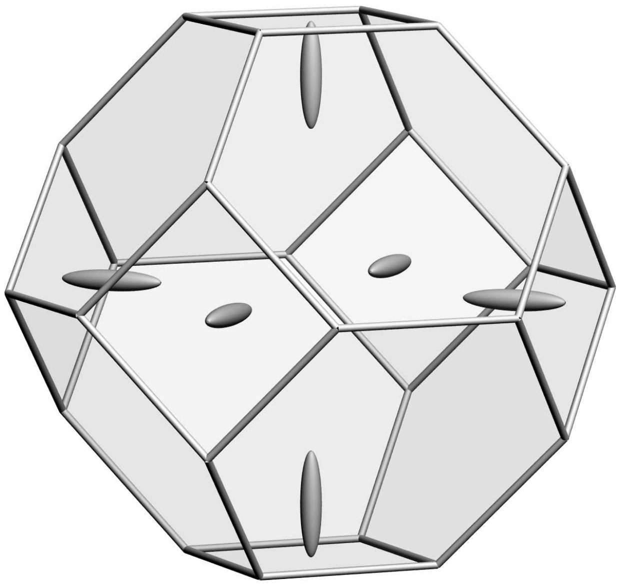
Electronics



Definitions

- ☞ Work Functions
- ☞ Schottky Barrier
- ☞ Intrinsic Semiconductors
- ☞ Doping
- ☞ Semiconductor Junctions
- ☞ Rectification
- ☞ Diodes and Transitors
- ☞ Heterostructures
- ☞ Two-Dimensional Electron Gas (2DEG)
- ☞ Quantum Point Contact
- ☞ Quantum Dot

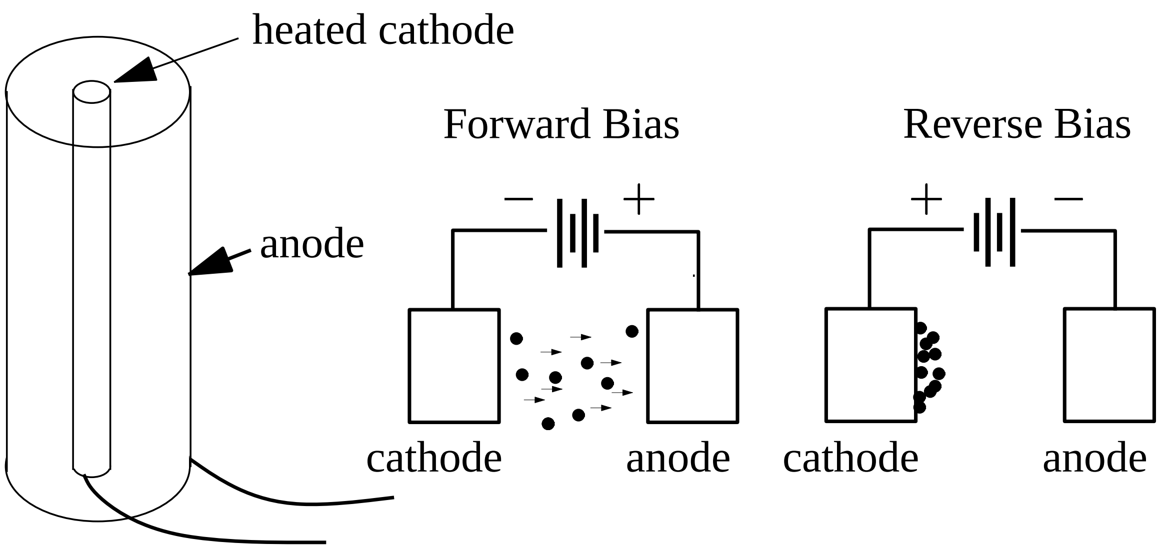


Figure 1: Operation of a diode.

