Physics 24 Final Exam

May 13, 2014

Exam Total
200 / 200

Printed Name: **Key**

Rec. Sec. Letter: **N/A**

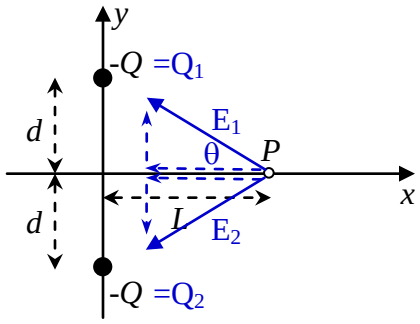
1 (40 points total) Two identical negative point charges $-Q$ are arranged on the y-axis at positions $y = d$ and $y = -d$, respectively.

(a) (15 points) Find the electric field produced by these two charges at point P on the positive x-axis at a distance L from the origin. Express your answer in unit vector notation.

$E_y = 0$ and $E_{1x} = E_{2x}$ by symmetry

$$E_{1x} = -E_1\cos\theta = -\frac{k|-Q|}{\left(\sqrt{L^2+d^2}\right)^2}\frac{L}{\sqrt{L^2+d^2}} = -\frac{kQL}{(L^2+d^2)^{3/2}}$$

$$\vec{E} = -\frac{2kQL}{(L^2+d^2)^{3/2}}\hat{\imath}$$



(b) (10 points) Find the electric potential produced by the two charges at point P.

$$V = V_1 + V_2 = \frac{k(-Q)}{\sqrt{L^2+d^2}} + \frac{k(-Q)}{\sqrt{L^2+d^2}} = -\frac{2kQ}{\sqrt{L^2+d^2}}$$

(c) (5 points) A particle of mass m and negative charge $-q_0$ is now released from rest at point P. Calculate its initial acceleration due to the two other charges. Express your answer in unit vector notation.

$$\vec{F} = m\vec{a} = q\vec{E} \qquad \vec{a} = \frac{q\vec{E}}{m} = \frac{(-q_0)}{m}\left(\frac{-2kQL}{(L^2+d^2)^{3/2}}\hat{\imath}\right) = \frac{2kq_0QL}{m(L^2+d^2)^{3/2}}\hat{\imath}$$

(d) (10 points) Find the speed this particle reaches at $x = \infty$.

$$E_f - E_i = \cancel{[W_{other}]_{i\to f}}^{\,0} \Rightarrow K_f + U_f - \cancel{K_i}^{\,0} - U_i = 0$$

$$K_f = -U_f + U_i = -\Delta U = -q\Delta V = -q(\cancel{V_f}^{\,0} - V_i)$$

$$\frac{1}{2}mv^2 = -q_0\left(-\frac{2kQ}{\sqrt{L^2+d^2}}\right) \Rightarrow v = \sqrt{\frac{4kq_0Q}{m\sqrt{L^2+d^2}}}$$

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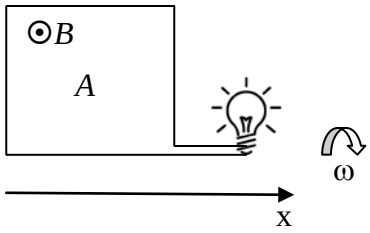
2. (20 points total) The diagram to the right shows a conducting loop of area A in a uniform magnetic field of magnitude B pointing out of the page. The loop is connected to a light bulb. The circuit acts as a generator as it rotates about the x-axis with angular frequency ω.



(a) (5 points) What is the magnetic flux $\Phi_B(t)$ through the loop as a function of time t? Express your answer in terms of B, A, ω, and t.

$\Phi_B = \vec{B}\cdot\vec{A} = \boxed{BA\cos\omega t}$ it is ok to begin with $\Phi_B = \int \vec{B}\cdot d\vec{A}$

(b) (10 points) Using a starting equation, derive an expression for the emf $\varepsilon(t)$ generated in the loop as a function of time t. Express your answer in terms of B, A, ω, and t.

