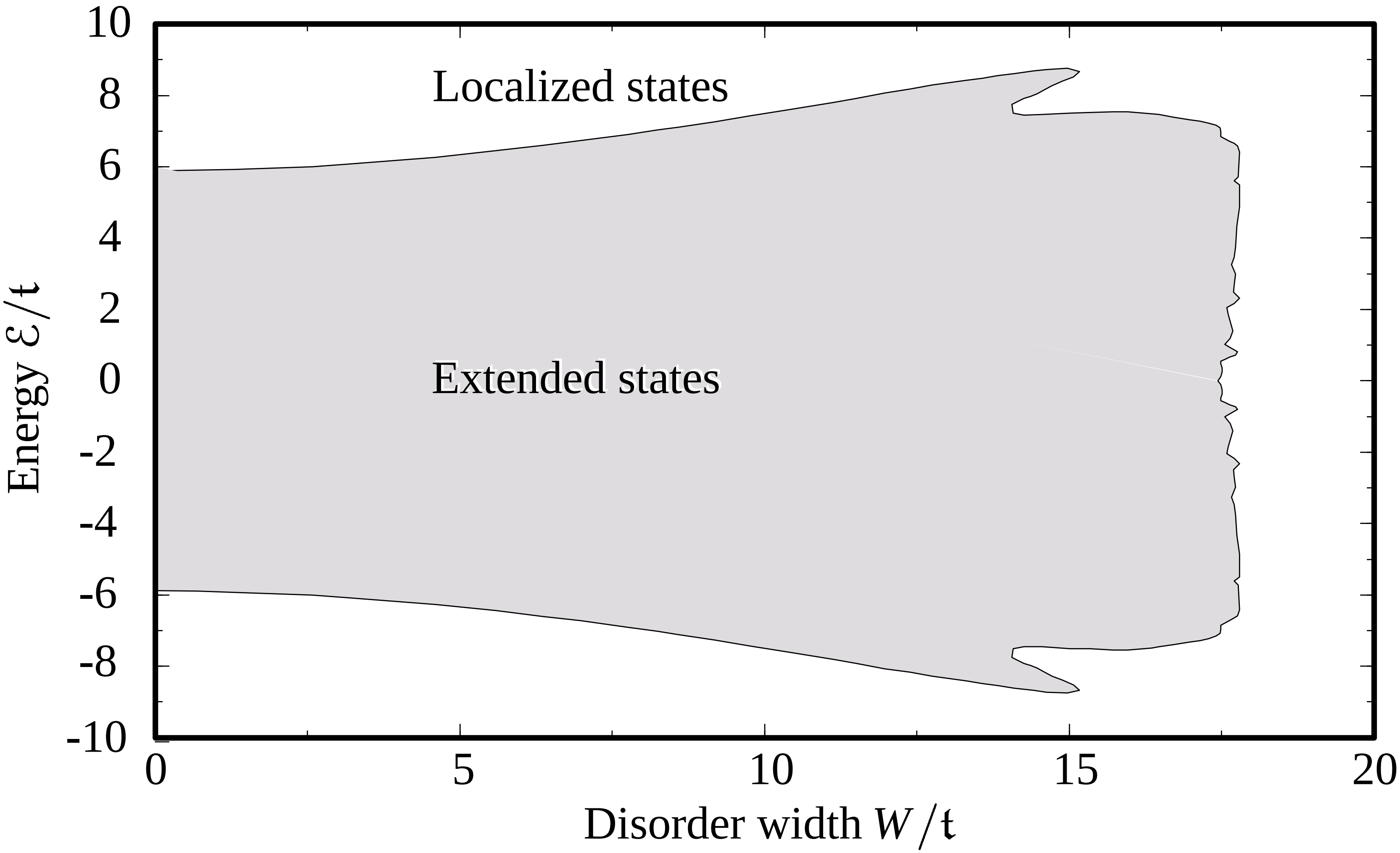
Microscopic Theories of Conduction



Definitions

- Weak Scattering Theory
- Noise
- Metal–Insulator Transitions
- Green's Functions
- Effects of Impurities
- Anderson Localization
- Mobility Edge and Localization Length
- Scaling Theory

Problem: A perfect crystal is a perfect electrical conductor

General Formula for Relaxation Time

$$\mathcal{P}(\vec{k} \to \vec{k}', t) = g_{\vec{k}} \left[1 - g_{\vec{k}'}\right] \delta_{\sigma\sigma'} W_{\vec{k}\vec{k}'}. \tag{L1}$$

$$\left.\frac{dg}{dt}\right|_{\text{coll.}} = \frac{\mathcal{V}}{2} \int [d\vec{k}']\, g_{\vec{k}'}[1 - g_{\vec{k}}] W_{\vec{k}'\vec{k}} - g_{\vec{k}}[1 - g_{\vec{k}'}] W_{\vec{k}\vec{k}'}. \tag{L2}$$

$$W_{\vec{k}\vec{k}'} = \frac{2\pi}{\hbar} \delta(\mathcal{E}_{\vec{k}} - \mathcal{E}_{\vec{k}'}) |\langle \vec{k} | \hat{U}_{\text{tot}} | \vec{k}' \rangle|^2, \tag{L3}$$

where

$$U_{\text{tot}}(\vec{r}) = \sum_{\vec{R}} U(r - \vec{R}). \tag{L4}$$

$$W_{\vec{k}\vec{k}'} = W_{\vec{k}'\vec{k}}. \tag{L5}$$

$$g_{\vec{k}} = f_{\vec{k}} + \vec{\mathcal{C}} \cdot \vec{k}. \tag{L6}$$

General Formula for Relaxation Time

$$\left.\frac{dg}{dt}\right|_{\text{coll.}} = -\vec{\mathcal{E}} \cdot \frac{1}{2}\mathcal{V} \int [d\vec{k}'] \, (\vec{k} - \vec{k}') W_{\vec{k}\vec{k}'}, \tag{L7}$$

and so

$$\left.\frac{dg}{dt}\right|_{\text{coll.}} = -\frac{g-f}{\tau_{\mathcal{E}}}, \tag{L8}$$

with

$$\frac{1}{\tau_{\mathcal{E}}} = ? \qquad\qquad ?. \tag{L9}$$

Ziman's Expression for Resistivity

$$\vec{q} = \vec{k} - \vec{k}' \tag{L10}$$

$$(1 - \hat{k}\cdot\hat{k}') = 2\left(\frac{q}{2k_F}\right)^2 \tag{L11}$$

$$W_{\vec{k}\vec{k}'} = \frac{2\pi}{\hbar}\left|\int \frac{d\vec{r}}{\mathcal{V}} e^{i\vec{q}\cdot\vec{r}} \sum_{\vec{R}} U(\vec{r}-\vec{R})\right|^2 \delta(\mathcal{E}_F - \mathcal{E}(\vec{k}')). \tag{L12}$$

$$W_{\vec{k}\vec{k}'} = \frac{2\pi}{\hbar}\frac{1}{\mathcal{V}^2}\left|\sum_{\vec{R}} e^{i\vec{q}\cdot\vec{R}} \int d\vec{r}\, e^{i\vec{q}\cdot\vec{r}} U(\vec{r})\right|^2 \delta(\mathcal{E}_F - \mathcal{E}(\vec{k}')) \tag{L13}$$

$$= \frac{2\pi}{\hbar} S(q)|U(q)|^2 \frac{N_s}{\mathcal{V}^2} \delta(\mathcal{E}_F - \mathcal{E}(|\vec{k}-\vec{q}|)), \tag{L14}$$

$$\int_{-1}^{1} d(\cos\theta)\,\delta(\mathcal{E}_F - \mathcal{E}(\sqrt{k_F^2 + q^2 - 2k_F q\cos\theta})) = \frac{\theta(2k_F - q)}{q\partial\mathcal{E}/\partial k_F} \tag{L15}$$

