Physics 2135 Final Exam

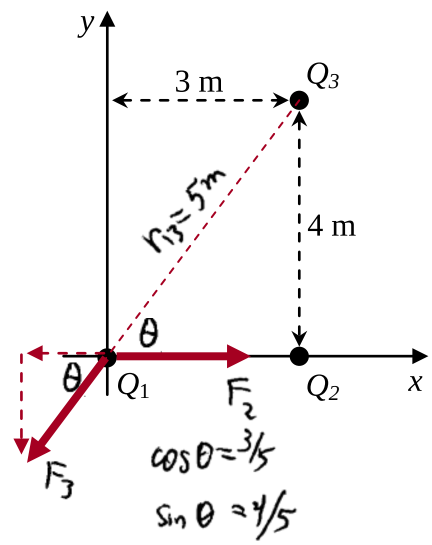
December 18, 2014

Exam Total
200 / 200

Printed Name: Key

Rec. Sec. Letter: N/A

1. (40 points total) Three point charges Q_1 = +5 C, Q_2 = -9 C, Q_3 = +25 C are located at the positions shown in the diagram. **Express all of your answers in unit vector notation.**



(a) (15 points) Find the Coulomb force on Q_1 due to Q_2.

$$\vec{F}_2 = F_2\hat{i} = \frac{k|Q_1Q_2|}{r_{12}^2}\hat{i}$$

$$\vec{F}_2 = 9\times10^9 \frac{|(+5)(-9)|}{3^2}\hat{i}$$

$$\boxed{\vec{F}_2 = 4.5\times10^{10}\,N\,\hat{i}}$$

(b) (15 points) Find the Coulomb force on Q_1 due to Q_3.

$$\vec{F}_3 = -F_3\cos\theta\,\hat{i} - F_3\sin\theta\,\hat{j} = -k\frac{|Q_1Q_3|}{r_{13}^2}\frac{3}{5}\hat{i} - k\frac{|Q_1Q_3|}{r_{13}^2}\frac{4}{5}\hat{j}$$

$$\vec{F}_3 = -9\times10^9\frac{(5\cdot25)}{5^2}\frac{3}{5}\hat{i} - 9\times10^9\left(\frac{5\cdot25}{5^2}\right)\frac{4}{5}\hat{j}$$

$$\boxed{\vec{F}_3 = -2.7\times10^{10}\,N\,\hat{i} - 3.6\times10^{10}\,N\,\hat{j}}$$

(c) (10 points) Find the net force on Q_1 due to the other two point charges.

$$\vec{F} = \vec{F}_2 + \vec{F}_3 = 4.5\times10^{10}\,N\,\hat{i} - 2.7\times10^{10}\,N\,\hat{i} - 3.6\times10^{10}\,N\,\hat{j}$$

$$\boxed{\vec{F} = 1.8\times10^{10}\,N\,\hat{i} - 3.6\times10^{10}\,N\,\hat{j}}$$

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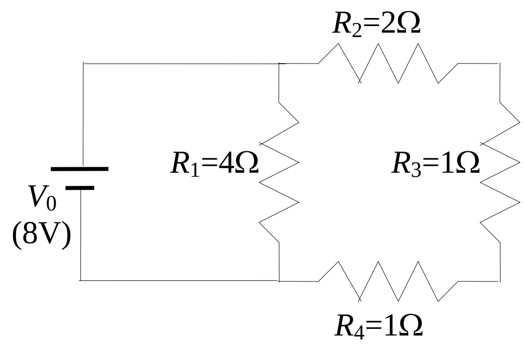


2. (20 points total) For the resistor circuit shown, $R_1 = 4\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 1\ \Omega$, $R_4 = 1\ \Omega$, and $V_0 = 8$ V.

(a) (10 points) Find the equivalent resistance of this circuit.

$R_{234} = R_2 + R_3 + R_4 = 2+1+1 = 4\Omega$

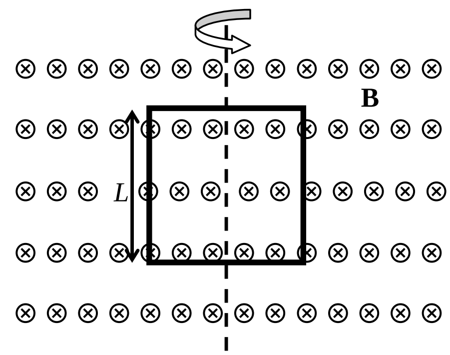
$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_{234}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}$ $\boxed{R_{eq} = 2\ \Omega}$

(b) (10 points) Determine the power dissipated by resistor R_2.

