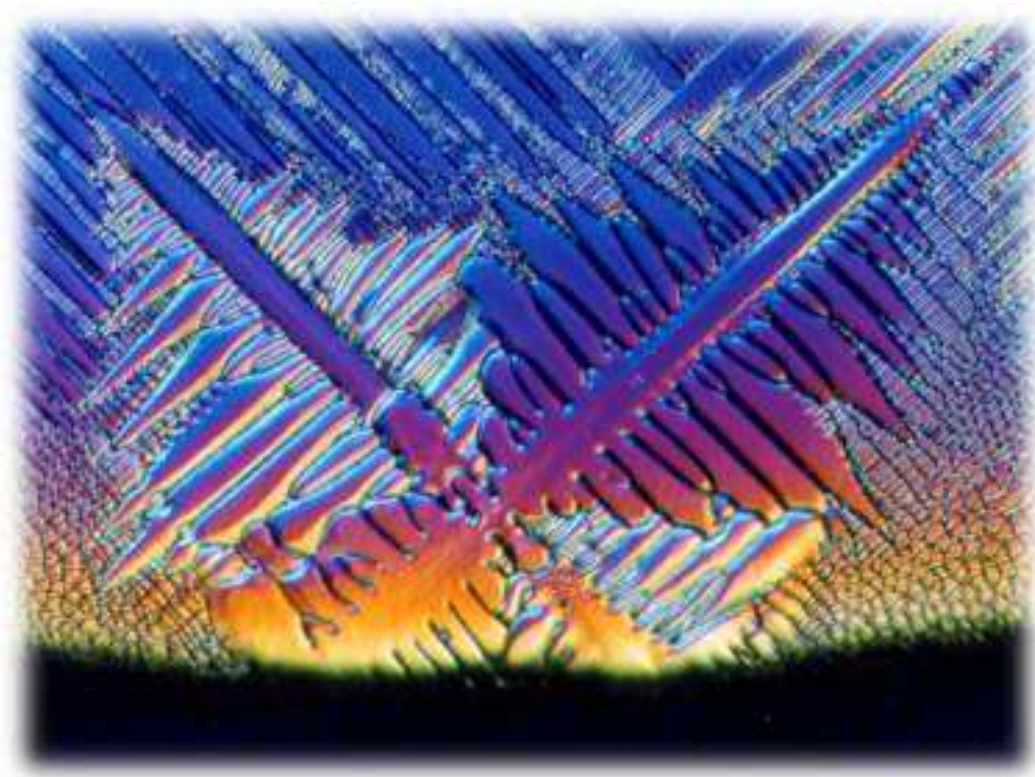


- Largest subfield of physics
- Link between atoms and everyday world.
- Unity obscured by tremendous variety of topics.



Historical Roots

- ➡ Atomic Structure
- ➡ Electronic Structure
- ➡ Mechanical Properties
- ➡ Electron Transport
- ➡ Optical Properties
- ➡ Magnetism

Concepts

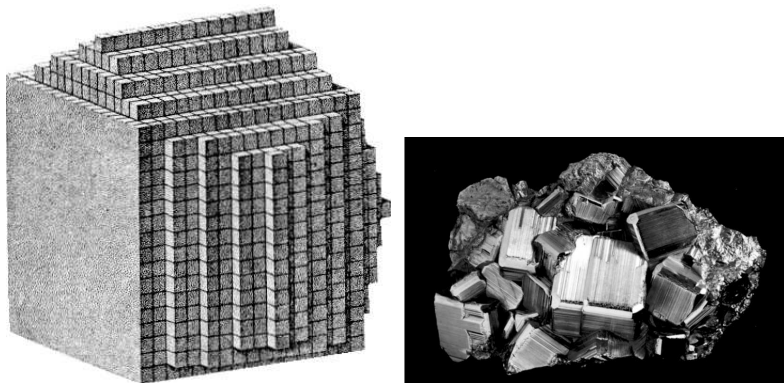
- ➡ Self-organization
- ➡ Form and Function
- ➡ Scaling and Symmetry
- ➡ Precision Measurement
- ➡ Fabrication
- ➡ Computation

Questions:

- ☞ What is the basic structure of matter?
- ☞ How do atoms spontaneously organize?

