

$\oint \vec{E}\cdot d\vec{A} = q_{encl}/\epsilon_0$

$E(4\pi r^2) = (+2Q-Q)/\epsilon_0$

$$\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$$

(d) (10 points) Find the **surface charge densities** σ_a and σ_b on the inner and outer surfaces of the conductor.

$Q_{conductor} = 2Q = Q_a + Q_b$

$Q_a = -(-Q) = +Q$ because $q_{encl}=0$ for Gaussian sphere of radius $a<r<b$

$Q_b = 2Q - Q_a = 2Q - Q = Q$

$\sigma_a = \frac{Q_a}{4\pi a^2}$ $\quad$ $\sigma_b = \frac{Q_b}{4\pi b^2}$

$\boxed{\sigma_a = \frac{Q}{4\pi a^2}}$ $\quad$ $\boxed{\sigma_b = \frac{Q}{4\pi b^2}}$

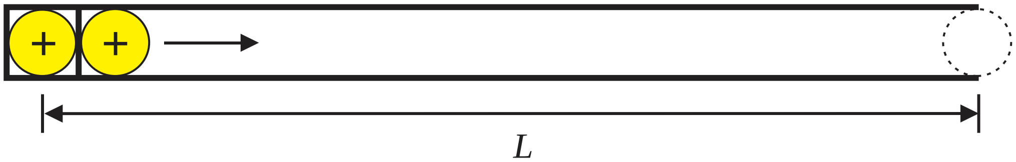
(e) (5 points) What are the directions of $\vec{E}$ for $r<a$ and $r>b$? You can show the directions in a diagram or describe the directions in words.

$r<a$ $\vec{E}$ is radially in $\vec{E}$ •

$r>b$ $\vec{E}$ is radially out $\vec{E}$ ⊙

40 /40 for page 3

8. (40 points total) You are building a Coulomb gun, that is, a device that uses the Coulomb force to fire a bullet. Initially, two identical charged spheres of radius R and mass m are held in place inside an insulating frictionless barrel of length L (where L is defined in the figure). They are separated by a thin insulating barrier and carry identical positive charges. When the trigger of the gun is pulled, the right sphere is released from rest.



(a) (20 points) Calculate the value Q of the spheres' charges such that the bullet (the right sphere) has a speed v when its center reaches the end of the barrel. You can treat the charges as point charges at the centers of their respective spheres.

$$E_f - E_i = [\cancel{W_{other}}]_{i\to f} \xrightarrow{} 0$$

$$K_f + U_f - \cancel{K_i} - U_i = 0$$

$$\frac{1}{2}mv^2 = U_i - U_f = \frac{kQ^2}{2R} - \frac{kQ^2}{L} = kQ^2\left(\frac{1}{2R} - \frac{1}{L}\right)$$

$$Q^2 = \frac{mv^2}{2k\left(\frac{1}{2R} - \frac{1}{L}\right)}$$

$$\boxed{Q = \sqrt{\frac{mv^2}{2k\left(\frac{1}{2R} - \frac{1}{L}\right)}}}$$

alternative, slightly simpler answer: $Q = \sqrt{\dfrac{mv^2R}{k(1-2R/L)}}$

(b) (10 points) Find the work done by the Coulomb force during the acceleration of the bullet inside the barrel.

$$W_{NET} = W_{COULOMB} + \cancel{W_{OTHER}} = \Delta K = K_f - \cancel{K_i} = \frac{1}{2}mv^2$$

$$\boxed{W_{COULOMB} = \frac{1}{2}mv^2}$$ Yes, that simple!

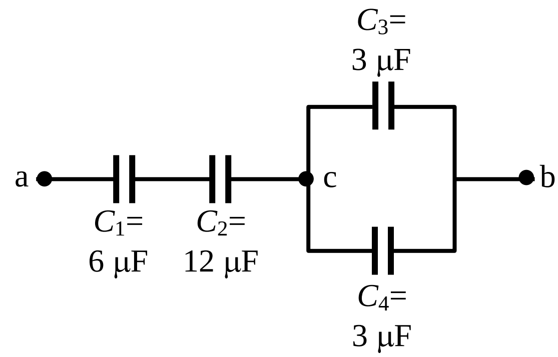
(c) (10 points) Determine the magnitude of the force required to hold the bullet in place before the trigger is pulled.

$$\boxed{F = \frac{kQ^2}{(2R)^2} = \frac{kQ^2}{4R^2}}$$ Yes, that simple!

40 /40 for page 4

9. (40 points total) In the capacitor circuit shown, V_b - V_a = +30 volts. Use starting equations for all calculations.

(a) (15 points) Calculate the total (equivalent) capacitance of this configuration of capacitors.



C_3 and C_4 are in parallel

$C_{34} = C_3 + C_4 = 6\,\mu F$

C_1 and C_2 and C_{34} are in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_{34}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{6} = \frac{5}{12}$$

$$\boxed{C_{eq} = 2.4\,\mu F}$$

(b) (15 points) Calculate the charge stored in each of the 3 microfarad capacitors (C_3 and C_4).

$Q_{eq} = C_{eq}\,\Delta V = (2.4)(30) = 72\,\mu C$

because C_1 and C_2 and C_{34} are in series $Q_{34} = Q_1 = Q_2 = Q_{eq} = 72\,\mu C$

$V_{34} = \frac{Q_{34}}{C_{34}} = \frac{72}{6} = 12\,V = V_3 = V_4$ because C_3 and C_4 are in parallel

$$\boxed{\begin{aligned} Q_3 &= C_3 V_3 = (3)(12) = 36\,\mu C \\ Q_4 &= C_4 V_4 = (3)(12) = 36\,\mu C \end{aligned}}$$

There are other correct ways to solve this part!

(c) (10 points) Calculate the magnitude of the potential difference between points *a* and *c*. Which point is at a higher potential, *a* or *c*?

method 1:

$|V_{ac}| + |V_{cb}| = |V_{ab}|$

$|V_{ac}| = |V_{ab}| - |V_{cb}| = 30 - 12$

$$\boxed{|V_{ac}| = 18\,V}$$

$\boxed{\text{c is at higher potential}}$ because b is at higher potential than a

method 2:

$|V_{ac}| = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} = \frac{72}{6} + \frac{72}{12} = 12 + 6$

$|V_{ac}| = 18V$

there are other valid ways to solve this!

40 /40 for page 5