## Physics 24 Exam 3

April 22, 2014

## Exam Total

Printed Name: $\qquad$
Key

Rec. Sec. Letter: $\qquad$ 200 / 200 N/A

Five multiple choice questions, 8 points each. Choose the best or most nearly correct answer.
$\qquad$ 1. Two long straight wires are parallel and a distance $D$ apart. Both wires carry constant current $I$ in opposite directions, as shown in the diagram. What is the magnitude of the magnetic field at point $P$, located a distance $D$ to the right of the right-hand wire?

[A] 0
[B] $\frac{\mu_{0} I}{4 \pi D}$
[C] $\frac{\mu_{0} I}{2 \pi D}$
[D] $\frac{\mu_{0} I}{\pi D}$

A 2. A long solenoid of $n$ turns per meter and radius $R$ carries a current $I$. If the radius $R$ is increased to $2 R$ while keeping $I$ constant, then the magnitude of the magnetic field at the center of the solenoid
[A] remains unchanged
[B] increases by a factor of 2
[C] decreases by a factor of 2
[D] decreases by a factor of 4 .

B 3. A long straight wire carries a constant current $I_{0}$. A conducting rectangular loop is pushed away from the wire as shown. The induced current in the loop is
[A] )
[B] $)$
[C] ©
[D] $\otimes$


D 4. The average radiation pressure on a totally absorbing solar panel is $P_{\text {rad }}$ when it is a distance $R$ from a source. What is the radiation pressure when the panel is a distance $2 R$ from the source? Assume the source radiates in all directions.
[A] $4 P_{\text {rad }}$
[B] $2 P_{\text {rad }}$
[C] $P_{\mathrm{rad}} / 2$
[D] $P_{\mathrm{rad}} / 4$

AB 5. True or false? Dogs prefer to align themselves along a magnetic north-south axis when barking.
[A] True
[B] False

You might want to read this:
http://www.frontiersinzoology.com/content/pdf/1742-9994-10-80.pdf

6. (40 points total) A long fixed wire carries a current of magnitude $I$ as shown. A distance $D$ below the wire is a conducting bar of length $L$ and mass $m$ which rests on conductive supports and carries the same current I. Both conductors are aligned parallel to the surface of the earth. If there is a large enough upward magnetic force on the bar, contact between the bar and its supports will
 be broken.
(a) (5 points) Which way does the current need to flow through the bar for this apparatus to function as a circuit breaker for the lower wire?

Circle one: $\qquad$

(b) (20 points) Find an expression for the magnetic force on the bar in terms of $I, L, D$, and any necessary fundamental constants.
Magnetic field of upper wire at position of bar:

$$
\vec{B}=\frac{\mu I}{2 \pi D}(-\hat{k})
$$



Magnetic force on bar:

$$
\begin{aligned}
& \overrightarrow{\vec{B}}=I \vec{L} \times \vec{B} \\
& \overrightarrow{F_{B}}=I L(+\hat{\imath}) \times \frac{\mu_{0} I}{2 \pi D}(-\hat{k}) \\
& \overrightarrow{F_{B}}=\frac{\mu_{0} I^{2} L}{2 \pi D} \hat{\jmath}
\end{aligned}
$$

(c) (15 points) If the bar is 0.100 m below the upper wire, 0.500 m long and has a mass of 0.500 kg , determine the current $I$ which will break the contact.

$$
\begin{aligned}
& W_{b a r}=m g=F_{B} \\
& m g=\frac{\mu_{0} I^{2} L}{2 \pi D} \\
& I=\sqrt{\frac{2 \pi m g D}{\mu_{0} L}}=\sqrt{\frac{2 \pi(0.5)(9.8)(0.1)}{\left(4 \pi \times 10^{-7}\right)(0.5)}} \\
& I=2.21 \times 10^{3} \mathrm{~A}
\end{aligned}
$$

7. (40 points total) Two parallel wires carry currents $I_{\mathrm{a}}$ and $I_{\mathrm{b}}$ from left to right. The wires form semicircles with radii $R_{\mathrm{a}}$ and $R_{\mathrm{b}}$ and center $P$ as shown in the figure. (The small distance between the parallel wires can be neglected.)
(a) (15 points) Calculate the magnetic field $\vec{B}_{a}$ that the current $I_{\mathrm{a}}$ produces at point $P$. Express your answer in unit vector notation. You must consider all three segments of wire a.