Ziman's Expression for Resistivity

$$\frac{1}{\tau_{\mathcal{E}}} = \frac{1}{4\pi\hbar^2 k_F^2 v_F} \frac{N_s}{\mathcal{V}} \int_0^{2k_F} dq\, q^3 S(q) |U(q)|^2 \tag{L16}$$

$$\Rightarrow \rho = \frac{m}{ne^2\tau_{\mathcal{E}}} = \frac{3\pi}{e^2\hbar v_F^2} \left(\frac{N_s}{\mathcal{V}}\right) \frac{1}{4k_F^4} \int_0^{2k_F} dq\, q^3 S(q) |U(q)|^2. \tag{L17}$$

Evidence that Liquid Metal Scatter Weakly

Metal:	Li	Na	Cu	Ag	Au	Zn	Hg	Al	Ga	Sn	Pb	Sb	Bi	Fe
l_T (Å):	45	157	34	51	27	15	5	20	17	5	6	4	4	3

Phonon Resistivity

$$\sum_l e^{i\vec{q}\cdot(\vec{R}^l+\hat{u}^l)} = \sum_l e^{i\vec{q}\cdot\vec{R}^l}[1+i\vec{q}\cdot\hat{u}^l+\ldots] \tag{L18}$$

$$\rightarrow \quad \frac{1}{\sqrt{N}}\sum_{l\vec{k}} e^{i\vec{q}\cdot\vec{R}^l} i[\hat{u}_{\vec{k}}\cdot\vec{q}e^{i\vec{k}\cdot R^l}+\hat{u}^*_{\vec{k}}\cdot\vec{q}e^{-i\vec{k}\cdot R^l}] \tag{L19}$$

$$= \quad \frac{1}{\sqrt{N}}\sum_{l\vec{k}} i[\hat{u}_{\vec{k}}\cdot\vec{q}e^{i(\vec{k}+\vec{q})\cdot R^l}+\hat{u}^*_{\vec{k}}\cdot\vec{q}e^{i(\vec{q}-\vec{k})\cdot R^l}] \tag{L20}$$

$$= \quad \sqrt{N}\sum_{\vec{K}\vec{k}} i[\hat{u}_{\vec{k}}\cdot\vec{q}\delta_{\vec{K},\vec{q}+\vec{k}}+\hat{u}^*_{\vec{k}}\cdot\vec{q}\delta_{\vec{K},\vec{q}-\vec{k}}]. \tag{L21}$$

$$S(\vec{q}) = \frac{1}{N}\left\langle|\sum_l e^{i\vec{q}\cdot(\vec{R}^l+\hat{u}^l)}|^2\right\rangle \tag{L22}$$

$$\approx \left\langle|\hat{u}^*_{\vec{q}}\cdot\vec{q}|^2\right\rangle \tag{L23}$$

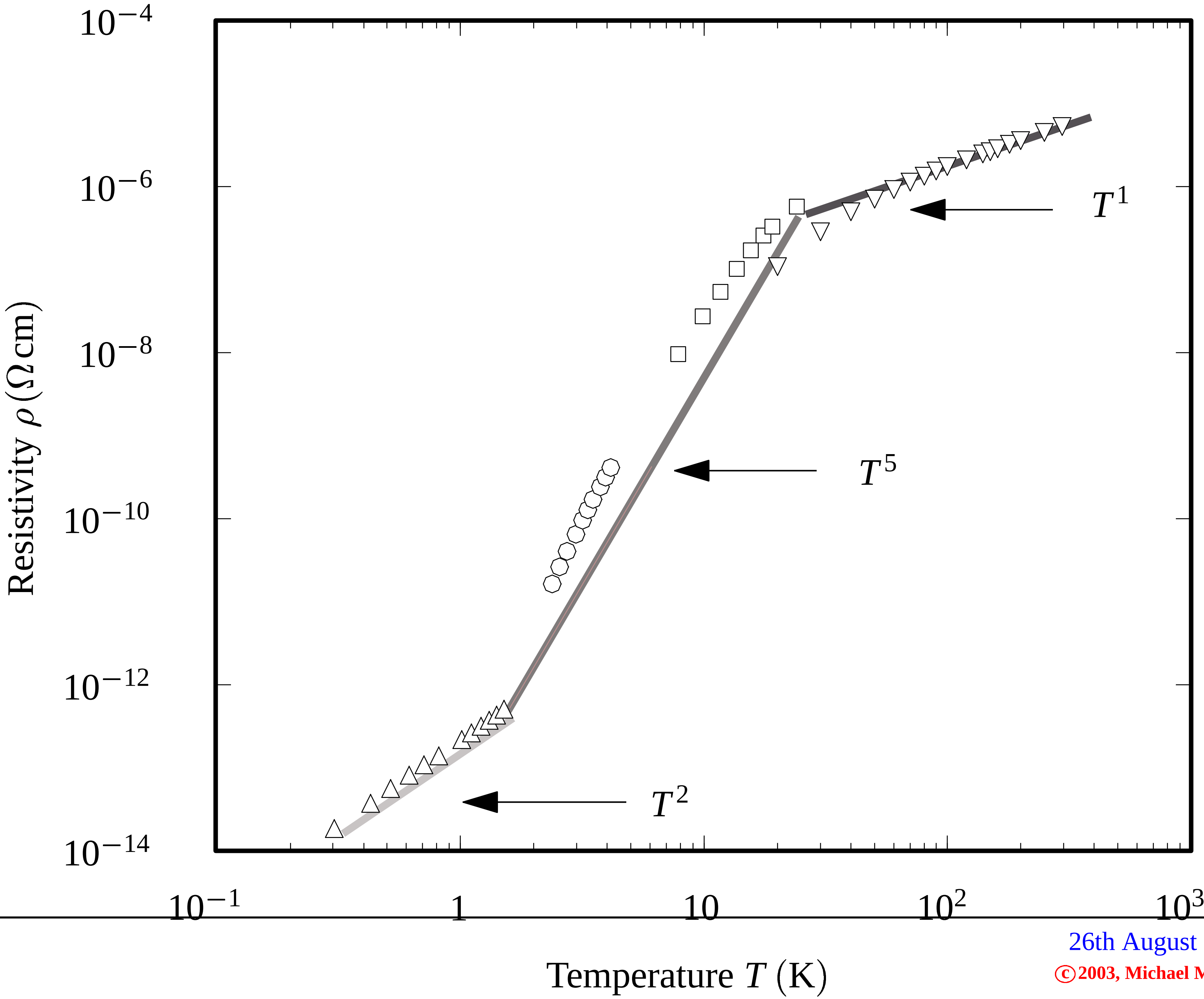
$$= \frac{\hbar}{2M\omega_{\vec{q}}}|\vec{\epsilon}\cdot\vec{q}|^2\left\langle\hat{a}_{\vec{k}}\hat{a}^*_{\vec{k}}+a^*_{\vec{k}}\hat{a}_{\vec{k}}\right\rangle \tag{L24}$$

$$= \frac{\hbar q^2}{2M\omega_{\vec{q}}}(2n_{\vec{q}}+1) \quad \text{(L25)}$$

$$\Rightarrow \rho = \frac{3\pi}{e^2\hbar v_F^2}\left(\frac{N_s}{\mathcal{V}}\right)\frac{1}{4k_F^4}\int_0^{2k_F} dq\, q^3 \frac{\hbar q^2}{2M\omega_{\vec{q}}}(2n_{\vec{q}}+1)|U(q)|^2, \quad \text{(L26)}$$

$$= \frac{3\pi}{e^2\hbar v_F^2}\left(\frac{N_s}{\mathcal{V}}\right)\frac{1}{4k_F^4}\frac{\hbar}{2Mc}\left(\frac{k_BT}{\hbar c}\right)^5\int_0^{2\Theta/T} dz\, z^4 \frac{e^z+1}{e^z-1}|U\left(\frac{k_F zT}{\Theta}\right)|^2, \quad \text{(L27)}$$

Phonon Resistivity



Matthiessen's Rule

When resistivity is small, add contributions from different sources.

