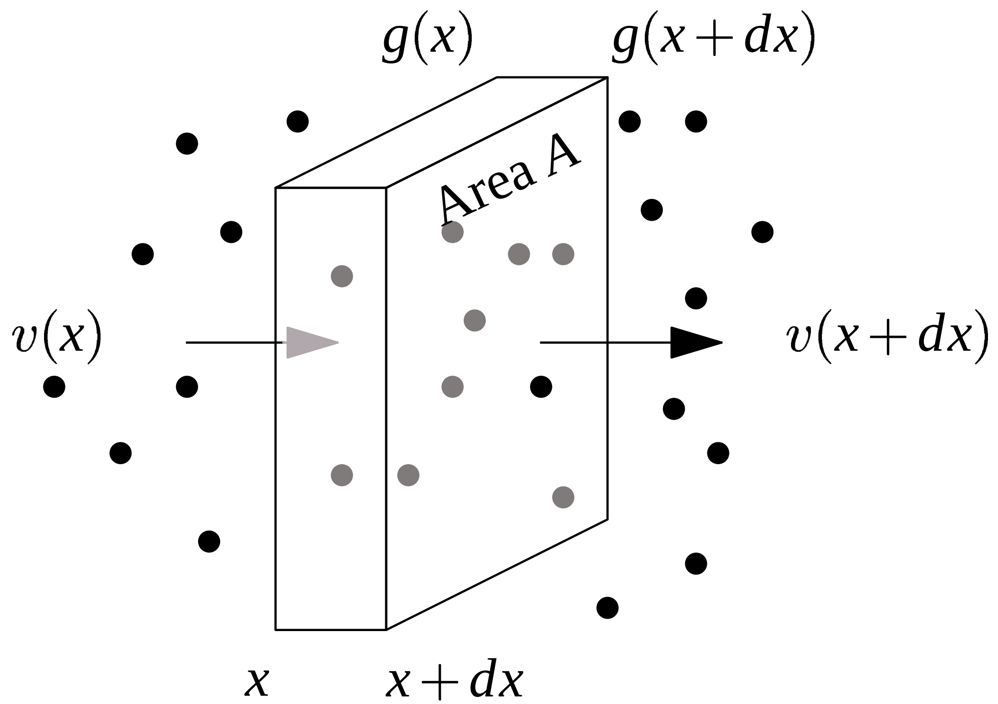
$g(x)$
$g(x+dx)$
Area A
$v(x)$
$v(x+dx)$
x
$x+dx$

Definitions

- Boltzmann Equation
- Relaxation Time Approximation
- Onsager Relations
- Holes
- Wiedemann–Franz Law
- Seebeck, Peltier, and Thomson Effects
- Classical Hall Effect
- Magnetoresistance
- Fermi Liquid Theory
- Quasi–Particles
- Zero Sound

Boltzmann Equation

Suppose have Hamiltonian structure:

$$\dot{\vec{r}} = \frac{\partial \mathcal{H}}{\partial \vec{p}}, \quad \dot{\vec{p}} = -\frac{\partial \mathcal{H}}{\partial \vec{r}}, \tag{L1}$$

In particular case of electrons in semi–classical approximation (discard anomalous velocity)

$$\mathcal{H}(\vec{r},\vec{p}) = \mathcal{E}(\vec{p}+\vec{A}e/c) - eV(\vec{r}), \tag{L2}$$

$$\dot{\vec{r}} = \frac{\partial \mathcal{E}}{\partial \hbar \vec{k}} \equiv \vec{v} \tag{L3a}$$

$$\hbar \dot{\vec{k}} = -e\vec{E} - \frac{e\vec{v}}{c} \times \vec{B}, \tag{L3b}$$

where $\hbar \vec{k}$ is defined by

$$\hbar \vec{k} = \vec{p} + e\vec{A}/c. \tag{L3c}$$

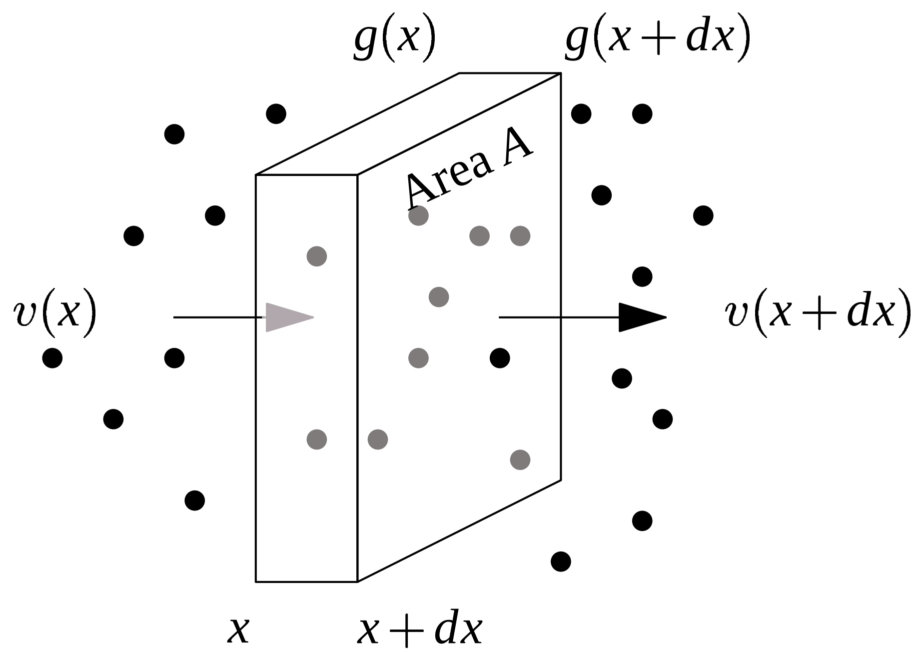

Figure 1:

Let $g(x)$ be the number of particles per volume. Then the number entering volume Adx minus the number leaving it is

$$? \qquad ? \tag{L4}$$

$$\frac{\partial g}{\partial t} = ? \qquad ? \tag{L5}$$

Continuity Equation

For a system with flows in more than one dimension,

$$\frac{\partial g}{\partial t} = -\sum_l \frac{\partial}{\partial x_l} v_l(\vec{x}) g(\vec{x}, t). \tag{L6}$$

$$g_{\vec{r}\vec{k}}(t)\, d\vec{r}\, D_{\vec{k}} d\vec{k} = 2\frac{d\vec{k}d\vec{r}}{(2\pi)^3} g_{\vec{r}\vec{k}}(t). \tag{L7}$$

$$G = \int [d\vec{k}]\, d\vec{r}\, g_{\vec{r}\vec{k}}\, G_{\vec{r}\vec{k}}. \tag{L8}$$

$$g_{\vec{r}\vec{k}} = f_{\vec{r}\vec{k}} + \text{corrections}, \tag{L9}$$

$$g_{\vec{r}\vec{k}} \approx f_{\vec{r}\vec{k}} - \tau e \frac{\partial f}{\partial \mu} \vec{v}_{\vec{k}} \cdot \vec{E}. \tag{L10}$$

Boltzmann Equation

$$\frac{\partial g}{\partial t} = -\frac{\partial}{\partial \vec{r}} \cdot \dot{\vec{r}} g - \frac{\partial}{\partial \vec{k}} \cdot \dot{\vec{k}} g. \tag{L11}$$

$$\frac{\partial g}{\partial t} = -\dot{\vec{r}} \frac{\partial}{\partial \vec{r}} g - \dot{\vec{k}} \frac{\partial}{\partial \vec{k}} g. \tag{L12}$$

$$\frac{\partial g}{\partial t} = -\dot{\vec{r}} \cdot \frac{\partial}{\partial \vec{r}} g - \dot{\vec{k}} \frac{\partial}{\partial \vec{k}} g + \left.\frac{dg}{dt}\right|_{\text{coll.}}, \tag{L13}$$

$$\left.\frac{dg}{dt}\right|_{\text{coll.}} \tag{L14}$$

Relaxation Time Approximation

$$\left.\frac{dg}{dt}\right|_{\text{coll.}} = -\frac{1}{\tau}\left[g_{\vec{r}\vec{k}} - f_{\vec{r}\vec{k}}\right], \qquad \text{(L15)}$$