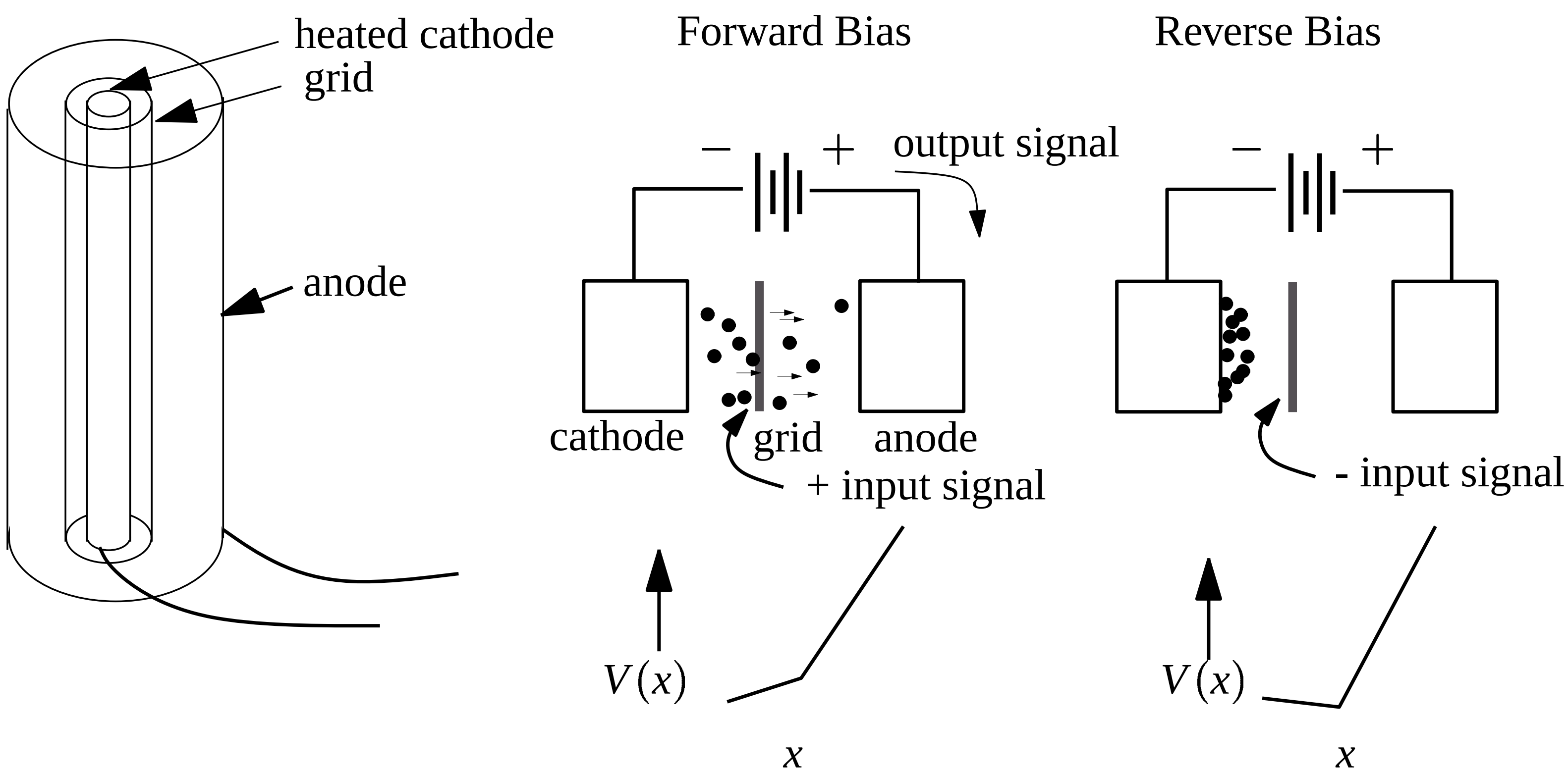


Figure 2: Operation of a triode.