Fluctuations

Thermal noise

$$\langle \delta V^2 \rangle = 4k_B T R\, d\omega. \tag{L28}$$

Shot noise

$$\langle \delta J^2 \rangle = 2eJ\, d\omega. \tag{L29}$$

$1/f$ noise.

Non–compensated impurities

$$a_* = \frac{\epsilon\hbar^2}{m^\star e^2} \quad \text{and} \quad \mathcal{E}_b = \frac{e^2}{2\epsilon a_*} = \frac{m^\star}{m}\frac{1}{\epsilon^2} \cdot 13.6\,\text{eV}. \tag{L30}$$

$$\alpha = \frac{9}{2}{a_*}^3. \tag{L31}$$

$$\frac{\epsilon - 1}{\epsilon + 2} = \frac{4\pi}{3} n_{\text{P}} \alpha \tag{L32}$$

$$\Rightarrow \epsilon = \frac{3 + 8\pi n_{\text{P}}\alpha}{3 - 4\pi n_{\text{P}}\alpha}, \tag{L33}$$

Metal–Insulator Transitions

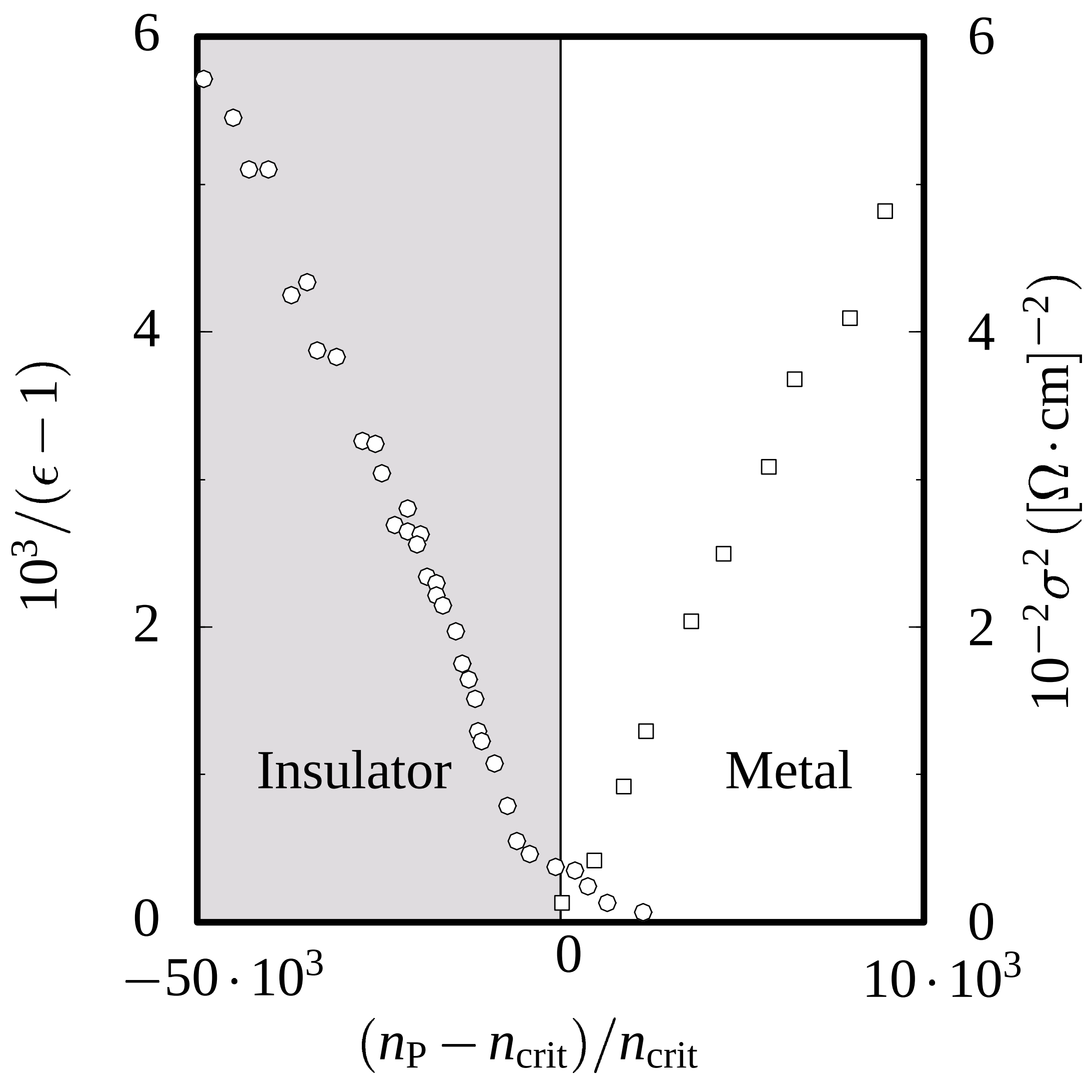


Figure 2: Metal–insulator transition in silicon doped with phosphorus. Rosenbaum (1985)

$$n_{\rm crit} = \frac{3}{4\pi\alpha} = \frac{0.053}{a_*^3} \quad \Rightarrow n_{\rm crit}^{1/3} a_* = 0.38, \tag{L34}$$

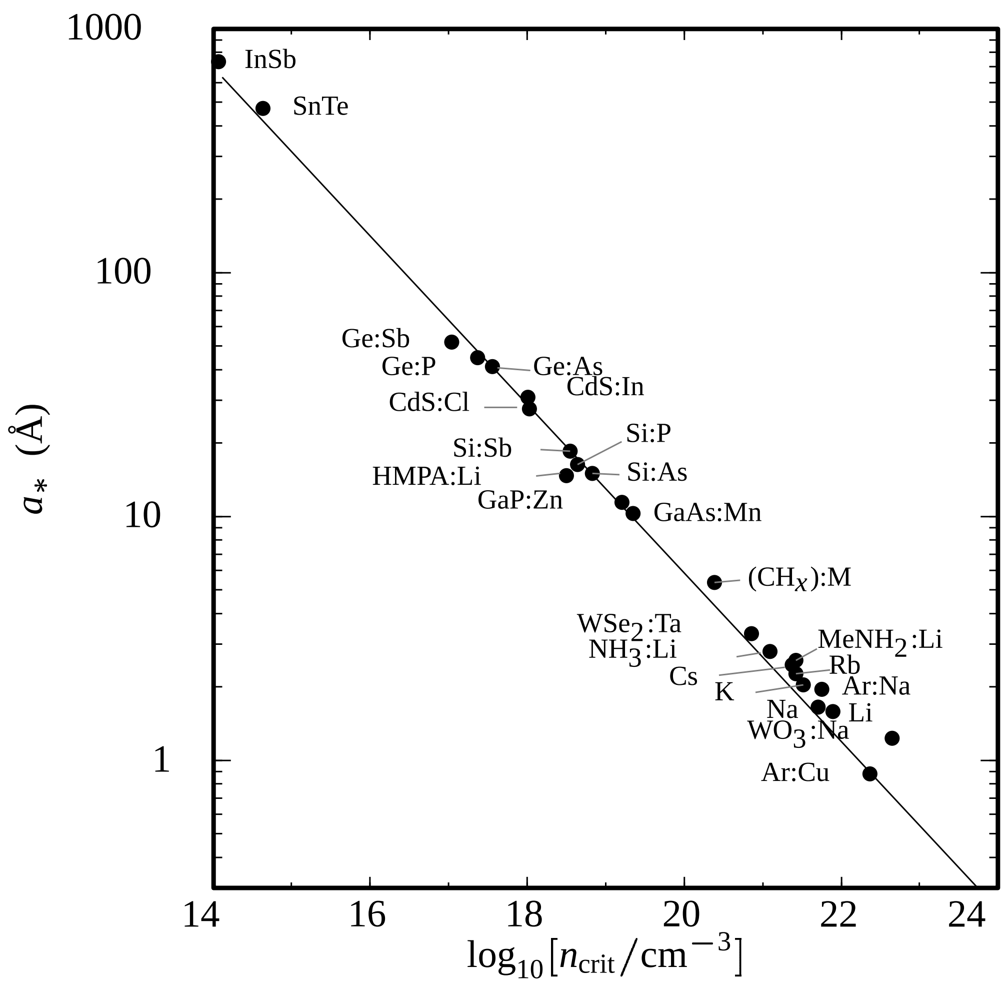


Figure 3: A host of different systems displays metal–insulator transitions when $a_* n^{1/3} = 0.26$, Edwards and Sienko (1982)