Expand about Fermi function appropriate for local conditions:

$$f_{\vec{r}\vec{k}} = \frac{1}{e^{\beta_{\vec{r}}(\mathcal{E}_{\vec{k}}-\mu_{\vec{r}})}+1} \qquad \text{(L16)}$$

$$\frac{dg}{dt} = \frac{\partial g}{\partial t} + \dot{\vec{r}}\cdot\frac{\partial g}{\partial \vec{r}} + \dot{\vec{k}}\cdot\frac{\partial g}{\partial \vec{k}}, \qquad \text{(L17)}$$

$$\frac{dg}{dt} = -\frac{g-f}{\tau_{\mathcal{E}}} \qquad \text{(L18)}$$

$$\Rightarrow g_{\vec{r}\vec{k}}(t) = \int_{-\infty}^{t} dt'\, f(t')\frac{e^{-(t-t')/\tau_{\mathcal{E}}}}{\tau_{\mathcal{E}}}. \qquad \text{(L19)}$$

Relaxation Time Approximation

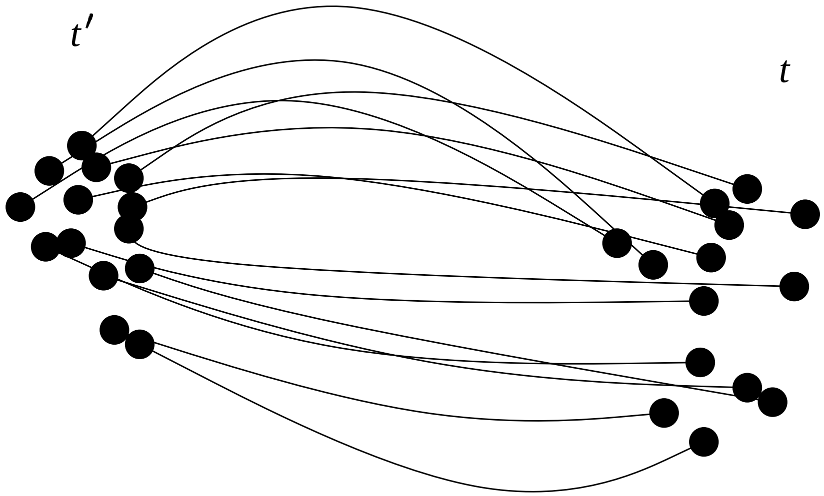

Figure 2: Electrons that at time t end up at $\vec{r}$ and $\vec{k}$.

$$g_{\vec{r}\vec{k}}(t) = f_{\vec{r}\vec{k}} - \int_{-\infty}^{t} dt' e^{-(t-t')/\tau_{\mathcal{E}}} \frac{d}{dt'} f(t'). \quad \text{(L20)}$$

$$g_{\vec{r}\vec{k}} = f_{\vec{r}\vec{k}} - \int_{-\infty}^{t} dt' e^{-(t-t')/\tau_{\mathcal{E}}} \left[\dot{\vec{r}}_{t'} \frac{\partial}{\partial \vec{r}} + \dot{\vec{k}}_{t'} \frac{\partial}{\partial \vec{k}}\right] f(t'). \quad \text{(L21)}$$

$$\frac{\partial f}{\partial \vec{r}} = \frac{\partial f}{\partial \mathcal{E}}[-\vec{\nabla}\mu - (\mathcal{E}-\mu)\frac{\vec{\nabla}T}{T}], \quad \text{(L22)}$$

and

$$\frac{\partial f}{\partial \vec{k}} = \frac{\partial f}{\partial \mathcal{E}} \frac{\partial \mathcal{E}}{\partial \vec{k}} = \frac{\partial f}{\partial \mathcal{E}} \hbar \vec{v}, \tag{L23}$$

$$g = f - \int_{-\infty}^{t} dt' e^{-(t-t')/\tau_{\mathcal{E}}} \vec{v}_{\vec{k}} \cdot \left\{ e\vec{E} + \vec{\nabla}\mu + \frac{\mathcal{E}_{\vec{k}} - \mu}{T} \vec{\nabla} T \right\} \frac{\partial f(t')}{\partial \mu}. \tag{L24}$$

$$g = f - \tau_{\mathcal{E}} \vec{v}_{\vec{k}} \cdot \left\{ e\vec{E} + \vec{\nabla}\mu + \frac{\mathcal{E}_{\vec{k}} - \mu}{T} \vec{\nabla} T \right\} \frac{\partial f}{\partial \mu}. \tag{L25}$$

Relation to Rate of Production of Entropy

$$T\frac{\partial S}{\partial t} = \frac{\partial \mathcal{E}}{\partial t} - \mu\frac{\partial N}{\partial t}. \tag{L26}$$

$$\vec{J}_N = N\vec{v} \text{ and } \vec{J}_{\mathcal{E}} = \mathcal{E}\frac{\vec{J}_N}{N} \tag{L27}$$

$$\frac{\partial N}{\partial t} + \vec{\nabla}\cdot\vec{J}_N = 0 \tag{L28}$$

$$\frac{\partial \mathcal{E}}{\partial t} + \vec{\nabla}\cdot\vec{J}_{\mathcal{E}} = \vec{F}\cdot\vec{J}_N, \tag{L29}$$

$$T\frac{\partial S}{\partial t} - \mu\vec{\nabla}\cdot\vec{J}_N + \vec{\nabla}\cdot\vec{J}_{\mathcal{E}} = \vec{F}\cdot\vec{J}_N \tag{L30}$$

so the rate $\dot{\mathcal{S}}$ at which entropy is generated is

$$\dot{\mathcal{S}} \equiv \frac{\partial S}{\partial t} + \vec{\nabla}\cdot\left[\frac{\vec{J}_{\mathcal{E}} - \mu\vec{J}_N}{T}\right] = \frac{\vec{F}\cdot\vec{J}_N}{T} - \vec{\nabla}\left(\frac{\mu}{T}\right)\cdot\vec{J}_N + \vec{\nabla}\left(\frac{1}{T}\right)\cdot\vec{J}_{\mathcal{E}} \tag{L31}$$

Relation to Rate of Production of Entropy

$$\Rightarrow \dot{Q} \equiv T\frac{\dot{\mathcal{S}}}{\mathcal{V}} = \left[-e\vec{E} - \vec{\nabla}\mu - \frac{\vec{\nabla}T}{T}\left(\frac{\mathcal{E}}{N} - \mu\right)\right] \cdot \frac{\vec{J}_N}{\mathcal{V}}. \tag{L32}$$

$$\dot{Q}_{\vec{r}\vec{k}} = \left[-e\vec{E} - \vec{\nabla}\mu - \frac{\vec{\nabla}T}{T}\left(\mathcal{E}_{\vec{k}} - \mu\right)\right] \cdot \vec{v}_{\vec{k}}\, f_{\vec{k}} \tag{L33}$$

$$g_{\vec{r}\vec{k}} = f_{\vec{r}\vec{k}} + \int_{-\infty}^{t} dt'\, e^{-(t-t')/\tau_{\mathcal{E}}}\, \dot{Q}(t')\frac{\partial}{\partial\mu}\ln f(t'). \tag{L34}$$

Forces and fluxes;

$$X_\alpha = \frac{d\dot{Q}}{dx_\alpha}. \tag{L35}$$

Flux associated with electric field is

$$-e\frac{\vec{J}_N}{\mathcal{V}} = \vec{j}. \tag{L36}$$

Flux associated with temperature gradient is

$$-\frac{1}{T}(\mathcal{E}-\mu)\frac{\vec{J}_N}{\mathcal{V}}. \tag{L37}$$

$$X_\alpha = \sum_\beta L_{\alpha\beta} x_\beta . \tag{L38}$$

$$L_{\alpha\beta}(B) = L_{\beta\alpha}(-B) . \tag{L39}$$

The flux of β in response to force α is the same as the flux of α in response to force β, (provided that one also reverses the sign of the magnetic induction B.)

Heat flux produced by electric field equals electric current produced by temperature gradient.

