

Physics 2135 Exam 2 - March 21, 2017

Printed Name: Solution

Rec. Sec. Letter: _____

Exam Total

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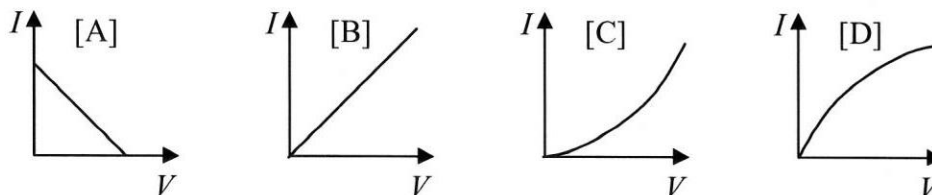
Solutions for problems 6 to 10 must start from official starting equations. Show your work to receive credit for your solution.

Five multiple choice questions, 8 points each. Choose the **best** or **most nearly correct** answer.

C 1. An air-filled parallel plate capacitor of capacitance C contains charge Q . With the capacitor disconnected from the voltage source, how much work must be done to triple the separation of the plates?

- [A] $\frac{Q^2}{3C}$ [B] $\frac{Q^2}{2C}$ [C] $\frac{Q^2}{C}$ [D] $\frac{3Q^2}{C}$

B 2. A resistor with resistance R is known to obey Ohm's law. It is connected to a variable voltage source and the current through the resistor as a function of applied voltage is measured. Which plot (A, B, C, or D) best describes the results of the measurements?



B 3. When an external resistor R is connected to a battery of nonzero internal resistance r , the terminal voltage of the battery is measured to be V . If a second identical external resistor is connected in parallel with the first one, the terminal voltage

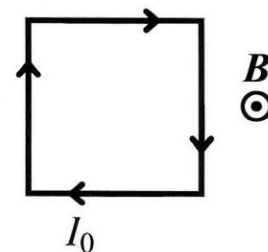
- [A] increases [B] decreases [C] stays the same.

D 4. The timing of your car's turn signal is based on an RC circuit. To achieve the largest blink frequency, you should

- [A] increase the resistance and increase the capacitance
 [B] decrease the resistance but increase the capacitance
 [C] increase the resistance but decrease the capacitance
 [D] decrease the resistance and decrease the capacitance

D 5. A square loop is placed in a uniform magnetic field directed as shown in the diagram. The entire loop is in the magnetic field. There is a current I_0 around the loop as shown. Which is true about the net force and net torque on the loop?

- [A] The net force is zero, but there is a net torque.
 [B] There is a net force, but the net torque is zero.
 [C] There is a net force and a net torque.
 [D] The net force is zero and the net torque is zero.



6. (20 points total) An air-filled parallel plate capacitor has plates of area $3.4 \times 10^{-4} \text{ m}^2$ that are separated by 1 mm.

(a) (10 points) A 12V battery is connected across the plates and the capacitor is allowed to charge fully. What is the energy stored in the capacitor?

$$C = \epsilon_0 \frac{A}{d} = 3.0 \text{ pF}$$
$$U = \frac{1}{2} C V^2 = 217 \text{ pJ}$$

(b) (10 points) The charged capacitor in part (a) is then disconnected from the battery and kept electrically isolated while the region between the plates is filled with an insulating material having dielectric constant $\kappa = 2.16$. What is the total energy stored in the capacitor after the dielectric has been inserted?

$$Q_i = C_i V_i = 36 \text{ pC} \qquad Q_f = Q_i, \quad C_f = \kappa C_i = 6.48 \text{ pF}$$
$$U_f = \frac{Q_f^2}{2C_f} = 100 \text{ pJ}$$

7. (20 points total) An 18-gauge copper wire having a radius of 0.5 mm carries 2.5A of current. The density of free electrons in copper is 8.5×10^{28} per cubic meter, and its resistivity is $1.72 \times 10^{-8} \Omega \text{ m}$.

(a) (10 points) Calculate the drift velocity of electrons in the wire.

$$v_d = \frac{j}{ne} = \frac{I}{\pi r^2} \frac{1}{ne} = 2.3 \times 10^{-4} \frac{\text{m}}{\text{s}}$$

(b) (10 points) Calculate the power dissipated in a 10m long segment of the wire.

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = 0.22 \Omega$$

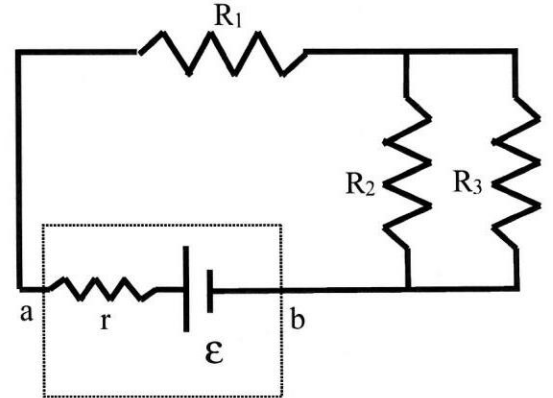
$$P = I^2 R = 1.37 \text{ W}$$

8. (40 points total) For the circuit shown $R_1 = 4.0 \Omega$, $R_2 = 12.0 \Omega$, $R_3 = 6.0 \Omega$, $r = 1.0 \Omega$, and $\mathcal{E} = 18.0 \text{ V}$.

(a) (10 points) Find the equivalent resistance of the circuit (including the internal resistance r of the battery)

$$R_{23}^{-1} = R_2^{-1} + R_3^{-1} \quad R_{23} = 4 \Omega$$

$$R_{eq} = r + R_1 + R_{23} = 9 \Omega$$



(b) (10 points) Find the current through R_1 and the voltage across R_1 .

$$I_1 = I = \frac{\mathcal{E}}{R_{eq}} = 2 \text{ A}$$

$$V_1 = I_1 R_1 = 8 \text{ V}$$

(c) (10 points) Find the current through R_3 and the voltage across R_3 .