Compensated impurities

$$\hat{\mathcal{H}}_{\text{TB}} = \sum_{\vec{R}} U_{\vec{R}} |\vec{R}\rangle\langle\vec{R}| + \sum_{\langle\vec{R}\vec{R}'\rangle} \mathfrak{t}|\vec{R}\rangle\langle\vec{R}'| + \mathfrak{t}|\vec{R}'\rangle\langle\vec{R}|, \tag{L35}$$

$$\hat{\mathcal{H}}_1 = U_0|0\rangle\langle 0|. \tag{L36}$$

$$\mathcal{E}|\psi\rangle = \left(\hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_1\right)|\psi\rangle. \tag{L37}$$

$$\langle\vec{R}|\hat{G}(t)|0\rangle = \langle\vec{R}|e^{-i\hat{\mathcal{H}}t/\hbar}|0\rangle. \tag{L38}$$

$$\hat{G}(\mathcal{E}) = \frac{1}{i\hbar}\int_0^\infty dt\, e^{i\mathcal{E}t/\hbar}\hat{G}(t) \tag{L39}$$

$$\Rightarrow \hat{G}(\mathcal{E}) = (\mathcal{E}-\hat{\mathcal{H}})^{-1}. \tag{L40}$$

$$\hat{G}(\mathcal{E}) = \frac{i}{\hbar}\int_{-\infty}^0 dt\, e^{i\mathcal{E}t/\hbar}\hat{G}(t), \tag{L41}$$

$$\hat{G} = (\mathcal{E}-\hat{\mathcal{H}})^{-1} = \sum_n (\mathcal{E}-\hat{\mathcal{H}})^{-1}|\mathcal{E}_n\rangle\langle\mathcal{E}_n| = \sum_n \frac{|\mathcal{E}_n\rangle\langle\mathcal{E}_n|}{\mathcal{E}-\mathcal{E}_n}. \tag{L42}$$

$$\hat{G}^{\pm}(\mathcal{E}) \sim \frac{|\mathcal{E}_n\rangle\langle\mathcal{E}_n|(\mathcal{E}_r-\mathcal{E}_n)}{(\mathcal{E}_r-\mathcal{E}_n)^2+\eta^2} \mp \frac{i\eta|\mathcal{E}_n\rangle\langle\mathcal{E}_n|}{(\mathcal{E}_r-\mathcal{E}_n)^2+\eta^2} \tag{L43}$$

$$= |\mathcal{E}_n\rangle\langle\mathcal{E}_n|\left\{\frac{1}{\mathcal{E}_r-\mathcal{E}_n} \mp i\pi\delta(\mathcal{E}_r-\mathcal{E}_n)\right\}. \tag{L44}$$

$$\mp\frac{1}{\pi}\,\mathrm{Im}[\langle\vec{R}|\hat{G}^{\pm}(\mathcal{E})|\vec{R}\rangle]=\sum_n\delta(\mathcal{E}_r-\mathcal{E}_n)|\langle\vec{R}|n\rangle|^2 \tag{L45}$$

$$\langle R|\hat{G}_0|R'\rangle = \sum_k\frac{\langle R|k\rangle\,\langle k|R'\rangle}{\mathcal{E}-\mathcal{E}_0(k)} \tag{L46}$$

$$= \sum_l\frac{1}{N}\frac{e^{2\pi il(R-R')/N}}{\mathcal{E}-2\mathfrak{t}\cos\left(2\pi l/N\right)}\rightarrow\int_0^{2\pi}\frac{dk}{2\pi}\frac{e^{ik(R-R')}}{\mathcal{E}-2\mathfrak{t}\cos(k)}. \tag{L47}$$

$$z=e^{ik}\Rightarrow dk=\frac{e^{-ik}}{i}dz \tag{L48}$$

$$\oint\frac{dz}{2\pi i}\frac{z^{R-R'}}{z\left(\mathcal{E}-\mathfrak{t}\left(z+z^{-1}\right)\right)} \tag{L49}$$

$$= \oint\frac{dz}{2\pi i}\frac{z^{R-R'}}{\mathcal{E}z-\mathfrak{t}z^2-\mathfrak{t}}. \tag{L50}$$

$$z = \frac{\mathcal{E} \pm \sqrt{\mathcal{E}^2 - 4\mathfrak{t}^2}}{2\mathfrak{t}} \equiv z_- \text{ or } z_+, \tag{L51}$$

$$\langle R|\hat{G}_0(\mathcal{E})|R'\rangle = \frac{\mathcal{E}}{|\mathcal{E}|}\frac{1}{\sqrt{\mathcal{E}^2 - 4\mathfrak{t}^2}}\left[\frac{\mathcal{E}}{2\mathfrak{t}} - \frac{\mathcal{E}}{|\mathcal{E}|}\sqrt{\left(\frac{\mathcal{E}}{2\mathfrak{t}}\right)^2 - 1}\right]^{|R-R'|}, \tag{L52}$$

$$\langle R|\hat{G}_0(\mathcal{E}_r \pm i\eta)|R\rangle = \frac{(-)\pm i}{\sqrt{4\mathfrak{t}^2 - \mathcal{E}_r^2}}\left[\left(\frac{\mathcal{E}_r}{2\mathfrak{t}}\right) \pm \frac{1}{i}\sqrt{1 - \left(\frac{\mathcal{E}_r}{2\mathfrak{t}}\right)^2}\right]^{|R-R'|}. \tag{L53}$$

$$\langle 0|\hat{G}_0(\mathcal{E})|0\rangle = \frac{1}{N}\sum_{k_1k_2}\frac{1}{\mathcal{E} - 2\mathfrak{t}[\cos 2\pi k_1/\sqrt{N} + \cos 2\pi k_2/\sqrt{N}]} \tag{L54}$$

$$= \frac{1}{(2\pi)^2}\int_0^{2\pi} dk_1 \int_0^{2\pi} dk_2 \frac{1}{\mathcal{E} - 2\mathfrak{t}[\cos k_1 + \cos k_2]} \tag{L55}$$

$$= \frac{1}{(2\pi)^2}\int dk_1 dk_2 \frac{1}{\delta\mathcal{E} - 4\mathfrak{t} - 2\mathfrak{t}[\cos k_1 + \cos k_2]} \tag{L56}$$

$$\sim \quad \frac{1}{(2\pi)}\int kdk\,\frac{1}{\delta\mathcal{E}-\mathfrak{t}k^2} \tag{L57}$$

$$\sim \quad \frac{\ln(-\delta\mathcal{E}/\mathfrak{t})}{4\pi\mathfrak{t}}. \tag{L58}$$