Derivation:

$$L_{\alpha\beta} = \int [d\vec{k}_t]\, d\vec{r}_t \int_{-\infty}^{t} dt' \frac{d\dot{Q}(t)}{dx_\alpha} e^{-(t-t')/\tau_\mathcal{E}} \left[\frac{\partial}{\partial\mu} \ln f(t') \right] \frac{d\dot{Q}(t')}{dx_\beta} . \tag{L40}$$

$$\mathcal{E}_{\vec{k}} = \mathcal{E}_{\vec{k}_{t'}} \Rightarrow f(t) = f(t'). \tag{L41}$$

Onsager Relations

$$\begin{aligned} t &\rightarrow t'; \; t' \rightarrow t; \\ \vec{B} &\rightarrow -\vec{B} \end{aligned} \tag{L42a}$$

$$\begin{aligned} \vec{k}_{t'} &\rightarrow -\vec{k}_{-t'} \\ \vec{r}_{t'} &\rightarrow \vec{r}_{-t'} \end{aligned} \quad , \tag{L42b}$$

Electrical Current

$$\vec{j} = \frac{\vec{J}}{\mathcal{V}} = -e \int [d\vec{k}]\, \vec{v}_{\vec{k}}\, g_{\vec{r}\vec{k}}. \tag{L43}$$

$$\frac{\partial j_\alpha}{\partial E_\beta} \equiv \sigma_{\alpha\beta} \tag{L44}$$

$$= \; ? \qquad\qquad ? \tag{L45}$$

Electrical Current

$$\sigma_{\alpha\beta} = e^2\tau \int [d\vec{k}]\, v_\alpha \left(-\frac{\partial f_{\vec{k}}}{\partial \hbar k_\beta}\right) \tag{L46}$$

$$\Rightarrow \sigma_{\alpha\beta} = e^2\tau \int [d\vec{k}]\, f_{\vec{k}} \frac{\partial v_\alpha}{\partial \hbar k_\beta} \tag{L47}$$

$$= e^2\tau \int [d\vec{k}]\, f_{\vec{k}} (\mathbf{M}^{-1})_{\alpha\beta}. \tag{L48}$$

$$\sigma = \frac{ne^2\tau}{m^\star}, \tag{L49}$$

in cubic crystals

$$\frac{1}{m^\star} = \frac{1}{3n} \int [d\vec{k}]\, f_{\vec{k}} \mathrm{Tr}(\mathbf{M}^{-1}). \tag{L50}$$

Conductivity related to effective mass

$$\sigma_{\alpha\beta} = e^2 \int \frac{d\Sigma}{4\pi^3 \hbar v} \tau_\varepsilon v_\alpha v_\beta, \tag{L51}$$

Alternative form as Fermi surface average.

Conductivity of filled bands is zero.

Effective Mass and Holes

$$\sigma_{\alpha\beta} = e^2\tau \int_{\substack{\text{occupied}\\ \text{levels}}} [d\vec{k}]\, (\mathbf{M}^{-1})_{\alpha\beta} \tag{L52}$$

$$= -e^2\tau \int_{\substack{\text{unoccupied}\\ \text{levels}}} [d\vec{k}]\, (\mathbf{M}^{-1})_{\alpha\beta}. \tag{L53}$$

$$\mathcal{E}_{\vec{k}} \approx \mathcal{E}_c + \frac{\hbar^2 k^2}{2m_n^\star}. \tag{L54}$$

$$\sigma = \frac{ne^2\tau}{m_n^\star}. \tag{L55}$$

$$\mathcal{E}_{\vec{k}} \approx \mathcal{E}_v - \frac{\hbar^2 k^2}{2m_p^\star}. \tag{L56}$$

$$\sigma = \frac{pe^2\tau}{m_p^\star}. \tag{L57}$$

$$\vec{G} = \vec{E} + \frac{\vec{\nabla}\mu}{e} \tag{L58}$$

Force	*Flux*	
$\vec{G}$	$\vec{j} = -e\vec{J}_N/\mathcal{V}$	$= -e\int \frac{d\vec{r}}{\mathcal{V}} \int [d\vec{k}]\, \vec{v}_{\vec{r}\vec{k}}\, g_{\vec{r}\vec{k}}$ (L59)
$\frac{-\vec{\nabla}T}{T}$	$\vec{j}_Q = (\vec{J}_\mathcal{E} - \mu\vec{J}_N)/\mathcal{V}$	$= \int \frac{d\vec{r}}{\mathcal{V}} \int [d\vec{k}]\, (\mathcal{E}_{\vec{k}} - \mu)\vec{v}_{\vec{r}\vec{k}} g_{\vec{r}\vec{k}}.$

$$\vec{j} \quad = \quad \mathbf{L}^{11}\vec{G} + \mathbf{L}^{12}\left(\frac{-\vec{\nabla}T}{T}\right) \tag{L60}$$

$$\vec{j}_Q \quad = \quad \mathbf{L}^{21}\vec{G} + \mathbf{L}^{22}\left(\frac{-\vec{\nabla}T}{T}\right). \tag{L61}$$

$$\mathbf{L}^{11} = \mathfrak{L}^{(0)},\ \mathbf{L}^{12} = \mathbf{L}^{21} = -\frac{1}{e}\mathfrak{L}^{(1)},\ \mathbf{L}^{22} = \frac{1}{e^2}\mathfrak{L}^{(2)}, \tag{L62}$$

$$\mathfrak{L}^{(\nu)}_{\alpha\beta} = e^2 \int [d\vec{k}]\, \tau_\mathcal{E} \frac{\partial f}{\partial \mu} v_\alpha v_\beta\, (\mathcal{E}_{\vec{k}} - \mu)^\nu. \tag{L63}$$

$$\sigma_{\alpha\beta}(\mathcal{E}) = \tau_{\mathcal{E}} e^2 \int [d\vec{k}]\, v_\alpha v_\beta\, \delta(\mathcal{E} - \mathcal{E}_{\vec{k}}), \tag{L64}$$

$$\mathcal{L}^{(\nu)}_{\alpha\beta} = \int d\mathcal{E}\, \frac{\partial f}{\partial \mu} (\mathcal{E} - \mu)^\nu \sigma_{\alpha\beta}(\mathcal{E}). \tag{L65}$$

$$\frac{\partial f}{\partial \mu} \approx \delta(\mathcal{E} - \mathcal{E}_F) \tag{L66}$$

$$\mathcal{L}^{(0)}_{\alpha\beta} = \sigma_{\alpha\beta}(\mathcal{E}_F) \tag{L67}$$

$$\mathcal{L}^{(1)}_{\alpha\beta} = \frac{\pi^2}{3}(k_B T)^2 \sigma'_{\alpha\beta}(\mathcal{E}_F) \tag{L68}$$

$$\mathcal{L}^{(2)}_{\alpha\beta} = \frac{\pi^2}{3}(k_B T)^2 \sigma_{\alpha\beta}(\mathcal{E}_F). \tag{L69}$$

$$\vec{j}_Q = \kappa\Big(-\vec{\nabla}T\Big) \tag{L70}$$

$$0 = \mathbf{L}^{11}\vec{G} + \mathbf{L}^{12}(\frac{-\vec{\nabla}T}{T}) \tag{L71}$$

$$\Rightarrow \vec{G} = (\mathbf{L}^{11})^{-1}\mathbf{L}^{12}\frac{\vec{\nabla}T}{T}, \tag{L72}$$

$$\vec{j}_Q = \Big[\mathbf{L}^{21}(\mathbf{L}^{11})^{-1}\mathbf{L}^{12} - \mathbf{L}^{22}\Big](\frac{\vec{\nabla}T}{T}) \tag{L73}$$

$$\Rightarrow \kappa = \frac{\mathbf{L}^{22}}{T} + \mathcal{O}(\frac{k_B T}{\mathcal{E}_F})^2 \tag{L74}$$

$$\Rightarrow \kappa_{\alpha\beta} = \frac{\pi^2}{3}\frac{k_B^2 T}{e^2}\sigma_{\alpha\beta}. \tag{L75}$$

$$L_0 = \frac{\pi^2}{3}\frac{k_B^2}{e^2} = 2.72\cdot 10^{-13}\,\mathrm{erg\,cm^{-1}\,K^{-2}} = 2.43\cdot 10^{-8}\,\mathrm{W\cdot\Omega\cdot K^{-2}} \tag{L76}$$

Thermopower—Seebeck Effect

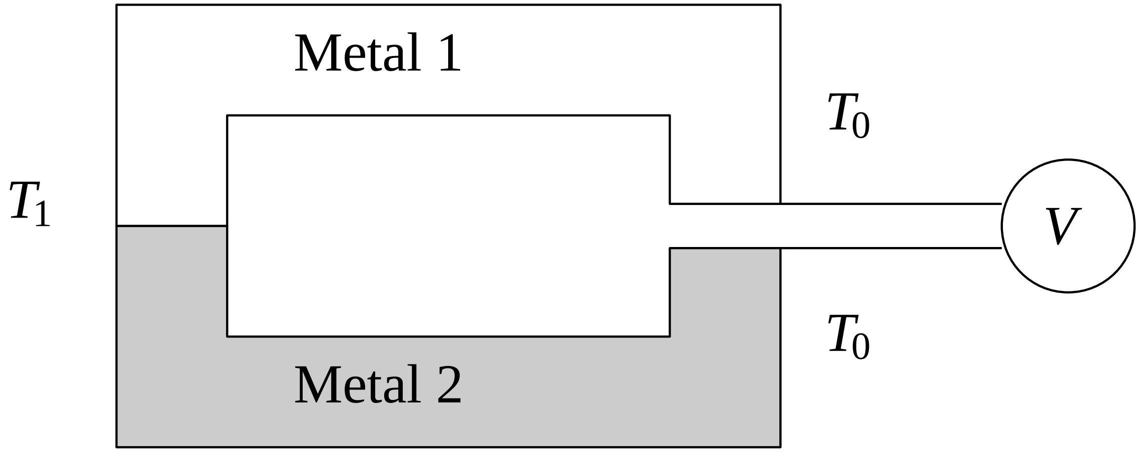

Figure 3: Geometry for Thermopower

$$\vec{G} = \alpha \vec{\nabla} T \tag{L77}$$

$$\Rightarrow \alpha = (\mathbf{L}^{11})^{-1} \frac{\mathbf{L}^{12}}{T} = -\frac{\pi^2}{3} \frac{k_B^2 T}{e} \sigma^{-1} \sigma'. \tag{L78}$$