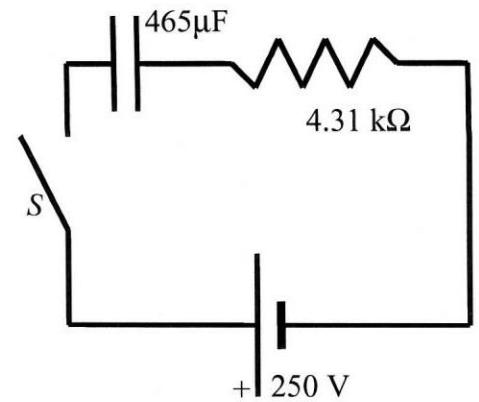
$$V_{23} = I R_{23} = 8 \text{ V} = V_3$$

$$I_3 = \frac{V_3}{R_3} = \frac{4}{3} \text{ A} = 1.333 \text{ A}$$

(d) (10 points) Find the terminal voltage V_{ab} of the battery.

$$V_{ab} = \mathcal{E} - I r = 16 \text{ V}$$

9. (40 points total) An initially uncharged $465 \mu\text{F}$ capacitor is connected in series with a $4.31 \text{ k}\Omega$ resistor, a 250 V battery with negligible internal resistance, and a switch.



(a) (10 points) At an instant after the switch is closed, what is the voltage drop across the resistor and the current in the resistor?

$$V_R(0) = \mathcal{E} - V_C(0) = \mathcal{E} = 250 \text{ V}$$

$$I(0) = \frac{\mathcal{E}}{R} = 58 \text{ mA}$$

(b) (10 points) What is the final voltage drop across the capacitor and final charge on the capacitor?

$$V_C(\infty) = \mathcal{E} - V_R(\infty) = \mathcal{E} = 250 \text{ V}$$

$$Q_f = Q(\infty) = C V_C(\infty) = C \mathcal{E} = 0.116 \text{ C}$$

(c) (10 points) What is the voltage drop across the capacitor 2 s after the switch is closed?

$$Q(t) = Q_f (1 - e^{-t/RC})$$

$$V_C(t) = \mathcal{E} (1 - e^{-t/RC}) = 0.632 \mathcal{E} = 158 \text{ V}$$

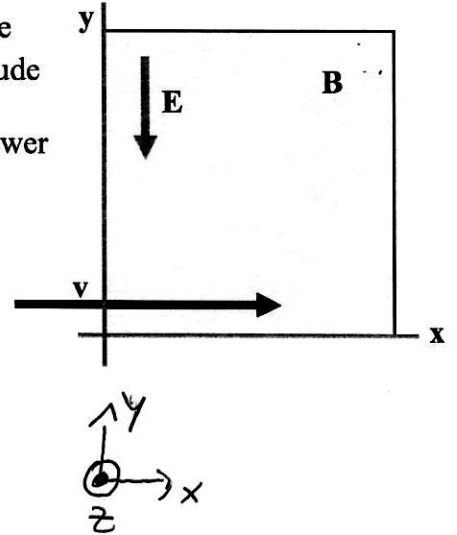
(d) (10 points) What is the current in the circuit 2 s after the switch is closed?

$$I(t) = \frac{d}{dt} Q(t) = \frac{Q_f}{RC} e^{-t/RC} = \frac{\mathcal{E}}{R} e^{-t/RC}$$

$$I(t) = I(0) e^{-t/RC} = 0.368 I(0) = 21.3 \text{ mA}$$

10. (40 points total) A beam of protons enters a region affected by a uniform magnetic field. The protons have an initial velocity $\vec{v} = v\hat{i}$ as shown in the diagrams below.

(a) (20 points) A uniform electric field $\vec{E} = -E\hat{j}$ is applied throughout the region. Find a symbolic expression for the magnetic field vector (magnitude and direction) of minimum magnitude that will allow the protons to pass through the region undeflected. Start from valid OSEs and leave your answer in terms of system parameters and any necessary fundamental constants.



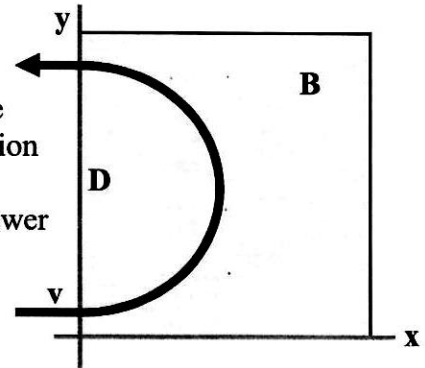
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{E} = -\vec{v} \times \vec{B}$$

$$|\vec{B}| = \frac{E}{v} \quad \vec{B} \otimes$$

$$\vec{B} = -\frac{E}{v} \hat{k}$$

(b) (20 points) The electric field is turned off and the protons follow a semicircular path, exiting the magnetic field some distance D directly above the point at which they entered. Starting from valid OSEs, find a symbolic expression for D in terms of system parameters and any necessary fundamental constants. You may use the magnetic field magnitude B as a system parameter in your answer to this question.



$$\vec{F} = q\vec{v} \times \vec{B} = m\vec{a}$$

$$qvB = m\frac{v^2}{r} = \frac{2mv^2}{D}$$

$$D = \frac{2mv^2}{qvB} = \frac{2mv}{eB} \quad \left(\text{or } \frac{2mv^2}{eE} \right)$$