$$
\begin{aligned}
& \vec{B}_{a}=\vec{B}_{1}+\vec{B}_{2}+\vec{B}_{3} \quad \vec{B}_{1}=\vec{B}_{3}=0 \text { because ts } \times \hat{r}=0 \\
& \overrightarrow{d B}_{2}=\frac{\mu_{0} I_{a}}{4 \pi R_{a}^{2}} d \vec{s} \times \hat{r}=\frac{\mu_{0} I_{a}}{4 \pi R_{a}^{2}} d s(-\hat{k})=\frac{\mu_{0} I_{a}}{4 \pi R_{a}^{2}} R_{a} d \phi(-\hat{k}) \\
& \vec{B}_{2}=\frac{\mu_{0} I_{a}}{4 \pi R_{a}} \int_{0}^{\pi} d \phi(-\hat{k})=-\frac{\mu_{0} I_{a}}{4 R_{a}} \hat{k}=\vec{B}_{a}
\end{aligned}
$$


(b) (15 points) Calculate the magnetic field $\vec{B}_{b}$ that the current $I_{\mathrm{b}}$ produces at point $P$. Express your answer in unit vector notation. You must consider all three segments of wire $b$.

$$
\begin{aligned}
& \vec{B}_{4}=\vec{B}_{6}=0 \text { because d } \overrightarrow{5} \times \hat{r}=0 \\
& {\overrightarrow{B B_{5}}}_{5}=\frac{\mu}{4 \pi I_{b}} d \phi(+\hat{k}) \quad \text { (same steps as in part) } \\
& \vec{B}_{5}=\frac{\mu_{b} I_{b}}{4 \pi R_{b}} \int_{0}^{\pi} d \phi(+\hat{k})=+\frac{\mu a I_{b}}{4 R_{b}} \hat{k}=\vec{B}_{b}
\end{aligned}
$$

(c) (5 points) Find the total magnetic field $\vec{B}$ at point $P$. Express your answer in unit vector notation.

$$
\vec{B}=\vec{B}_{a}+\vec{B}_{b}=-\frac{\mu_{a} I_{a}}{4 R_{a}} \hat{k}+\frac{\mu_{0} I_{b}}{4 R_{b}} \hat{k}=\frac{\mu_{0}}{4}\left(\frac{I_{b}}{R_{b}}-\frac{I_{a}}{R_{a}}\right) \hat{k}
$$

(d) (5 points) At what value of the ratio $I_{a} / I_{b}$, if any, does the total magnetic field $\vec{B}$ vanish?

$$
\begin{aligned}
& B=0 \Rightarrow \frac{I_{b}}{R_{b}}-\frac{I_{a}}{R_{a}}=0 \Rightarrow \frac{I_{b}}{R_{b}}=\frac{I_{a}}{R_{a}} \\
& I_{a} \\
& \frac{I_{b}}{I_{b}}=\frac{R_{a}}{R_{b}}
\end{aligned}
$$

8. (30 points total) A metal bar is being pulled with constant speed $v$ along two parallel horizontal frictionless metal tracks separated by a distance $L$, as shown in the diagram. Throughout the region of the tracks there is a constant magnetic field of magnitude $B$, directed out of the page. A light bulb of resistance $R$ is attached across the two tracks as shown.
(a) (10 points) The direction of the induced current in the apparatus is:

(b) (20 points) Derive an algebraic expression for the speed $v$ with which the bar must be pulled to generate power $P_{0}$ in the light bulb. Start the problem only with equations from the equation sheet. Express your answer in terms of the parameters $B, L, R$, and $P_{0}$.

$$
\begin{aligned}
|\varepsilon| & =\left|-N \frac{d \bar{q}_{B}}{d t}\right|=\left|\frac{d(B A)}{d t}\right|=B\left|\frac{d t}{d t}\right| \\
& =B\left|\frac{d(L)}{d t}\right|=B C\left|\frac{d x}{d t}\right|=B L \\
P_{0} & =\frac{\varepsilon^{2}}{R}=\frac{B^{2} L^{2} N^{2}}{R} \\
v^{2} & =\frac{P_{0} R}{B^{2} L^{2}} \\
v & =\sqrt{\frac{P_{0} R}{B^{2} L^{2}}} \text { or } \frac{\sqrt{G B^{R}}}{B L}
\end{aligned}
$$