Thermopower—Seebeck Effect

Element	Z	L/L_0 300 K	L/L_0 20 K	α (μVK^{-1}) 300 K	α (μVK^{-1}) 100 K	$\mathcal{R}nec$ 300 K	$\mathcal{R}nec$ 100 K
Li	1	0.90	0.22	10.6	4.3	−1.02	−0.16
Na	1	0.91	0.30	−5.8	−2.6	−0.54	−0.50
K	1	0.92		−13.7	−5.2	−0.89	−0.95
Rb	1			−10.2	−3.6	−0.86	−0.91
Cs	1			−0.9		−0.99	
Cu	1	0.91	0.31	1.9	1.2	−0.72	−0.78
Ag	1	0.96	0.70	1.5	0.7	−0.84	−0.84
Au	1	0.96	0.76	1.9	0.8	−0.69	−0.68
Be	2	0.97	0.23	1.7	−2.5	−30.49	−30.49
Mg	2	0.97	0.78	−1.5	−2.1	−1.15	
Ca	2			10.3	1.1		
Sr	2			1.1	−3.0		
Ba	2			12.1	−4.0		
Zn	2	0.92	0.67	2.4	0.7	3.03	3.89
Cd	2	0.97	0.65	2.6	−0.1	2.06	1.48
Hg	2	1.49	0.65			−1.97	
Al	3	0.89	0.72	−1.7	−2.2	−0.96	−0.84
Ga	3			1.8	0.5	−0.96	
In	3			1.7	0.6	−1.00	−0.50
Sn	4			−0.9	−0.0	−0.05	
Pb	4			−1.3	−0.6	0.21	
Sb	5	1.58					
Bi	5	1.07					
Mn	4			−10.0	−2.5	4.41	−23.51
Fe	2	1.36	0.98	16.2	11.6		
Co	2			−30.8	−8.4		
Ni	2	0.83		−19.2	−8.5		

Peltier Effect

$$\vec{j}_Q = \Pi \vec{j}. \tag{L79}$$

$$\Pi = \mathbf{L}^{21}(\mathbf{L}^{11})^{-1} = T\alpha. \tag{L80}$$

$$Z = \frac{\alpha^2}{R\kappa}, \tag{L81}$$

Thomson Effect

$$-T\frac{d\alpha}{dT}\vec{\nabla}T\cdot\vec{j}\equiv-\mu\vec{\nabla}T\cdot\vec{j}, \tag{L82}$$

$$T\frac{d\alpha}{dT}, \tag{L83}$$

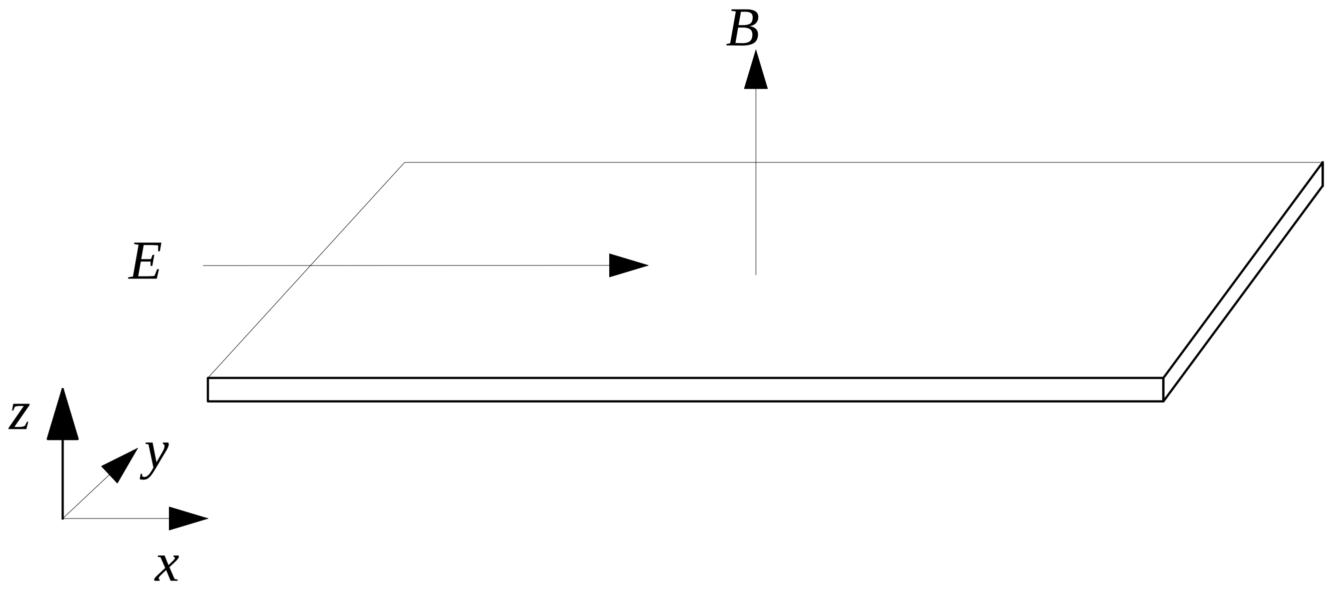

Figure 4: Geometry of the Hall effect.

$$\hbar\dot{\vec{k}} = -e\frac{\vec{v}}{c}\times\vec{B} - e\vec{E} \tag{L84}$$

$$\Rightarrow \vec{B}\times\hbar\dot{\vec{k}} + e\vec{B}\times\vec{E} = -e\vec{B}\times(\frac{\vec{v}}{c}\times\vec{B}) = -\frac{e}{c}\vec{v}_\perp B^2 \tag{L85}$$

$$\Rightarrow \vec{v}_\perp = -\frac{\hbar c}{e}\frac{\vec{B}\times\dot{\vec{k}}}{B^2} - c\frac{\vec{B}\times\vec{E}}{B^2}. \tag{L86}$$

$$g - f = \int_{-\infty}^{t} dt'\, e^{-(t-t')/\tau_{\mathcal{E}}} \left[\frac{c\hbar}{e} \frac{\vec{B} \times \dot{\vec{k}}}{B^2} \right] \cdot e\vec{E} \frac{\partial f}{\partial \mu} \qquad \text{(L87)}$$

$$= \int_{-\infty}^{t} dt'\, e^{-(t-t')/\tau_{\mathcal{E}}} \frac{c\hbar}{B^2} \dot{\vec{k}} \cdot \left[\vec{E} \times \vec{B} \right] \frac{\partial f}{\partial \mu} \qquad \text{(L88)}$$

$$= \frac{c\hbar}{B^2} \left(\vec{k} - \langle \vec{k} \rangle \right) \cdot \left[\vec{E} \times \vec{B} \right] \frac{\partial f}{\partial \mu} \qquad \text{(L89)}$$

where

$$\langle \vec{k} \rangle = \frac{1}{\tau_{\mathcal{E}}} \int_{-\infty}^{t} dt'\, e^{-(t-t')/\tau_{\mathcal{E}}} \vec{k}_{(t')}. \qquad \text{(L90)}$$