Basic Answer:

- ☞ Scaling theory relates atom-scale units to macroscopic solids.
- ☞ Atoms form crystalline arrays.
- ☞ Idea comes from special class of solids: minerals.

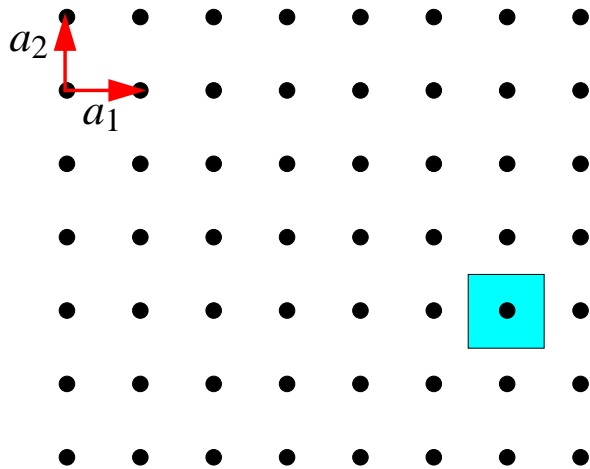


See vast numbers of minerals at <http://webmineral.com/>

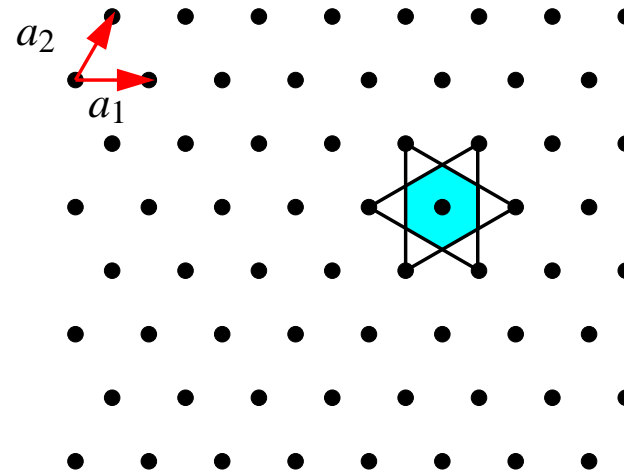
Definitions:

- ☞ Bravais lattice
- ☞ primitive vector
- ☞ basis vector
- ☞ unit cell (primitive or not)
- ☞ Wigner–Seitz cell (Voronoi polyhedron)
- ☞ translation, space, and point groups