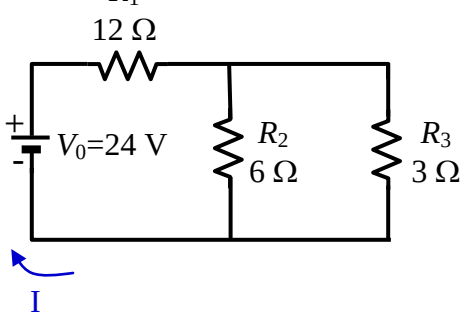
$\varepsilon = -N\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA\cos\omega t) = \boxed{BA\omega\sin\omega t}$

(c) (5 points) At what frequency f_{bulb} does the light bulb emit flashes of light?

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$ bulb will flash twice per cycle (when $\omega = \pi/2$ and $\omega = 3\pi/2$) $\Rightarrow \boxed{f_{bulb} = 2f = \frac{\omega}{\pi}}$

3. (20 points total) In the circuit on the right, R_1 = 12 Ω, R_2 = 6 Ω, R_3 = 3 Ω, and V_0 = 24V. How much power is dissipated in resistor R_1?



$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6} + \frac{1}{3} = \frac{3}{6} = \frac{1}{2} \Rightarrow R_{23} = 2\Omega$

$R_{eq} = R_1 + R_{23} = 14\Omega$

$V_0 = IR_{eq} \Rightarrow I = \frac{V_0}{R_{eq}} = \frac{24}{14} = \frac{12}{7}$

All of I flows through R_1, so

$P_1 = I^2 R_1 = \left(\frac{12}{7}\right)^2 (12)$

$\boxed{P_1 = 35.3\text{ W}}$

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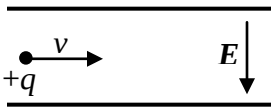
4. (40 points total) A particle with a positive charge q and mass m moves through a velocity selector consisting of a magnetic field $\vec{B}$ and an electric field $\vec{E}$. The electric field points downward, as shown.



(a) (10 points) In which direction must $\vec{B}$ point for the magnetic force to oppose the electric force? (Circle one)

$\rightarrow$ $\quad$ $\leftarrow$ $\quad$ $\uparrow$ $\quad$ $\downarrow$ $\quad$ $\otimes$ $\quad$ $\odot$

(b) (15 points) Beginning with official starting equations, find an expression for the speed at which the particle will pass undeflected through the velocity selector. Assume that the gravitational force on the particle is negligible.

$\vec{F}_B = q\vec{v} \times \vec{B}$ and $\vec{F}_E = q\vec{E}$ $\quad$ $|\vec{v} \times \vec{B}| = vB$ because $\vec{B} \perp \vec{v}$

$F_B = F_E$ for particle to pass undeflected

$qvB = qE$

$$\boxed{v = \frac{E}{B}}$$

(c) (15 points) Suppose the electric field is turned off. Now, affected only by the magnetic field, the particle moves in a circular orbit. Beginning with official starting equations, find an expression for the radius of the particle's orbit.

$\vec{F} = q\vec{v} \times \vec{B}$

$F = qvB = ma = m\frac{v^2}{R}$

$R = \frac{mv^2}{qvB}$

$$\boxed{R = \frac{mv}{qB}}$$

Note: it is better to not replace v by E/B because E has been turned off in part (c)!

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5. (30 points total) An inverted image is formed by a lens with focal length f satisfying $|f| = 18$ cm. The image is three times the size of the object.

(a) (5 points) The magnification m of the image is: (circle one) +3 (-3)

(b) (5 points) The image distance s' is: (circle one) (POSITIVE) NEGATIVE

(c) (5 points) The image is: (circle one) VIRTUAL (REAL)

(d) (15 points) Begin with starting equations and determine the object distance s and image distance s'.