Hall Effect

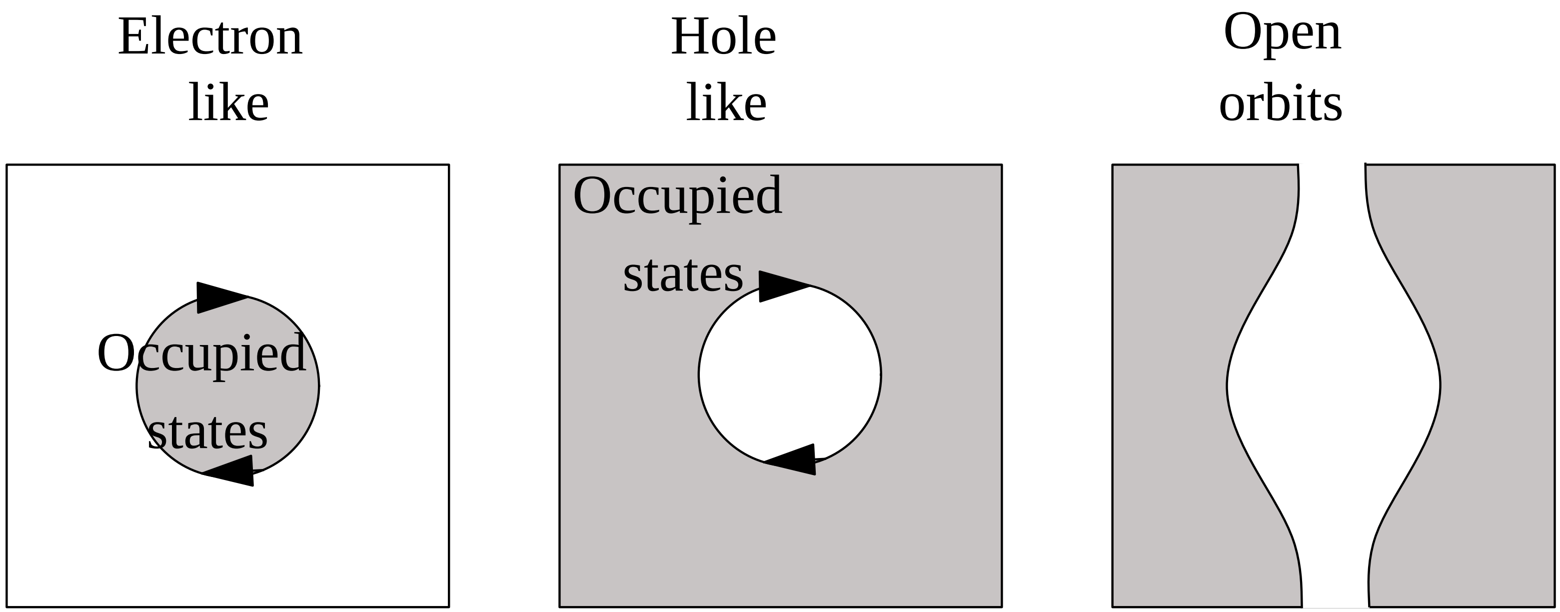

Figure 5: Electron-like, hole-like, and open orbits for the Hall effect.

$$\vec{j} = -e\int [d\vec{k}]\,\vec{v}_{\vec{k}}\frac{\partial f}{\partial \mu}\frac{\hbar c}{B^2}\vec{k}\cdot(\vec{E}\times\vec{B}) \quad \text{(L91)}$$

$$= e\int [d\vec{k}]\,\frac{\partial f}{\partial \hbar\vec{k}}\frac{\hbar c}{B^2}\vec{k}\cdot(\vec{E}\times\vec{B}) \quad \text{(L92)}$$

$$= \left\{\frac{ec}{B^2}\int [d\vec{k}]\,\frac{\partial}{\partial\vec{k}}\left(f\vec{k}\cdot(\vec{E}\times\vec{B})\right)\right\} - \frac{nec}{B^2}(\vec{E}\times\vec{B}) \quad \text{(L93)}$$

Hall Effect

$$\vec{j} = -\frac{nec}{B^2}(\vec{E} \times \vec{B}). \tag{L94}$$

$$\vec{j} = \frac{pec}{B^2}(\vec{E} \times \vec{B}), \tag{L95}$$

$$p = \int [d\vec{k}]\,(1 - f_{\vec{k}}) \tag{L96}$$

$$\mathcal{R} = -\frac{E_x}{B j_y}. \tag{L97}$$

Magnetoresistance

$$\vec{E} = \rho \vec{j} \tag{L98}$$

$$\sigma \propto \begin{pmatrix} \mathcal{C}\frac{\mathcal{T}}{\tau_\mathcal{E}}\frac{\mathcal{R}}{B} & \frac{\mathcal{R}}{B} \\ -\frac{\mathcal{R}}{B} & \mathcal{C}\frac{\mathcal{T}}{\tau_\mathcal{E}}\frac{\mathcal{R}}{B} \end{pmatrix} \tag{L99}$$

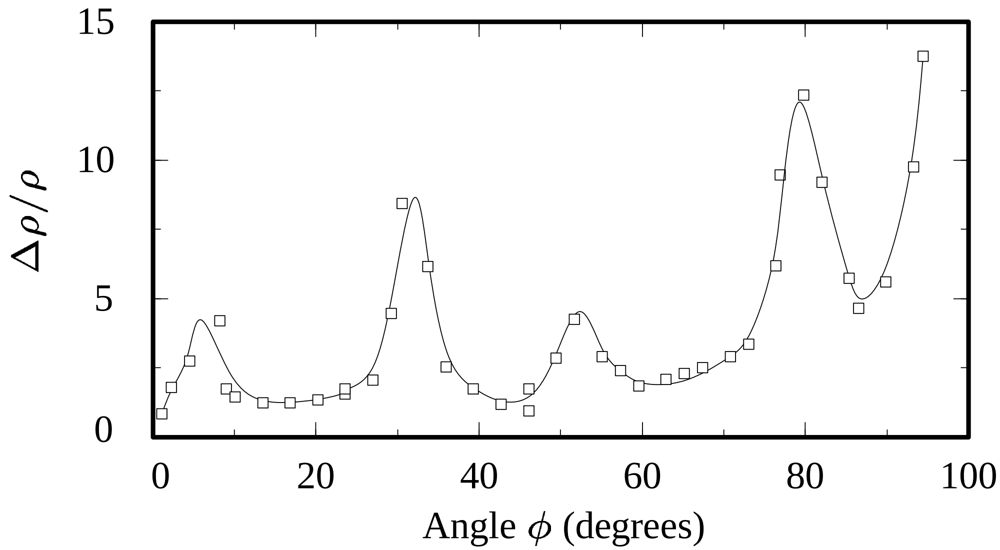

Figure 6: [Source: Alekseevskii and Gaidukhov (1960), p. 673.]

Giant Magnetoresistance (GMR) and Collossal Magnetoresistance (CMR)...new read heads.

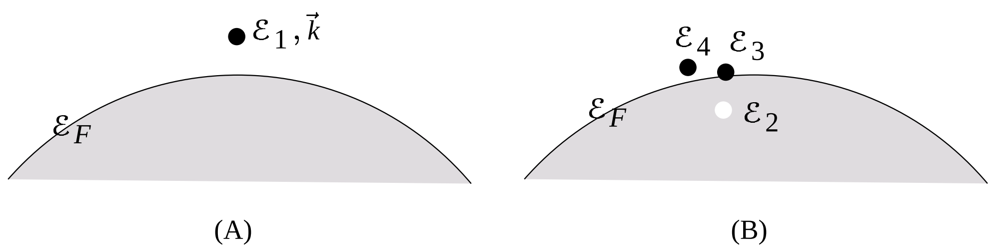

Figure 7: Fermi sea

Lifetime of quasiparticles large near Fermi surface

$$\hat{U}_{\text{int}} = \sum_{\substack{\vec{k}'\vec{q}\vec{k} \\ \sigma\sigma'}} U_{\vec{k}'\vec{q}\vec{k}} \hat{c}^{\dagger}_{\vec{k}'-\vec{q},\sigma'} \hat{c}^{\dagger}_{\vec{k}+\vec{q},\sigma} \hat{c}_{\vec{k},\sigma} \hat{c}_{\vec{k}',\sigma'}. \tag{L100}$$

$$\mathcal{P}(\vec{k} \to \vec{k}') = \int \left(\prod_{l=2}^{4} d\mathcal{E}_l D(\mathcal{E}_l) \right) \frac{2\pi}{\hbar} \delta(\mathcal{E}_1 + \mathcal{E}_2 - \mathcal{E}_3 - \mathcal{E}_4) |\langle \Psi^{\text{f}} | \hat{U}_{\text{int}} | \Psi^{\text{i}} \rangle|^2. \tag{L101}$$

$$\mathcal{P}(\vec{k} \to \vec{k}') \propto \int_{2\mathcal{E}_F - \mathcal{E}_1}^{\mathcal{E}_F} d\mathcal{E}_2 \int_{\mathcal{E}_F}^{\mathcal{E}_1 + \mathcal{E}_2 - \mathcal{E}_F} d\mathcal{E}_3 \propto (\mathcal{E}_1 - \mathcal{E}_F)^2 \propto \tau^{-1}. \tag{L102}$$

$$\mathcal{E}[\delta f] = \mathcal{E}_0 + \sum_{\vec{k}\sigma} \mathcal{E}^{(0)}_{\vec{k}} \delta f_{\vec{k}} + \frac{1}{2} \sum_{\substack{\vec{k},\vec{k}' \\ \sigma,\sigma'}} \delta f_{\vec{k}} u_{\vec{k}\vec{k}'} \delta f_{\vec{k}'} \ldots . \quad \text{(L103)}$$