(c) (10 points) Assume the bar is welded to the tracks so that it no longer moves. Instead, the magnetic field is decreased at a constant rate. During the time the magnetic field is decreasing, the direction of the current in the apparatus is:
circle one:

9. (40 points total) The power delivered into the cylindrical beam of a helium-neon laser of the type often found in physics laboratories is 10.0 mW . The light produced by the laser has a wavelength of 633 nm . Using a lens, the cylindrical beam emerging from the laser is focused onto a circular spot of a radius $r=800 \mathrm{~nm}$ on the surface of a black absorbing sheet of plastic whose surface is oriented at right angles to the beam. The plastic sheet has a cross sectional area much larger than that of the beam. All solutions must start with an Official Starting Equation, and numerical answers must include units.
(a)(8 points) Calculate the frequency $f$ of the light produced by this laser.

OLE: $C=f \lambda \quad f=\frac{c}{\lambda}=\frac{3 \times 10^{8}}{633 \times 10^{9}}=4.74 \times 10^{14} \mathrm{~Hz}$
(b) (8 points) Determine the intensity $I$ of the laser light at the point where it is focused onto the plastic sheet.
OLE: $I=\frac{P}{\text { Area }} \quad I=\frac{P}{\pi r^{2}}=\frac{10 \times 10^{-3}}{\pi\left(800 \times 10^{-9}\right)^{2}}=4.97 \times 10^{9} \mathrm{~W} / \mathrm{m}^{2}$
(c) (8 points) Determine the electric field amplitude of the laser light at the point where it is focused

ODE: $I=\frac{1}{2} c \epsilon_{0} E_{\text {max }}^{2} \quad E_{\max }=\sqrt{\frac{2 I}{C \epsilon_{0}}}=\sqrt{\frac{2\left(4.97 \times 10^{9}\right)}{\left(3 \times 10^{8}\right)\left(8.85 \times 10^{-12}\right)}}$
$I=\frac{1}{2} \frac{E_{\text {max }}^{2}}{\mu_{0} c}$

$$
E_{\text {max }}=1.94 \times 10^{6} \mathrm{~V} / \mathrm{m}
$$

(d) (8 points) What is the average total energy-per-unit-volume in the laser beam at the point where it is focused onto the plastic sheet?

ODE:

$$
\langle u\rangle=\frac{1}{2} G E_{\max }^{2}
$$

$$
\langle u\rangle=\frac{1}{2}\left(8.85 \times 10^{-12}\right)\left(1.94 \times 10^{6}\right)^{2}=6.6 \mathrm{~J} / \mathrm{m}^{3}
$$

$$
\begin{gathered}
\text { or } \\
I=c\langle u\rangle
\end{gathered}
$$

$$
\langle u\rangle=\frac{I}{c}=\frac{4.97 \times 10^{9}}{3 \times 10^{8}}=16.6 \mathrm{~J} / \mathrm{m}^{3}
$$

is also OK
(e) (8 points) What average force $F$ is exerted on the plastic sheet by the laser light focused on it?

OLE: $\left\langle P_{\text {rad }}\right\rangle=\frac{I}{c} \quad\left\langle P_{\text {rad }}\right\rangle=\frac{4.74 \times 10^{9}}{3 \times 10^{8}}=16.6 \mathrm{~N} / \mathrm{m}^{2}$
Pressure $=F / A$ canes
from physics 23

$$
F=\left\langle\text { I rad }^{3 \times 10^{8}}\right\rangle \cdot \text { Area }=(16.6) \pi\left(800 \times 10^{-9}\right)^{2}=3.33 \times 10^{-11} \mathrm{~N}
$$

This also works:
$F=\frac{I}{C} \cdot A_{\text {lea }}=\frac{P / \text { Area }}{C} \cdot$ Area $=\frac{P}{C}=\frac{10 \times 10^{-3}}{3 \times 108}=3.33 \times 10^{-11} \mathrm{~N}$