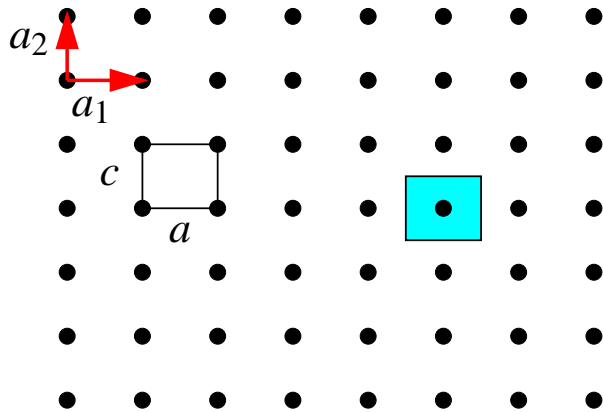
Square



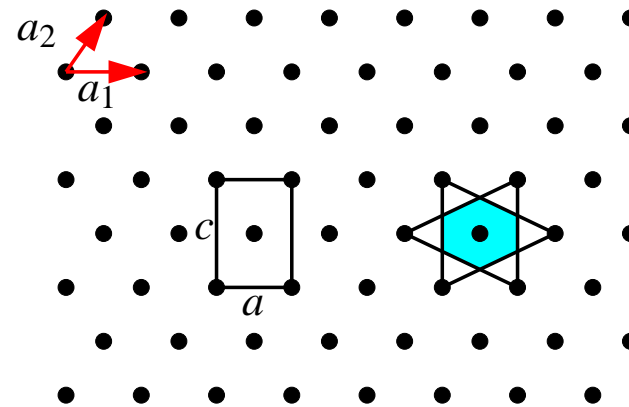
Hexagonal



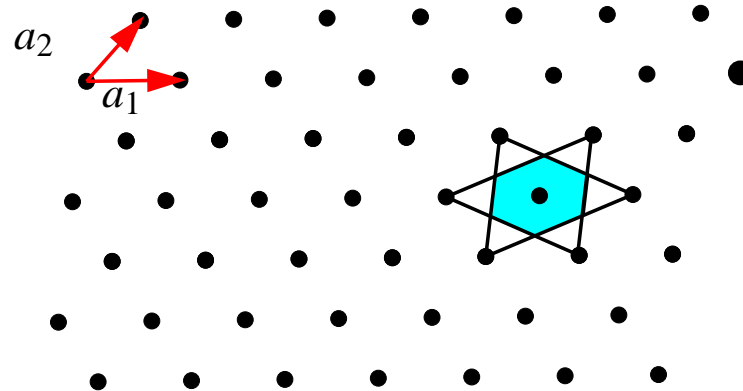
Rectangular



Centered Rectangular



Oblique



Q: Are primitive vectors unique?

A: No..for hexagonal lattice

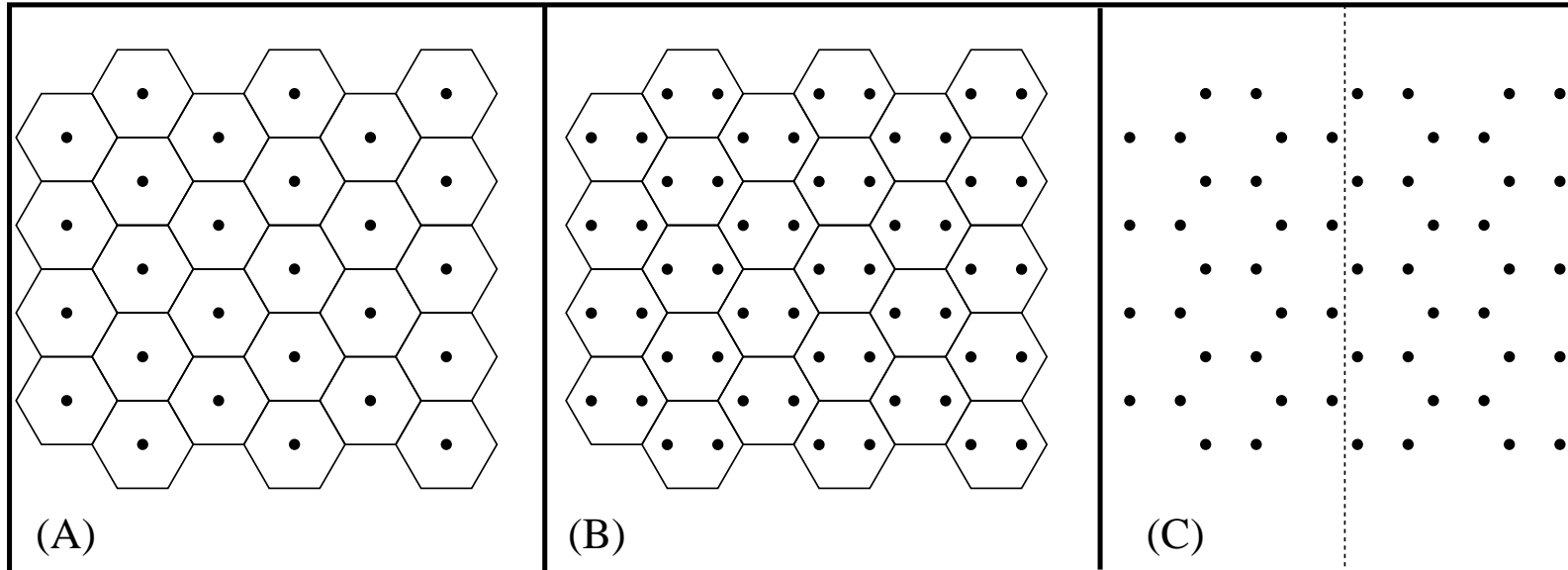
$$\vec{a}_1 = a(1 \ 0) \quad (\text{L1a})$$

$$\vec{a}_2 = a \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}. \quad (\text{L1b})$$

However, one could equally well choose

$$\vec{a}'_1 = a \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \quad (\text{L2a})$$