$$\mathcal{E}_{\vec{k}} \equiv \mathcal{E}^{(0)}_{\vec{k}} + \sum_{\vec{k}'\sigma'} u_{\vec{k}\vec{k}'} \delta f_{\vec{k}'} . \quad \text{(L104)}$$

$$f^{(0)}_{\vec{k}} \equiv \theta(\mathcal{E}_F - \mathcal{E}_{\vec{k}}). \quad \text{(L105)}$$

$$N = \sum_{\vec{k}\sigma} f_{\vec{k}} \Rightarrow n = \frac{N}{\mathcal{V}} = \frac{1}{3\pi^2} k_F^3 . \quad \text{(L106)}$$

$$Z_{\text{gr}} = \sum_{\delta n_{\vec{k}_1} \cdots \delta n_{\vec{k}_N}} \exp \Big\{ -\beta [\sum_{\vec{k}\sigma} (\mathcal{E}^{(0)}_{\vec{k}} - \mu) \delta n_{\vec{k}} + \frac{1}{2} \sum_{\substack{\vec{k}\vec{k}' \\ \sigma\sigma'}} \delta n_{\vec{k}} u_{\vec{k}\vec{k}'} \delta n_{\vec{k}'}] \Big\} . \quad \text{(L107)}$$

$$\delta n_{\vec{k}} = \delta f_{\vec{k}} + (\delta n_{\vec{k}} - \delta f_{\vec{k}}) \quad \text{(L108)}$$

$$Z_{\rm gr} = \sum_{\delta n_{\vec k_1}=0,1\ldots} e^{-\beta[\sum_{\vec k\sigma}\mathcal{E}^{(0)}_{\vec k}-\mu+\sum_{\vec k'\sigma'}u_{\vec k\vec k'}\delta f_{\vec k'}]\delta n_{\vec k}+\beta\frac{1}{2}\sum_{\substack{\vec k\vec k'\\ \sigma\sigma'}}\delta f_{\vec k}u_{\vec k\vec k'}\delta f_{\vec k'}} \quad \text{(L109)}$$

$$= \prod_{\substack{\vec k\vec k'\\ \sigma\sigma'}} e^{\frac{1}{2}\beta\delta f_{\vec k}u_{\vec k\vec k'}\delta f_{\vec k'}} \sum_{\delta n_{\vec k_1}\ldots}\prod_{\vec k\sigma} e^{-\beta[\mathcal{E}_{\vec k}-\mu]\delta n_{\vec k}} \quad \text{(L110)}$$

$$= \prod_{\substack{\vec k\vec k'\\ \sigma\sigma'}} e^{\frac{1}{2}\beta\delta f_{\vec k}u_{\vec k\vec k'}\delta f_{\vec k'}} \prod_{\vec k\sigma}(1+e^{-\beta[\mathcal{E}_{\vec k}-\mu]h_{\vec k}}), \quad \text{(L111)}$$

$$\delta f_{\vec k} = \prod_{\substack{\vec k\vec k'\\ \sigma\sigma'}} e^{\frac{1}{2}\beta\delta f_{\vec k}u_{\vec k\vec k'}\delta f_{\vec k'}} \left[\sum_{\delta n_{\vec k_1}=0,1\ldots}\right]\delta n_{\vec k}\prod_{\vec k'\sigma'} e^{-\beta[\mathcal{E}_{\vec k'}-\mu]\delta n_{\vec k'}}/Z_{\rm gr}, \quad \text{(L112)}$$

$$\delta f_{\vec k} = \frac{h_{\vec k}}{e^{\beta h_{\vec k}(\mathcal{E}_{\vec k}-\mu)}+1} = \frac{1}{e^{\beta(\mathcal{E}_{\vec k}-\mu)}+1} - f^{(0)}_{\vec k}. \quad \text{(L113)}$$

$$v_F \equiv \left| \frac{\partial \mathcal{E}_{\vec{k}}}{\partial \hbar \vec{k}} |_{k_F} \right| \equiv \frac{\hbar k_F}{m^\star}. \quad \text{(L114)}$$

$$\vec{J}_N = \sum_\alpha \langle \Psi | \frac{\hat{P}_\alpha}{m} | \Psi \rangle \quad \text{(L115)}$$

$$= \sum_{\vec{k}\sigma} \frac{\vec{k}\hbar}{m} f_{\vec{k}} = \sum_{\vec{k}\sigma} \frac{\vec{k}\hbar}{m} \delta f_{\vec{k}}. \quad \text{(L116)}$$

$$1 + \sum_l \vec{p} \cdot \frac{\partial}{\partial \hat{P}_l} = 1 + i \sum_l \vec{p} \cdot \hat{R}_l / \hbar. \quad \text{(L117)}$$

$$[1 - i \sum_l \vec{p} \cdot \hat{R}_l/\hbar] \left\{ \sum_l \frac{\hat{P}_l^2}{2m} + \frac{1}{2} \sum_\beta \hat{U}_{\text{int}}(\hat{R}_l, \hat{R}_\beta) \right\} [1 + i \sum_l \vec{p} \cdot \hat{R}_l/\hbar] \quad \text{(L118)}$$

$$= \hat{\mathcal{H}} + \sum_l \vec{p} \cdot \frac{\hat{P}_l}{m}. \quad \text{(L119)}$$

Effective Mass

$$\sum_{\vec{k}\sigma} \mathcal{E}^{(0)}_{\vec{k}} [f_{\vec{k}-d\vec{k}} - f^{(0)}_{\vec{k}}] + \frac{1}{2} \sum_{\substack{\vec{k}\vec{k}' \\ \sigma\sigma'}} [f_{\vec{k}-d\vec{k}} - f^{(0)}_{\vec{k}}] u_{\vec{k}\vec{k}'} [f_{\vec{k}'-d\vec{k}} - f^{(0)}_{\vec{k}'}] \tag{L120}$$

$$= d\vec{k} \cdot \sum_{\vec{k}\sigma} f_{\vec{k}} \frac{\partial \mathcal{E}_{\vec{k}}}{\partial \vec{k}} + \sum_{\vec{k}\sigma} \mathcal{E}^{(0)}_{\vec{k}} \delta f_{\vec{k}} + \frac{1}{2} \sum_{\substack{\vec{k}\vec{k}' \\ \sigma\sigma'}} \delta f_{\vec{k}} u_{\vec{k}\vec{k}'} \delta f_{\vec{k}'}. \tag{L121}$$

$$\vec{J}_N = \sum_{\vec{k}\sigma} v_{\vec{k}} f_{\vec{k}} \tag{L122}$$

with

$$\vec{v}_k = \frac{\partial \mathcal{E}_{\vec{k}}}{\partial \hbar \vec{k}}. \tag{L123}$$

Effective Mass

$$\vec{J}_N = \sum_{\vec{k}\sigma} \frac{\partial \mathcal{E}^{(0)}_{\vec{k}}}{\partial \hbar \vec{k}} f_{\vec{k}} + \sum_{\substack{\vec{k}\vec{k}' \\ \sigma\sigma'}} f_{\vec{k}} \frac{\partial}{\partial \hbar \vec{k}} u_{\vec{k}\vec{k}'} \delta f_{\vec{k}'} \quad \text{(L124)}$$

$$= \sum_{\vec{k}\sigma} \frac{\partial \mathcal{E}^{(0)}_{\vec{k}}}{\partial \hbar \vec{k}} \delta f_{\vec{k}} + \sum_{\substack{\vec{k}\vec{k}' \\ \sigma\sigma'}} [\delta f_{\vec{k}} + f^{(0)}_{\vec{k}}] \frac{\partial}{\partial \hbar \vec{k}} u_{\vec{k}\vec{k}'} \delta f_{\vec{k}'} \quad \text{(L125)}$$

$$= \sum_{\vec{k}\sigma} \frac{\partial \mathcal{E}_{\vec{k}}}{\partial \hbar \vec{k}} \delta f_{\vec{k}} - \sum_{\substack{\vec{k}\vec{k}' \\ \sigma\sigma'}} \frac{\partial f^{(0)}_{\vec{k}}}{\partial \hbar \vec{k}} u_{\vec{k}\vec{k}'} \delta f_{\vec{k}'} \quad \text{(L126)}$$

$$= \sum_{\vec{k}\sigma} \vec{v}_{\vec{k}} \delta f_{\vec{k}} + \sum_{\substack{\vec{k}\vec{k}' \\ \sigma\sigma'}} u_{\vec{k}\vec{k}'} \vec{v}_{\vec{k}'} \delta(\mathcal{E}^{(0)}_{\vec{k}'} - \mathcal{E}_F) \delta f_{\vec{k}}. \quad \text{(L127)}$$