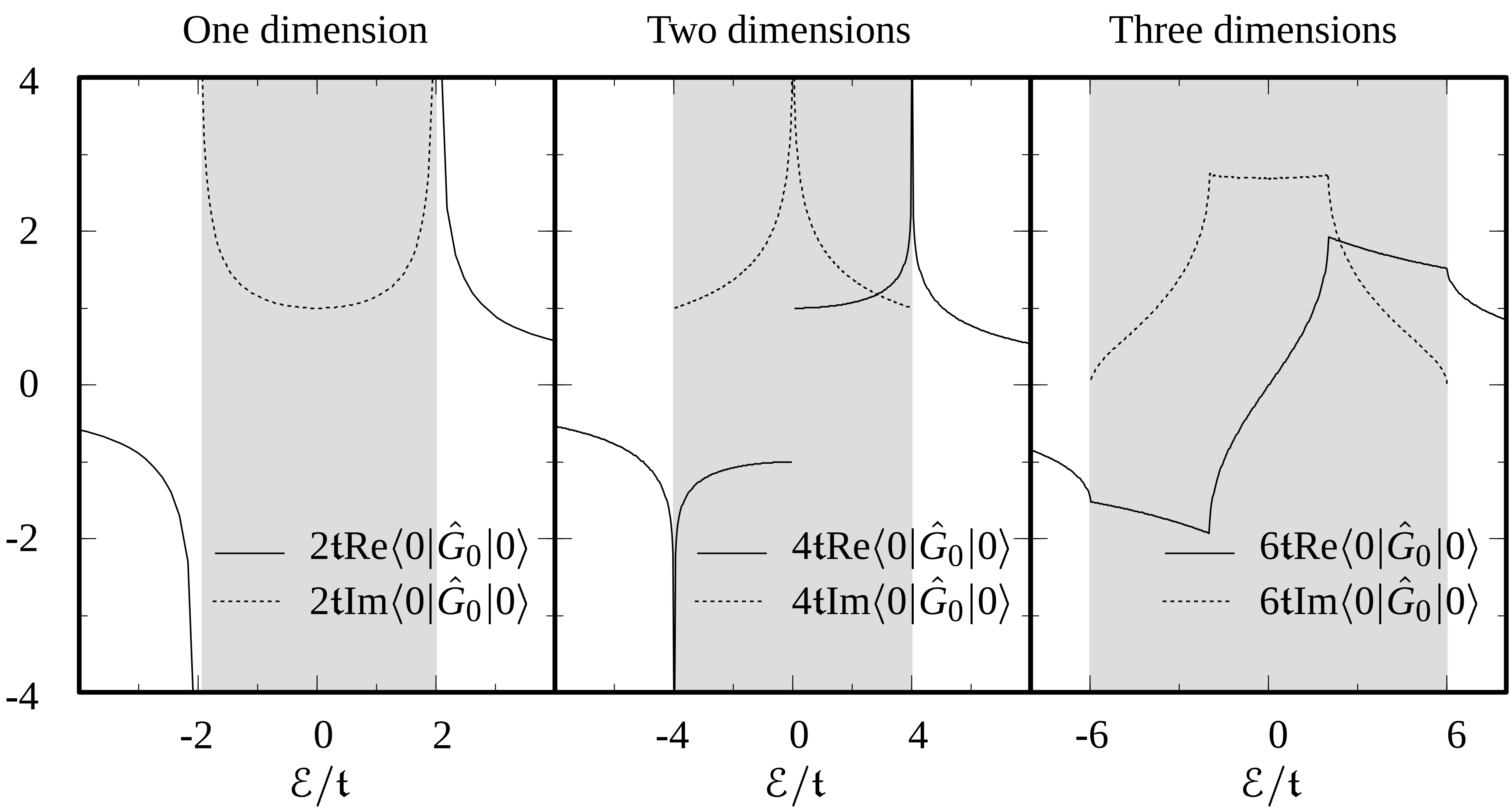


Figure 4: Green's functions for perfect square tight-binding lattice in one, two and three dimensions.

Adding Impurities

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_1, \tag{L59}$$

$$\hat{G}_0 = (\mathcal{E} - \hat{\mathcal{H}}_0)^{-1} \tag{L60}$$

$$\begin{aligned}
\hat{G} &= (\mathcal{E} - \hat{\mathcal{H}})^{-1} = (\mathcal{E} - \hat{\mathcal{H}}_0 - \hat{\mathcal{H}}_1)^{-1} && \text{(L61)} \\
&= ((\mathcal{E} - \hat{\mathcal{H}}_0)(1 - (\mathcal{E} - \hat{\mathcal{H}}_0)^{-1}\hat{\mathcal{H}}_1))^{-1} = (1 - \hat{G}_0\hat{\mathcal{H}}_1)^{-1}\hat{G}_0 && \text{(L62)} \\
&= \sum_{j=0}^{\infty}(\hat{G}_0\hat{\mathcal{H}}_1)^j\hat{G}_0 = \hat{G}_0 + \hat{G}_0\hat{\mathcal{H}}_1\hat{G}_0 + \hat{G}_0\hat{\mathcal{H}}_1\hat{G}_0\hat{\mathcal{H}}_1\hat{G}_0 + \dots && \text{(L63)} \\
&= \hat{G}_0 + \hat{G}_0\hat{\mathcal{H}}_1\hat{G} = \hat{G}_0 + \hat{G}\hat{\mathcal{H}}_1\hat{G}_0. && \text{(L64)}
\end{aligned}$$

$$\hat{G} \equiv \hat{G}_0 + \hat{G}_0\hat{T}\hat{G}_0. \tag{L65}$$

Single Impurity

$$\hat{G} = \hat{G}_0 + \hat{G}_0|0\rangle U_0\langle 0|\hat{G}_0 + \hat{G}_0|0\rangle U_0\langle 0|\hat{G}_0|0\rangle U_0\langle 0|\hat{G}_0 + \ldots \quad \text{(L66)}$$

$$= \hat{G}_0 + \hat{G}_0|0\rangle U_0\langle 0|\hat{G}_0 \sum_{p=0}^{\infty} \Big(U_0\langle 0|\hat{G}_0|0\rangle \Big)^p \quad \text{(L67)}$$

$$= \hat{G}_0 + \frac{\hat{G}_0|0\rangle U_0\langle 0|\hat{G}_0}{1 - U_0\langle 0|\hat{G}_0|0\rangle}. \quad \text{(L68)}$$

$$\hat{G}_0 \sim \frac{|\mathcal{E}_{n0}\rangle\langle\mathcal{E}_{n0}|}{\mathcal{E} - \mathcal{E}_{n0}} \quad \text{(L69)}$$

$$\Rightarrow \hat{G} \sim \frac{|\mathcal{E}_{n0}\rangle\langle\mathcal{E}_{n0}|}{\mathcal{E} - \mathcal{E}_{n0}} - \frac{|\mathcal{E}_{n0}\rangle\langle\mathcal{E}_{n0}|0\rangle\langle 0|\mathcal{E}_{n0}\rangle\langle\mathcal{E}_{n0}|}{(\mathcal{E} - \mathcal{E}_{n0})\langle 0|\mathcal{E}_{n0}\rangle\langle\mathcal{E}_{n0}|0\rangle}, \quad \text{(L70)}$$

$$1 - U_0\langle 0|\hat{G}_0(\mathcal{E})|0\rangle = 0. \quad \text{(L71)}$$

$$1 - U_0\langle 0|\hat{G}_0(\mathcal{E})|0\rangle \approx -U_0\langle 0|\hat{G}_0'(\mathcal{E}_n)|0\rangle(\mathcal{E} - \mathcal{E}_n) \quad \text{(L72)}$$

with

$$\hat{G}_0' = \frac{\partial \hat{G}_0}{\partial \mathcal{E}}, \tag{L73}$$

$$\Rightarrow \frac{|\mathcal{E}_n\rangle\langle\mathcal{E}_n|}{\mathcal{E}-\mathcal{E}_n} \sim \frac{\hat{G}_0(\mathcal{E}_n)|0\rangle\langle 0|\hat{G}_0(\mathcal{E}_n)}{-\langle 0|\hat{G}_0'(\mathcal{E}_n)|0\rangle}\frac{1}{\mathcal{E}-\mathcal{E}_n} \tag{L74}$$

$$\Rightarrow |\mathcal{E}_n\rangle = \frac{\hat{G}_0(\mathcal{E}_n)|0\rangle}{\sqrt{-\langle 0|\hat{G}_0'(\mathcal{E}_n)|0\rangle}}. \tag{L75}$$

$$\mathcal{E} = \pm\sqrt{4\mathfrak{t}^2 + U_0^2}. \tag{L76}$$

$$\mathcal{E} = -4\mathfrak{t} - \mathfrak{t}e^{-4\pi\mathfrak{t}/|U_0|}. \tag{L77}$$