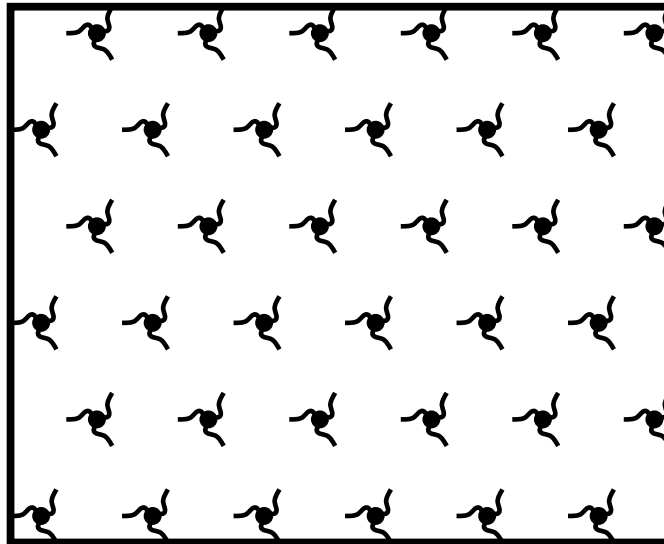
$$\vec{a}'_2 = a \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}. \quad (\text{L2b})$$



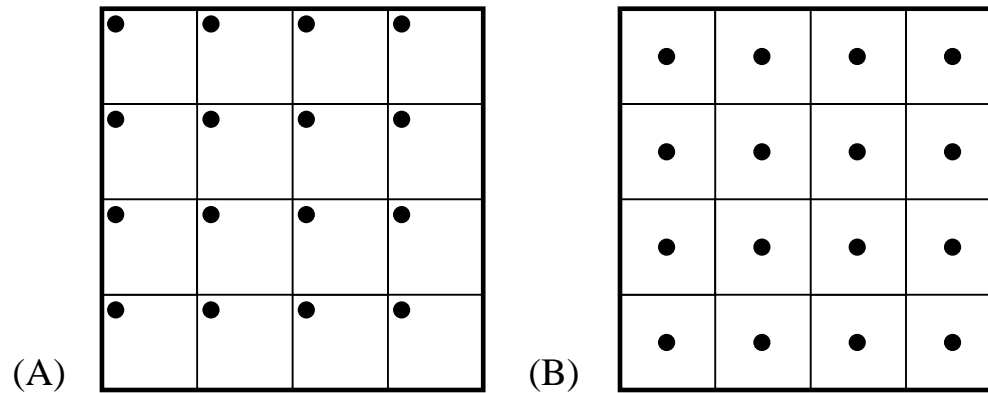
Note presence of **glide plane**, showing that **space group** is not the same as the product of **translation group** and **point group**.

Selective Destruction of Symmetry by Basis 9

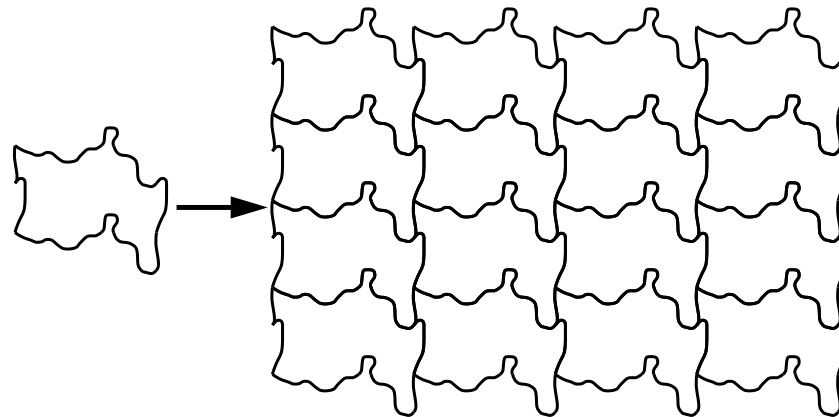
Some, but not all symmetries of triangular lattice destroyed.



Unit cells are **not** unique.