$V_{234} = V_0 \quad I_{234} = \frac{V_{234}}{R_{234}} = \frac{8}{4} = 2A = I_2$

because R_2 is in series with R_3 & R_4

$P_2 = I_2^2 R_2 = 2^2(2) = \boxed{8\ W}$



3. (20 points total) A square conducting coil with sides of length L contains N turns of wire. The coil is rotated about a vertical axis through its center (see figure) in a region of uniform magnetic field B directed into the plane of the paper. The coil is initially ($t = 0$) oriented so that its plane coincides with that of the paper. At a **short** time t later, the coil has rotated through a **small** angle $\theta = \omega t$ from its initial orientation, and the emf in the coil is ε.

(a) (10 points) What best describes the direction of the induced current in the coil at time t?

CLOCKWISE COUNTERCLOCKWISE (circle one)

(b) (10 points) Derive the expression for the angular frequency ω of the coil in terms of L, N, B, t and ε. Begin with a starting equation, and show all the steps leading to your answer. Use the small-angle approximation $\sin\theta \approx \theta$ in your calculation.

$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\frac{d\vec{B}\cdot\vec{A}}{dt} = -N\frac{d BA\cos\omega t}{dt} = -NBA\frac{d\cos\omega t}{dt} = +NBA\omega\sin\omega t$

$A = L^2$ and $\sin\omega t \approx \omega t$ for small angles

$\Rightarrow \mathcal{E} = NBL^2\omega^2 t$

$\boxed{\omega = \sqrt{\frac{\mathcal{E}}{NBL^2 t}}}$

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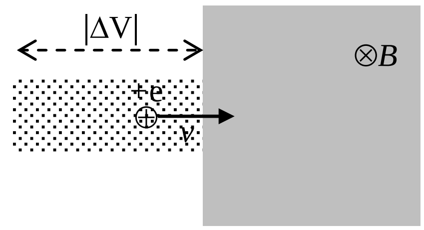
4. (40 points total) A mass spectrometer as seen in the figure applies a potential difference of 2.00 kV to accelerate a singly charged ion with a charge of +e and mass $M = 5.95\text{x}10^{-25}$ kg. After the acceleration, the ion enters a perpendicular magnetic field region where a uniform magnetic field of B = 0.400 T causes the ion to follow a circular path.



(a) (15 points) Calculate the speed v of the ion when it leaves the region of potential difference where it was accelerated.

$$E_f - E_i = \cancel{[W_{other}]_{i\to f}}^{0} \Rightarrow E_f = E_i$$

$$K_f + U_f = \cancel{K_i}^{0} + U_i$$

$$K_f = \tfrac{1}{2}mv^2 = -U_f + U_i = -\Delta U = -q\Delta V = -e\Delta V$$

$$v = \sqrt{\frac{-2e\Delta V}{m}} = \sqrt{\frac{-2(1.6\times10^{-19})(-2000)}{5.95\times10^{-25}}}$$

note: $\Delta V < 0$ to accelerate the positive charge

$$\boxed{v = 3.28\times10^{4}\ m/s}$$

(b) (20 points) Calculate the radius of the circular path the ion follows while it moves in the magnetic field region.

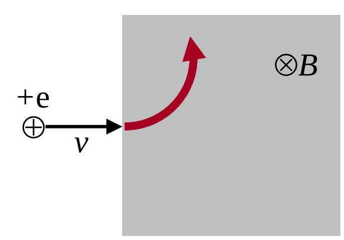
$$\vec{F}_B = q\vec{v}\times\vec{B}$$

$$F_B = qvB = ma = m\frac{v^2}{R}$$

$$qB = \frac{mv}{R}$$

$$R = \frac{mv}{qB} = \frac{(5.95\times10^{-25})(3.28\times10^{4})}{(1.6\times10^{-19})(0.4)} = \boxed{0.305\,m}$$

(c) (5 points) Suppose the magnetic field exists everywhere in the shaded region in the figure and is directed into the page. Draw in the figure the path the ion follows. Support your answer with a ***brief*** explanation or physical relation.



$\vec{F} = q\vec{v}\times\vec{B} \Rightarrow$ initial force is ↑

$\Rightarrow$ ion follows ↺ path

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5. (40 points total) A diverging lens has a focal length of ***magnitude*** 20.0 cm. The lens forms an image which is one fourth as tall as the object. The image appears on the same side of the lens as the object.