Coherent Potential Approximation

$$\hat{\mathcal{H}} = \mathcal{H}_0 + \sum_m (U_m - \Sigma)|m\rangle\langle m| + \Sigma, \tag{L78}$$

$$\hat{\mathcal{H}}_0^\Sigma = \mathcal{H}_0 + \Sigma, \quad \hat{\mathcal{H}}_1^\Sigma = \sum_m (U_m - \Sigma)|m\rangle\langle m| \tag{L79}$$

$$\hat{G}_0^\Sigma(\mathcal{E}) = \hat{G}_0(\mathcal{E} - \Sigma). \tag{L80}$$

$$\hat{G} = \hat{G}_0^\Sigma + \hat{G}_0^\Sigma \hat{T}^\Sigma \hat{G}_0^\Sigma, \tag{L81}$$

$$\hat{T}^\Sigma \approx \sum_m \hat{T}_m^\Sigma, \tag{L82}$$

$$\hat{T}_m^\Sigma = \frac{|m\rangle(U_m - \Sigma)\langle m|}{1 - (U_m - \Sigma)\langle m|\hat{G}_0^\Sigma|m\rangle} \tag{L83}$$

$$\hat{G} = \hat{G}_0^\Sigma + \hat{G}_0^\Sigma \hat{T}^\Sigma \hat{G}_0^\Sigma. \tag{L84}$$

$$\overline{\hat{G}} = \hat{G}_0^{\Sigma}(\mathcal{E}) = \hat{G}_0(\mathcal{E} - \Sigma), \tag{L85}$$

$$\overline{T_m} = 0 \tag{L86}$$

$$\Rightarrow \int dU \mathcal{P}(U) \frac{(U - \Sigma)}{1 - (U - \Sigma)\langle 0|\hat{G}_0^{\Sigma}|0\rangle} = 0. \tag{L87}$$

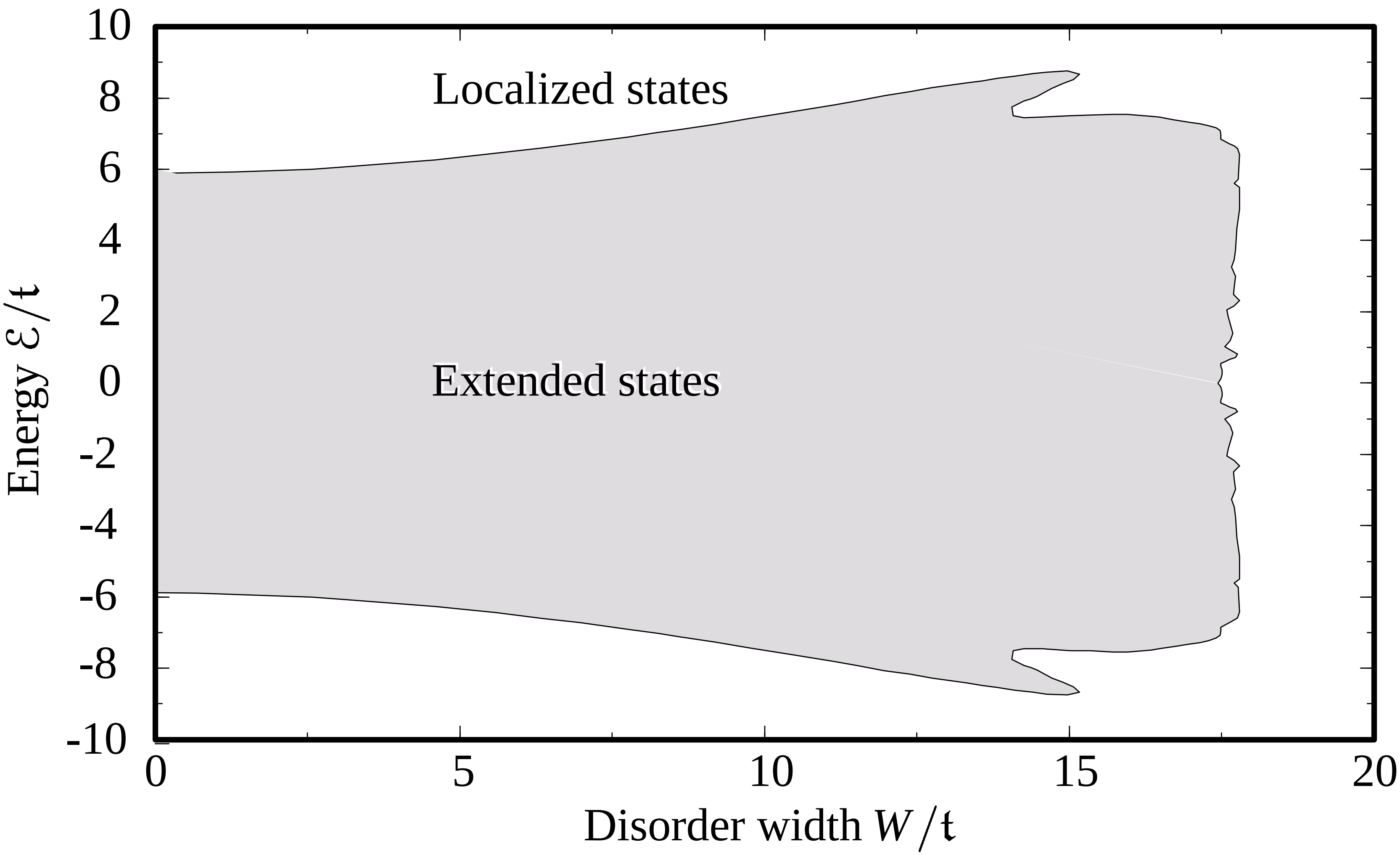


Figure 5: Calculation of the mobility edge.

Exact Results in One Dimension

$$\mathcal{P}(U) = \frac{1}{W}\theta\left(\frac{W}{2} - U\right)\theta\left(\frac{W}{2} + U\right). \tag{L88}$$

$$\langle l|\hat{G}_0|m\rangle = \frac{\delta_{lm}}{\mathcal{E} - U_l}; \tag{L89}$$

$$\lambda^{-1} \equiv \lim_{m\to\infty} -\frac{1}{2m}\overline{\ln|\langle 0|\hat{G}|m\rangle|^2}, \tag{L90}$$

$$\langle m|\hat{G}(\mathcal{E} - i\eta)|0\rangle = \sum_n \frac{\langle m|\mathcal{E}_n\rangle\langle\mathcal{E}_n|0\rangle}{\mathcal{E} - \mathcal{E}_n - i\eta}. \tag{L91}$$

$$\langle m|\hat{G}(\mathcal{E} - i\eta)|0\rangle = \mathcal{V}\int d\mathcal{E}'\, D(\mathcal{E}')\frac{\langle m|\mathcal{E}'\rangle\langle\mathcal{E}'|0\rangle}{\mathcal{E} - \mathcal{E}' - i\eta} \tag{L92}$$

$$\approx \quad \frac{\mathcal{V}\lambda}{N} D(\mathcal{E}) i\pi\langle m|\mathcal{E}\rangle \tag{L93}$$

$$\approx \quad \frac{\mathcal{V}\lambda}{N} D(\mathcal{E}) i\pi e^{i\phi} e^{-m/\lambda}, \tag{L94}$$