$m = -\frac{s'}{s} = -3 \Rightarrow s' = 3s$

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$

$\frac{1}{s} + \frac{1}{3s} = \frac{1}{f}$ note: $s > 0$ and $s' > 0 \Rightarrow f > 0$

$\frac{3+1}{3s} = \frac{1}{f}$

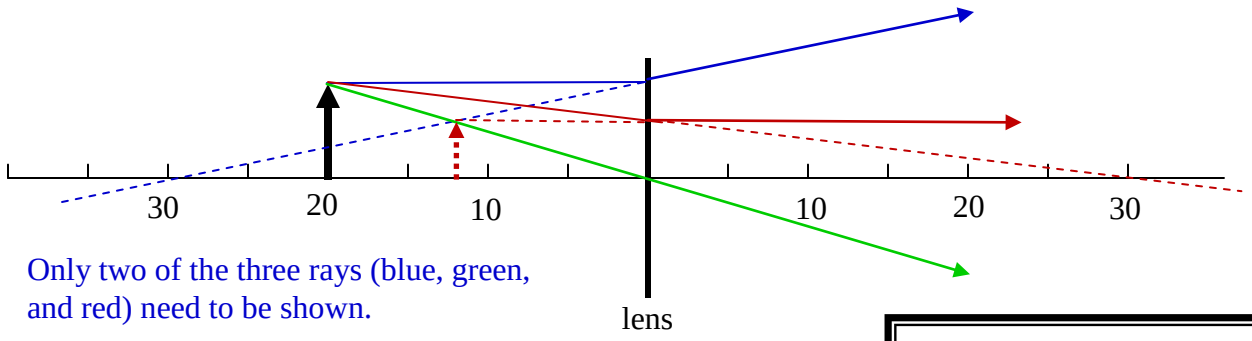
$\frac{3s}{4} = f$

$s = \frac{4f}{3} = \frac{4(18)}{3}$

$\boxed{s = 24\text{ cm}}$

$\boxed{s' = 3s = 72\text{ cm}}$

6. (10 points total) An object is placed 20 cm in front of a lens with f = - 30 cm as shown below. Draw a ray diagram for this lens-image-object, showing two of the principal rays and the location of the image.



Only two of the three rays (blue, green, and red) need to be shown.

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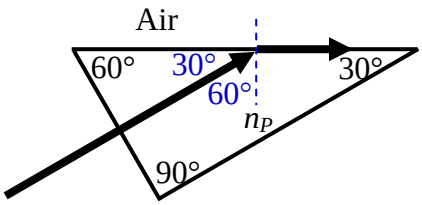
7. (20 points total) Light is incident perpendicular to the short face of a 30°-60°-90° glass prism, as shown. Inside the prism, the light striking the long surface arrives at exactly the critical angle for total internal reflection. What is the index of refraction of the prism?

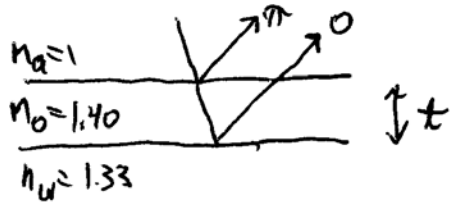


$n_p \sin 60^\circ = n_{air} \sin 90^\circ = 1$

$n_p = \frac{1}{\sin 60^\circ} = \frac{1}{\sqrt{3}/2}$

$\boxed{n_p = 1.15}$

8. (20 points total) After a short rain shower, a thin film of oil washed off the road floats on top of water in a puddle by the side of the road. In the bright sunlight following the storm, swirls of colored light are observed reflecting from the puddle's surface. Motor oil has an index of refraction of 1.4. Water has an index of refraction of 1.33. What is the minimum thickness of the oil film at a point on the surface where green light of wavelength 530 nm in air is strongly reflected?



for constructive interference:

$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_o}$

$m = 0$ for minimum thickness

$t = \frac{\lambda}{4n_o} = \frac{530}{4(1.4)} = \boxed{94.6\text{nm}}$

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