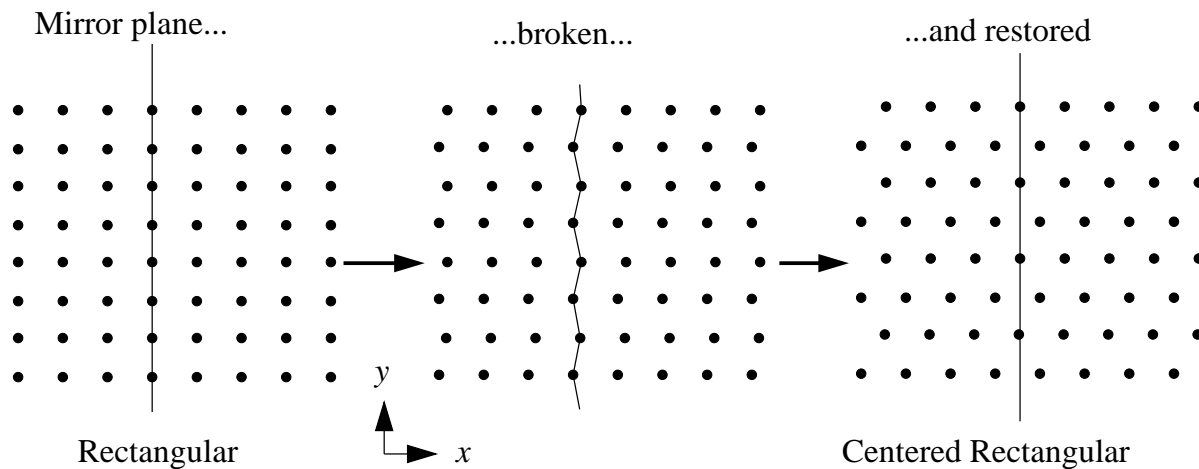


Puzzler: how does one construct bizarre-shaped cells that tile the plane?



Q: What makes lattices the same or different?

A: Two lattices are the same if one can be transformed continuously into the other without changing any symmetry operations along the way.

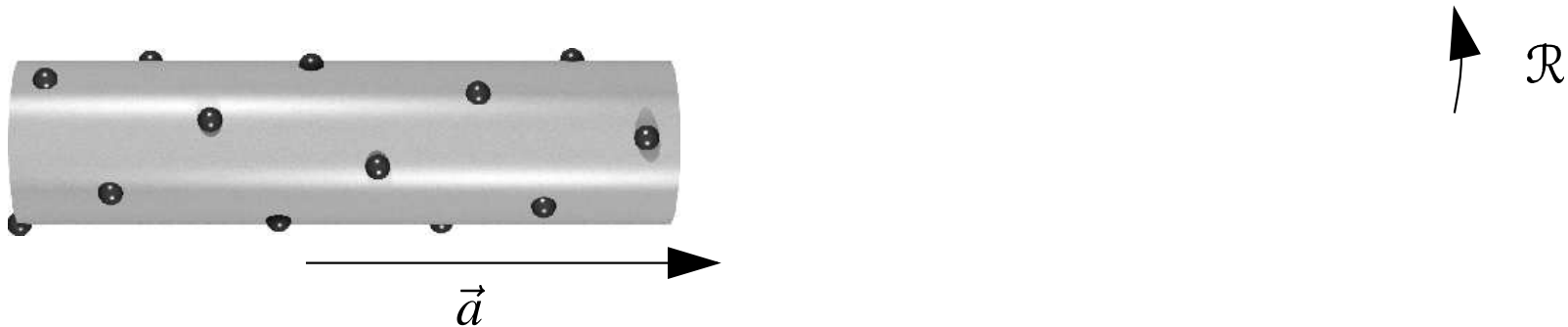


Operations

$$\mathbf{G} = \vec{a} + \mathcal{R}(\hat{n}, \theta). \quad (\text{L3})$$

that leave lattice invariant.

Two important subgroups: **translation** and **point** groups. The full space group cannot be formed from these because of **glide lines** and **Screw axes**.



$$S\mathcal{R}S^{-1} + S^{-1}\vec{a} = \mathcal{R}' + \vec{a}'. \quad (\text{L4})$$

$$S_t = (1 - t) + St, \quad (\text{L5})$$

Q: How many distinct Bravais lattices are there?

A: Five

Q: How many distinct two-dimensional lattices are there?

A: Seventeen. They are enumerated at

<http://www2.spsu.edu/math/tile/index.htm> or

<http://www.clarku.edu/~djoyce/wallpaper/>