Exact Results in One Dimension

$$\mathfrak{t} \sum_{\langle l'm' \rangle} |l'\rangle\langle m'|, \tag{L95}$$

$$\langle l|\hat{G}|m\rangle = \langle l|\hat{G}_0|m\rangle + \langle l|\hat{G}_0 \sum_{\langle l_1 m_1 \rangle} |l_1\rangle \mathfrak{t} \langle m_1|\hat{G}_0|m\rangle + \ldots. \tag{L96}$$

$$l = l_1 \to m_1 = l_2 \to m_2 \ldots \to m \tag{L97}$$

$$\langle l|\hat{G}_0|l\rangle \mathfrak{t} \langle l+1|\hat{G}_0|l+1\rangle \mathfrak{t} \ldots \mathfrak{t} \langle m|\hat{G}_0|m\rangle. \tag{L98}$$

$$\langle l|\hat{G}|m\rangle = \langle l|\hat{G}|l\rangle \mathfrak{t} \langle l+1|\hat{G}^l|m\rangle. \tag{L99}$$

$$\langle l|\hat{G}|m\rangle = \langle l|\hat{G}|l\rangle \mathfrak{t} \langle l+1|\hat{G}^l|l+1\rangle \mathfrak{t} \langle l+2|\hat{G}^{l+1}|l+2\rangle \ldots \mathfrak{t} \langle m|\hat{G}^{m-1}|m\rangle. \tag{L100}$$

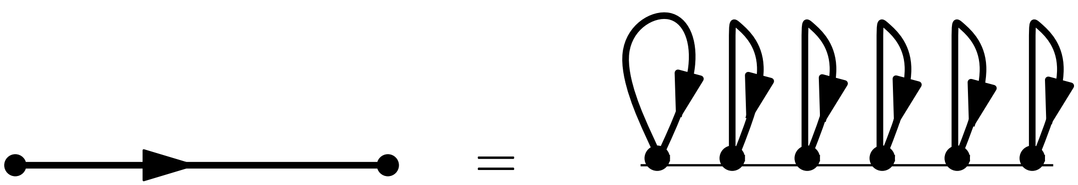


Figure 6: Diagram corresponding to Eq. (L100).

$$\lambda^{-1} = -\overline{\ln[|\langle l+1|\mathfrak{t}\hat{G}^l|l+1\rangle|]}; \tag{L101}$$

$$\langle l+1|\hat{G}^l|l+1\rangle = \langle l+1|\hat{G}_0|l+1\rangle + \begin{bmatrix} & \langle l+1|\hat{G}_0|l+1\rangle \\ \times\mathfrak{t} & \langle l+2|\hat{G}^{l+1}|l+2\rangle \\ \times\mathfrak{t} & \langle l+1|\hat{G}^l|l+1\rangle \end{bmatrix} \tag{L102}$$

$$\Rightarrow \quad \langle l+1|\hat{G}^l|l+1\rangle = \frac{1}{\mathcal{E} - U_{l+1} - \mathfrak{t}^2\langle l+2|\hat{G}^{l+1}|l+2\rangle}. \tag{L103}$$

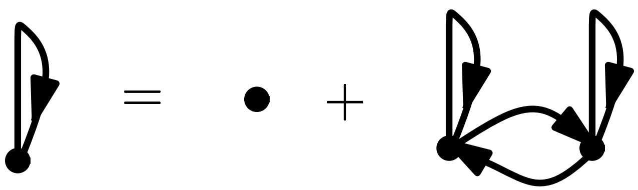


Figure 7: Diagram corresponding to Eq. (L103).

$$\mathcal{F}(g,\mathcal{E}) \quad = \quad \int \prod_m [dU_m \mathcal{P}(U_m)]\delta(g - \langle 1|\mathfrak{t}\hat{G}^0(\mathcal{E})|1\rangle) \tag{L104}$$

$$= \int \prod_m [dU_m \mathcal{P}(U_m)] \delta\left(g - \frac{\mathfrak{t}}{\mathcal{E} - U_1 - \mathfrak{t}^2 \langle 2|\hat{G}^1|2\rangle}\right) \tag{L105}$$

$$= \int \frac{\mathfrak{t}}{g^2} \prod_{m \neq 1} [dU_m \mathcal{P}(U_m)] \mathcal{P}\left(\mathcal{E} - \frac{\mathfrak{t}}{g} - \mathfrak{t}^2 \langle 2|\hat{G}^1|2\rangle\right) \tag{L106}$$

$$= \frac{\mathfrak{t}}{g^2} \int \prod_m [dU_m \mathcal{P}(U_m)] \int dg' \, \mathcal{P}\left(\mathcal{E} - \frac{\mathfrak{t}}{g} - \mathfrak{t} g'\right) \delta(g' - \mathfrak{t}\langle 2|\hat{G}^1|2\rangle) \tag{L107}$$

$$= \frac{\mathfrak{t}}{g^2} \int dg' \, \mathcal{P}\left(\mathcal{E} - \frac{\mathfrak{t}}{g} - \mathfrak{t} g'\right) \mathcal{F}(g', \mathcal{E}). \tag{L108}$$

$$\lambda^{-1} = 0.1142 \frac{\overline{U^2}}{\mathfrak{t}^2}. \tag{L109}$$

$$\lambda = \frac{105.045\mathfrak{t}^2}{W^2}. \tag{L110}$$

$$R_H \equiv h/e^2 = 25813\,\Omega. \tag{L111}$$

$$R_1(l) = R_1(L/L_0). \tag{L112}$$

$$R_2(l) = R_2(L/L_0), \tag{L113}$$

$$R_d \sim L^{2-d} \tag{L114}$$

$$R \sim e^{A_d L/L_0}. \tag{L115}$$

$$\beta_d(R) = \frac{L}{R}\frac{\partial R}{\partial L} = \frac{\partial \ln R}{\partial \ln L} = L\frac{\partial \ln R_d(L/L_0)}{\partial L} \tag{L116}$$