(a) (5 points) Is the image REAL or VIRTUAL (circle one)? image is on incoming light side

(b) (5 points) Is the image UPRIGHT or INVERTED (circle one)? $s' < 0 \Rightarrow m = -\frac{s'}{s} > 0 \Rightarrow$ upright

(c) (10 points) How far from the lens is the object located?

$$m = -\frac{s'}{s} = +\frac{1}{4} \Rightarrow s' = -\frac{s}{4} \quad \text{and} \quad f = -20\text{ cm (diverging)}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{s} - \frac{1}{s/4} = \frac{1}{s} - \frac{4}{s} = -\frac{3}{s} = \frac{1}{f}$$

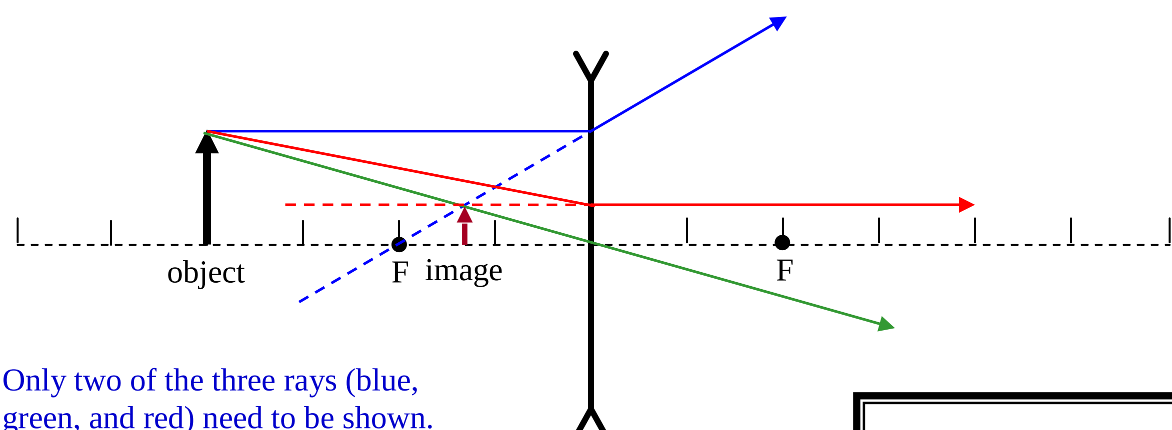
$$s = -3f = -3(-20) = \boxed{60\text{ cm}}$$

(d) (10 points) Determine the image distance s'.

$$s' = -\frac{s}{4} = -\frac{60}{4}$$

$$\boxed{s' = -15\text{cm}}$$

(e) (10 points) Suppose the object is placed 40 cm from the lens. For this object location, draw a ray diagram on the figure provided below, showing both the object and image positions. Adjacent marks on the principal axis are separated by 10.0 cm. You need show only two rays.



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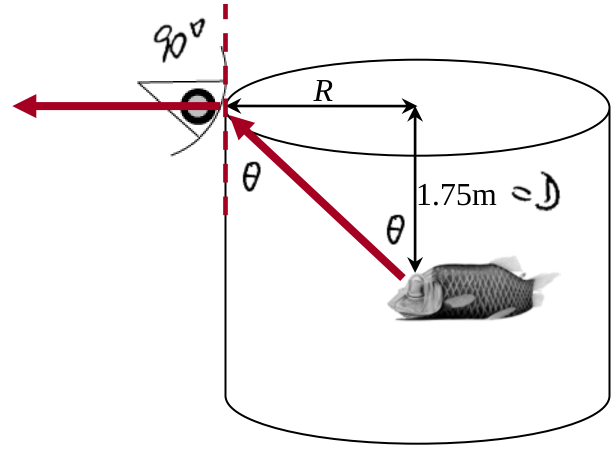
6. (20 points) You observe your pet fish swimming at the center of a cylindrical tank full of fresh water (n=1.33). When you put your eye at the top rim of the tank you can't see the fish until it swims to a depth of 1.75 m. Below that depth you are able to see the fish. What is the radius *R* of the tank?



$n_w \sin\theta = n_{air} \sin 90° = 1$

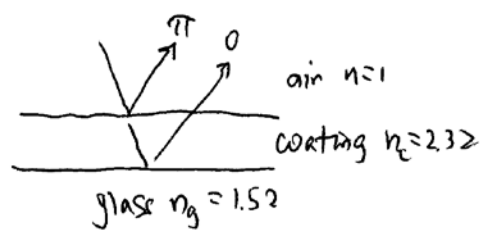
$\sin\theta = \frac{1}{1.33} \Rightarrow \theta = 48.75°$

$\tan\theta = \frac{R}{D}$

$R = D \tan\theta = 1.75 \tan 48.75°$

$\boxed{R = 2m}$

7. (20 points) When light shines into a crocodile's eye, the reflected light appears as a red glow, referred to as eyeshine. You are tasked with designing glass eyes that mimic this effect. To do so, you plan to apply a thin coating of zinc sulfide (n_c=2.32) to crown glass (n_g=1.52) in order to strongly reflect light of wavelength 633 nm in air. What is the minimum thickness that will work?



want constructive interference for $\lambda = 633$ nm in air

$2t = (m + \frac{1}{2}) \frac{\lambda}{n_c}$

m=0 for minimum t

$2t = \frac{\lambda}{2n_c}$

$t = \frac{\lambda}{4n_c} = \frac{633}{4(2.32)}$

$\boxed{t = 68.2 nm}$

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