$$\frac{\hbar \vec{k}}{m} = \vec{v}_{\vec{k}} + \sum_{\vec{k}'\sigma'} u_{\vec{k}\vec{k}'} \vec{v}_{\vec{k}'} \delta(\mathcal{E}^{(0)}_{\vec{k}'} - \mathcal{E}_F). \quad \text{(L128)}$$

$$= \frac{\hbar\vec{k}}{m^\star} + \sum_{\vec{k}'\sigma'} u_{\vec{k}\vec{k}'} \frac{\hbar\vec{k}'}{m^\star}\delta(\mathcal{E}^{(0)}_{\vec{k}'} - \mathcal{E}_F) \quad \text{(L129)}$$

$$\frac{m^\star}{m} = 1 + \sum_{\vec{k}'\sigma'} u_{\vec{k}\vec{k}'} \frac{\vec{k}'\cdot\vec{k}}{k_F^2}\delta(\mathcal{E}^{(0)}_{\vec{k}'} - \mathcal{E}_F) \quad \text{(L130)}$$

$$= 1 + \mathcal{V}\int dk' D_{\vec{k}'}\, d\Sigma\, \delta(\mathcal{E}^{(0)}_{\vec{k}'} - \mathcal{E}_F)\, u_{\vec{k}\vec{k}'}\hat{k}\cdot\hat{k}' \quad \text{(L131)}$$

$$= 1 + \mathcal{V}\int d\Sigma\, \frac{D(\mathcal{E}_F)}{4\pi} u_{\vec{k}\vec{k}'} \cos\theta \quad \text{(L132)}$$

$$= 1 + \mathcal{V}D(\mathcal{E}_F)\frac{1}{2}\int_{-1}^{1} d(\cos\theta)\, u_{\vec{k}\vec{k}'} \cos\theta. \quad \text{(L133)}$$

Specific Heat

$$C_{\mathcal{V}} = \frac{\partial \mathcal{E}}{\partial T}\Big|_{\mathcal{V}} = \frac{\partial}{\partial T}\Big[\sum_{\vec{k}\sigma} \mathcal{E}^{(0)}_{\vec{k}} \delta f_{\vec{k}} + \frac{1}{2}\sum_{\substack{\vec{k}\vec{k}' \\ \sigma\sigma'}} \delta f_{\vec{k}} u_{\vec{k}\vec{k}'} \delta f_{\vec{k}'}\Big] \tag{L134}$$

$$= \sum_{\vec{k}\sigma} \mathcal{E}_{\vec{k}} \frac{\partial \delta f_{\vec{k}}}{\partial T}. \tag{L135}$$

$$\frac{\partial \delta f_{\vec{k}}}{\partial T} = \frac{h_{\vec{k}} e^{\beta h_{\vec{k}}(\mathcal{E}_{\vec{k}}-\mu)}}{[e^{h_{\vec{k}}\beta(\mathcal{E}_{\vec{k}}-\mu)}+1]^2} \left\{ \frac{h_{\vec{k}}}{k_B T^2}(\mathcal{E}_{\vec{k}}-\mu) - \frac{h_{\vec{k}}}{k_B T} \sum_{\vec{k}'\sigma'} u_{\vec{k}\vec{k}'} \frac{\partial \delta f_{\vec{k}'}}{\partial T} + \frac{h_{\vec{k}}}{k_B T}\frac{\partial \mu}{\partial T} \right\}. \tag{L136}$$

$$C_{\mathcal{V}} = \mathcal{V}\int [d\vec{k}]\, \frac{1}{k_B T^2}(\mathcal{E}_{\vec{k}}-\mu)^2 \frac{e^{\beta(\mathcal{E}_{\vec{k}}-\mu)}}{[e^{\beta(\mathcal{E}_{\vec{k}}-\mu)}+1]^2} \tag{L137}$$

$$= \mathcal{V}\int d\mathcal{E}\, D(\mathcal{E}) \frac{1}{k_B T^2}(\mathcal{E}-\mu)^2 \frac{e^{\beta(\mathcal{E}-\mu)}}{[e^{\beta(\mathcal{E}-\mu)}+1]^2} \tag{L138}$$

$$\Rightarrow c_{\mathcal{V}} = \frac{\pi^2}{3} k_B^2 T D(\mathcal{E}_F). \tag{L139}$$

$$D(\mathcal{E}_F) = \int [d\vec{k}]\,\delta(\mathcal{E}_F - \mathcal{E}_{\vec{k}}) = \int d\mathcal{E}\,\frac{k^2\delta(\mathcal{E}_F - \mathcal{E})}{\pi^2\hbar|\partial\mathcal{E}_{\vec{k}}/\partial\hbar\vec{k}|} = \frac{k_F^2}{\pi^2\hbar v_F} = \frac{m^\star k_F}{\pi^2\hbar^2}, \qquad \text{(L140)}$$

Fermi Liquid Parameters

$$u_{\vec{k}\uparrow\vec{k}'\uparrow} = u_{\vec{k}\downarrow\vec{k}'\downarrow} = u^s_{\vec{k}\vec{k}'} + u^a_{\vec{k}\vec{k}'} \tag{L141}$$

$$u_{\vec{k}\uparrow\vec{k}'\downarrow} = u_{\vec{k}\downarrow\vec{k}'\uparrow} = u^s_{\vec{k}\vec{k}'} - u^a_{\vec{k}\vec{k}'}, \tag{L142}$$

$$u^s_{\vec{k}\vec{k}'} = \sum_{l=0}^{\infty} u^s_l P_l(\cos\theta) \tag{L143}$$

$$u^a_{\vec{k}\vec{k}'} = \sum_{l=0}^{\infty} u^a_l P_l(\cos\theta). \tag{L144}$$

$$u^s_l = \frac{2l+1}{2}\int_{-1}^{1} d(\cos\theta) P_l(\cos\theta) \frac{u_{\vec{k}\uparrow\vec{k}'\uparrow} + u_{\vec{k}\uparrow\vec{k}'\downarrow}}{2} \tag{L145}$$

$$u^a_l = \frac{2l+1}{2}\int_{-1}^{1} d(\cos\theta) P_l(\cos\theta) \frac{u_{\vec{k}\uparrow\vec{k}'\uparrow} - u_{\vec{k}\uparrow\vec{k}'\downarrow}}{2}. \tag{L146}$$

$$F^a_l \equiv \mathcal{V} D(\mathcal{E}_F)\, u^a_l, \quad F^s_l \equiv \mathcal{V} D(\mathcal{E}_F)\, u^s_l. \tag{L147}$$

$$\mathcal{V}D(\mathcal{E}_F)\frac{1}{2}\int_{-1}^{1} d(\cos\theta)\,\cos\theta\, u_{\vec{k}\vec{k}'} \tag{L148}$$

$$= \left(\frac{1}{3}\right)\left(\frac{3}{2}\right)\int_{-1}^{1} d(\cos\theta)\, P_1(\cos\theta)\mathcal{V}D(\mathcal{E}_F)\,\frac{u_{\vec{k}\uparrow\vec{k}'\uparrow} + u_{\vec{k}\uparrow\vec{k}'\downarrow}}{2} \tag{L149}$$

$$= \frac{1}{3}F_1^s. \tag{L150}$$

$$\frac{m^\star}{m} = 1 + \frac{1}{3}F_1^s. \tag{L151}$$

Traveling Waves

$$c^2 = \frac{\partial P}{\partial \rho}\,|_S\,. \tag{L152}$$

$$\begin{aligned} c^2 &= \frac{\mathcal{V}}{m}\frac{\partial P}{\partial N}\,|_{T\mathcal{V}} = -\frac{\mathcal{V}}{m}\frac{\partial}{\partial N}\frac{\partial F}{\partial \mathcal{V}} = \frac{-\mathcal{V}}{m}\frac{\partial \mu}{\partial \mathcal{V}}\,|_N & \text{(L153)}\\ &= \frac{N}{m}\frac{\partial \mu}{\partial N}\,|_{\mathcal{V}} = \frac{N}{m}\frac{\partial^2 F}{\partial N^2}. & \text{(L154)}\end{aligned}$$