$$\beta_d(R) \sim 2 - d, \tag{L117}$$

$$\beta_d(R) \sim \frac{A_d L}{L_0} \sim \ln R. \tag{L118}$$

$$\ln(L/L_0) = \int \frac{d \ln R}{\beta_d(\ln R)} \tag{L119}$$

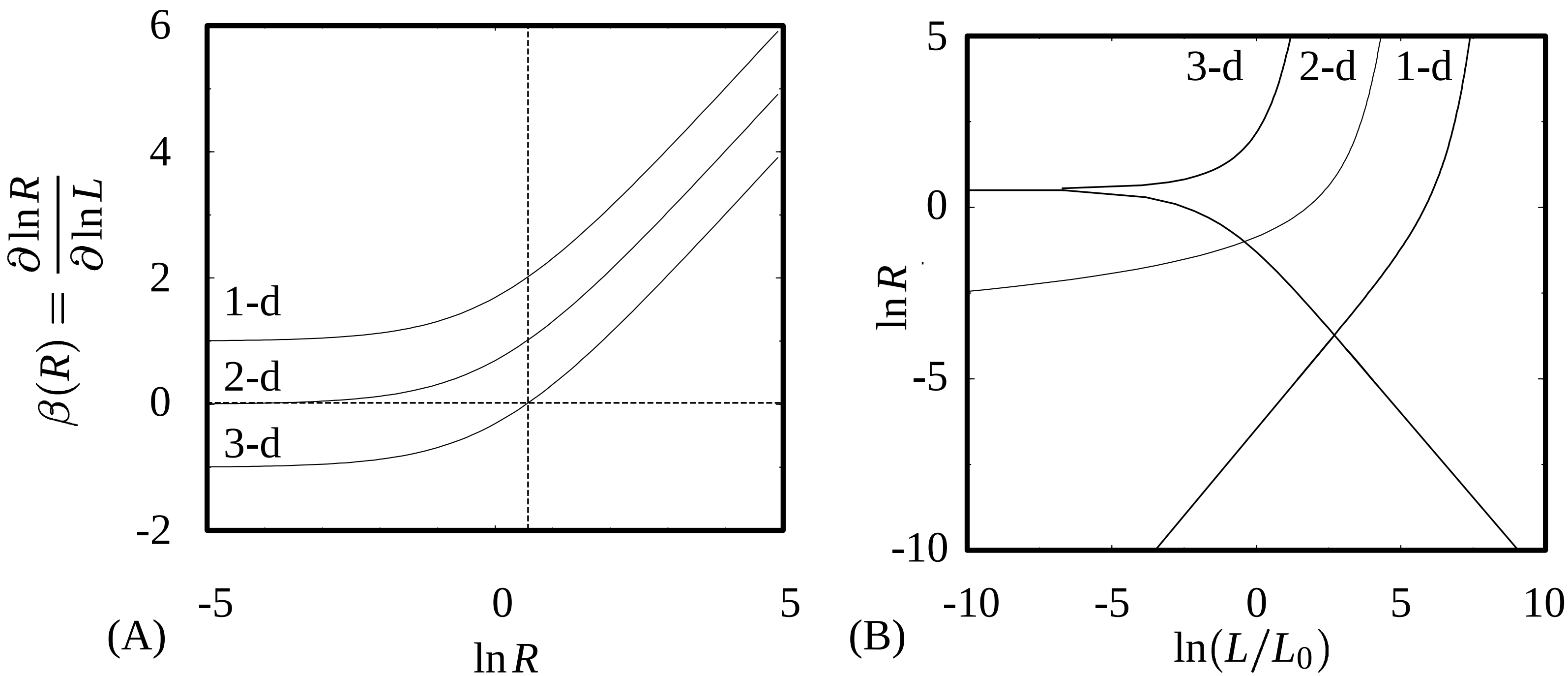


Figure 8: Scaling functions

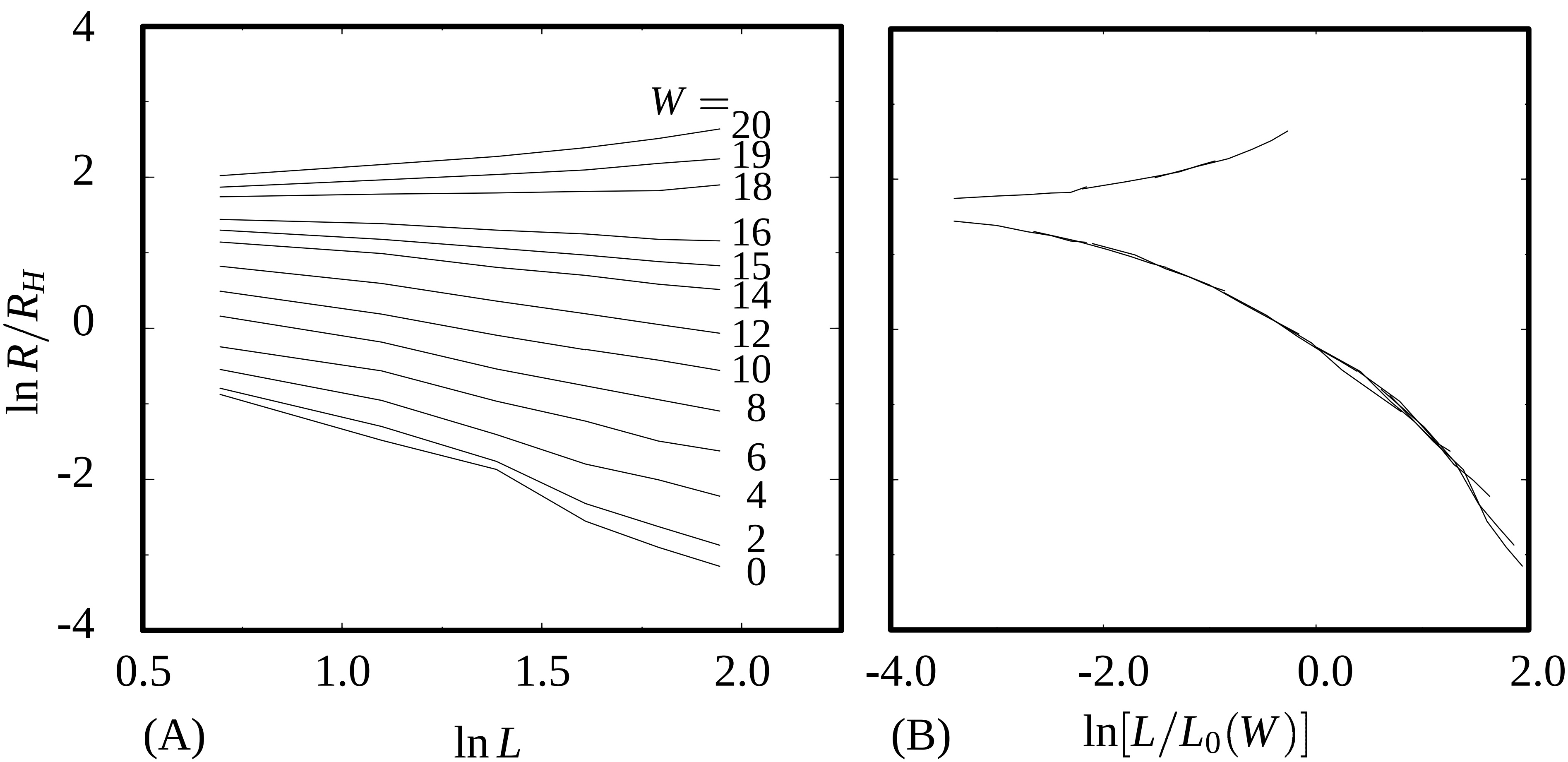


Figure 9: Three-dimensional scaling function for square lattice with diagonal disorder.

Comparison with Experiment

$$R = R_3\left(\frac{l_T}{L_0}\right). \tag{L120}$$

$$R = R_3\left(\frac{l_T}{L_0}\right)\frac{l_T}{L}. \tag{L121}$$

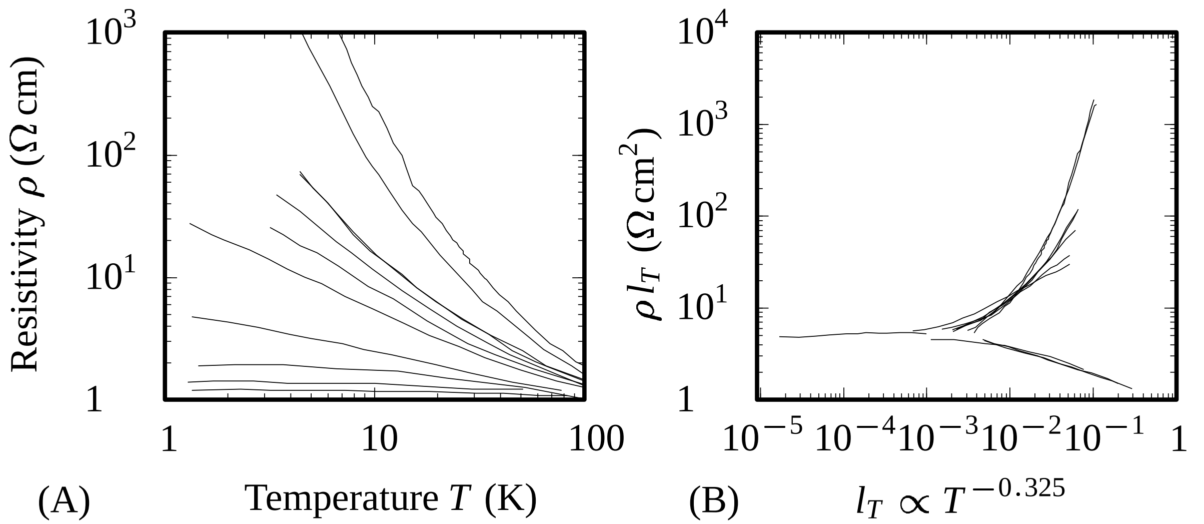


Figure 10: (A) Measurement of resistivity versus temperature in PPV, Ahlskog et al. (1997), p. 6779.