$$\begin{aligned} \delta f_{\vec{k}} &= \theta(\mu - \mathcal{E}_{\vec{k}}) - \theta(\mathcal{E}_F - \mathcal{E}_{\vec{k}}|_{\mu=\mathcal{E}_F}) & \text{(L155)}\\ \Rightarrow \frac{\partial \delta f_{\vec{k}}}{\partial \mu} &= \delta(\mathcal{E}_{\vec{k}} - \mu)(1 - \frac{\partial \mathcal{E}_{\vec{k}}}{\partial \mu}) & \text{(L156)}\\ &= \delta(\mathcal{E}_{\vec{k}} - \mu)\Big[1 - \sum_{\vec{k}'\sigma'} u_{\vec{k}\vec{k}'}\frac{\partial \delta f_{\vec{k}'}}{\partial \mu}\Big]. & \text{(L157)}\end{aligned}$$

$$A = \sum_{\vec{k}'\sigma'} u_{\vec{k}\vec{k}'} \frac{\partial \delta f_{\vec{k}'}}{\partial \mu}. \tag{L158}$$

$$A = \int [d\vec{k}']\, u_{\vec{k}\vec{k}'} \delta(\mathcal{E}_{\vec{k}'} - \mu)(1 - A) \tag{L159}$$

$$= B(1-A), \quad \text{where } B = \sum_{\vec{k}'\sigma'} u_{\vec{k}\vec{k}'} \delta(\mathcal{E}_{\vec{k}'} - \mu) = F_0^s \tag{L160}$$

$$\Rightarrow A = \frac{B}{1+B} = \frac{F_0^s}{1+F_0^s}. \tag{L161}$$

$$\frac{\partial N}{\partial \mu} = \sum_{\vec{k}\sigma} \frac{\partial \delta f_{\vec{k}}}{\partial \mu} \tag{L162}$$

$$= \sum_{\vec{k}\sigma} \delta(\mathcal{E}_{\vec{k}} - \mu)(1-A) \tag{L163}$$

$$= \mathcal{V} D(\mathcal{E}_F) \frac{1}{1+F_0^s} \tag{L164}$$

$$\Rightarrow c = \sqrt{\frac{n}{mD(\mathcal{E}_F)}(1+F_0^s)} \tag{L165}$$

$$= v_F\sqrt{\frac{m^\star}{3m}(1+F_0^s)}. \tag{L166}$$

$$\frac{\partial \delta f_{\vec{r}\vec{k}}}{\partial t} + \vec{v}_{\vec{k}} \cdot \frac{\partial}{\partial \vec{r}}\left\{\delta f_{\vec{r}\vec{k}} - \frac{\partial f^{(0)}_{\vec{k}}}{\partial \mathcal{E}_{\vec{k}}}\mathcal{E}_{\vec{r}\vec{k}}\right\} = \frac{dg}{dt}\bigg|_{\text{coll.}}. \tag{L167}$$

$$(\omega - \vec{q}\cdot\vec{v}_{\vec{k}})\delta f_{\vec{k}} - \delta(\mathcal{E}_{\vec{k}} - \mathcal{E}_F)\vec{q}\cdot\vec{v}_{\vec{k}}(\sum_{\vec{k}'\sigma'} u_{\vec{k}\vec{k}'}\delta f_{\vec{k}'}) = 0. \tag{L168}$$

$$\phi_{\vec{k}}\delta(\mathcal{E}_F - \mathcal{E}_{\vec{k}}) = \delta f_{\vec{k}}. \tag{L169}$$

$$[\omega - \vec{q}\cdot\vec{v}_{\vec{k}}]\phi_{\vec{k}} - \vec{q}\cdot\vec{v}_{\vec{k}}\sum_{\vec{k}'\sigma'} u_{\vec{k}\vec{k}'}\phi_{\vec{k}'} = 0. \tag{L170}$$

$$[\omega - \vec{q}\cdot\vec{v}_{\vec{k}}]\phi_{\vec{k}} - \vec{q}\cdot\vec{v}_{\vec{k}}F_0^s \int \frac{d\Sigma}{4\pi}\phi_{\vec{k}'} = 0. \quad \text{(L171)}$$

$$\Rightarrow \phi_{\vec{k}} = \frac{\vec{q}\cdot\vec{v}_{\vec{k}}}{\omega - \vec{q}\cdot\vec{v}_{\vec{k}}}F_0^s \int \frac{d\Sigma}{4\pi}\phi_{\vec{k}'}. \quad \text{(L172)}$$

$$\phi(\cos\theta) = \sum_{l=0}^{\infty} P_l(\cos\theta)\phi_l \quad \text{(L173)}$$

$$\Rightarrow \phi(\cos\theta) = -\left[1 - \frac{\omega}{\omega - qv_F\cos\theta}\right]F_0^s\phi_0 \quad \text{(L174)}$$

$$\Rightarrow \phi_0 = -F_0^s\phi_0\frac{1}{2}\int_{-1}^{1} d(\cos\theta)\left[1 - \frac{\omega}{\omega - qv_F\cos\theta}\right] \quad \text{(L175)}$$

$$= -F_0^s\phi_0\left[1 + \frac{1}{2}\frac{\omega}{qv_F}\ln\left(\frac{\omega - qv_F}{\omega + qv_F}\right)\right] \quad \text{(L176)}$$

$$\Rightarrow 1 + F_0^s\left[1 + \frac{1}{2}\frac{\omega}{qv_F}\ln\left(\frac{\omega - qv_F}{\omega + qv_F}\right)\right] = 0. \quad \text{(L177)}$$

$$\left\{F_0^s(1+\frac{1}{3}F_1^s)+(\frac{\omega}{qv_F})^2F_1^s\right\}\left[1+\frac{1}{2}\frac{\omega}{qv_F}\ln\left(\frac{\omega-qv_F}{\omega+qv_F}\right)\right]+1+\frac{F_1^s}{3}=0. \tag{L178}$$

Comparison with Experiment in ^{3}He

P(bar)	F_0^s	F_1^s	F_0^a	F_1^a	$m^\star/m$	v_F (m s^{-1})
0	9.15	5.27	−0.700	−0.55	2.76	59.7
3	15.83	6.40	−0.725	−0.73	3.13	54.3

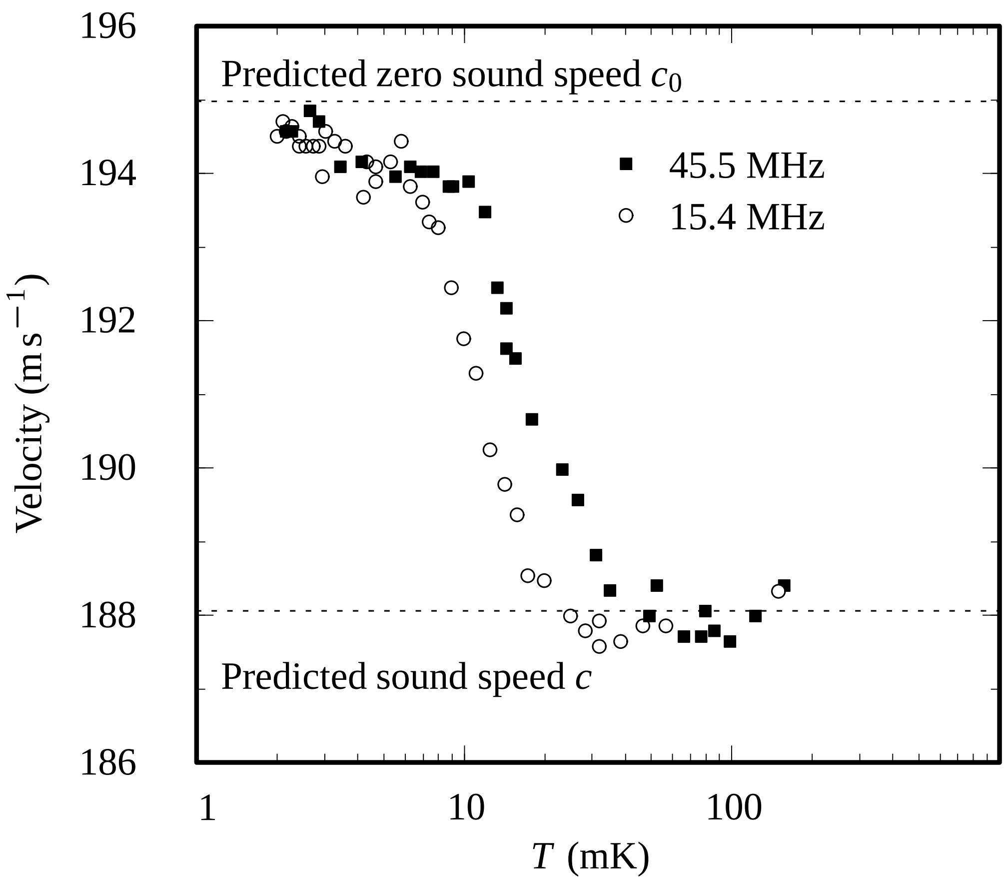

Figure 8: [Source: Abel et al. (1966), p. 76.]