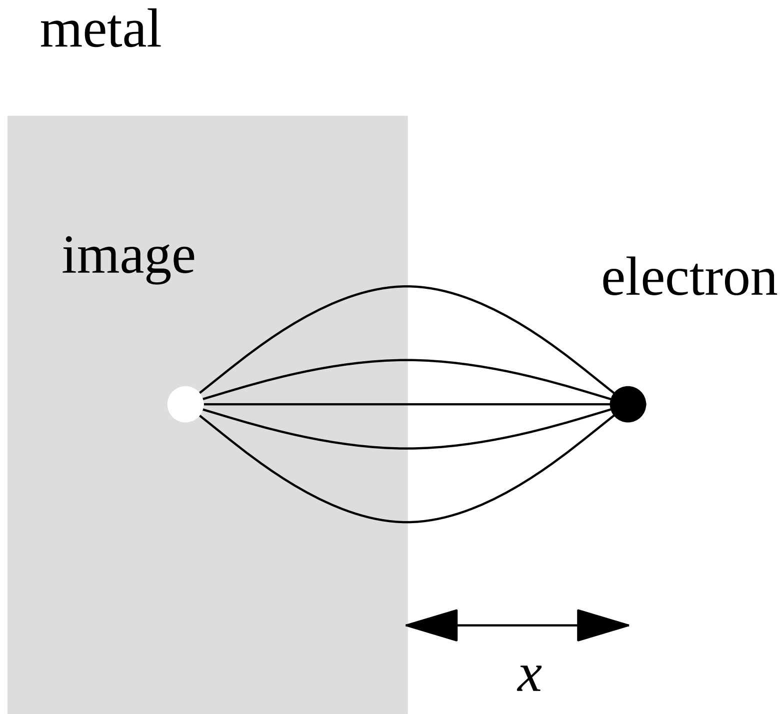


Figure 3: An electron attracted to metal surface.

$$F = \frac{e^2}{(2x)^2}, \tag{L1}$$

$$U(x) = -\frac{e^2}{4x} = -\frac{1}{x} 3.6 \cdot 10^{-4} \mu\text{m eV}. \tag{L2}$$

Work Functions

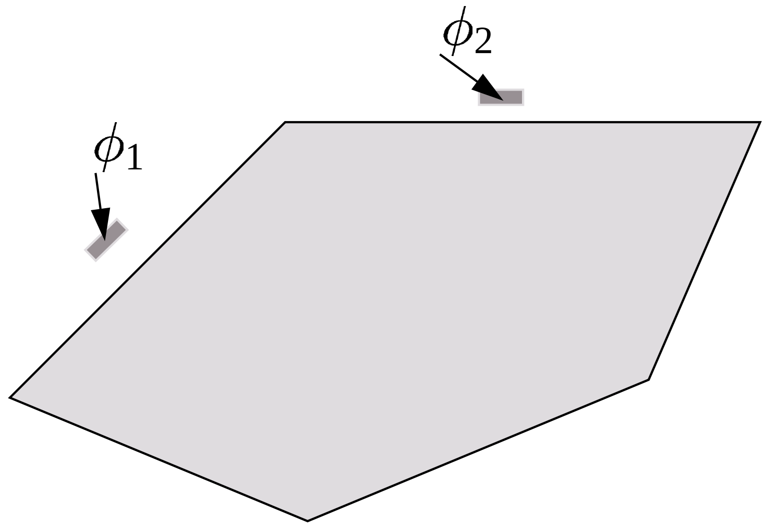


Figure 4: Work function.

Schottky Barrier

$$U(x) = -\frac{e^2}{4x} - e|E|x, \tag{L3}$$

$$x_0 = \sqrt{\frac{e}{4|E|}} \Rightarrow U(x_0) = -e\sqrt{e|E|}. \tag{L4}$$

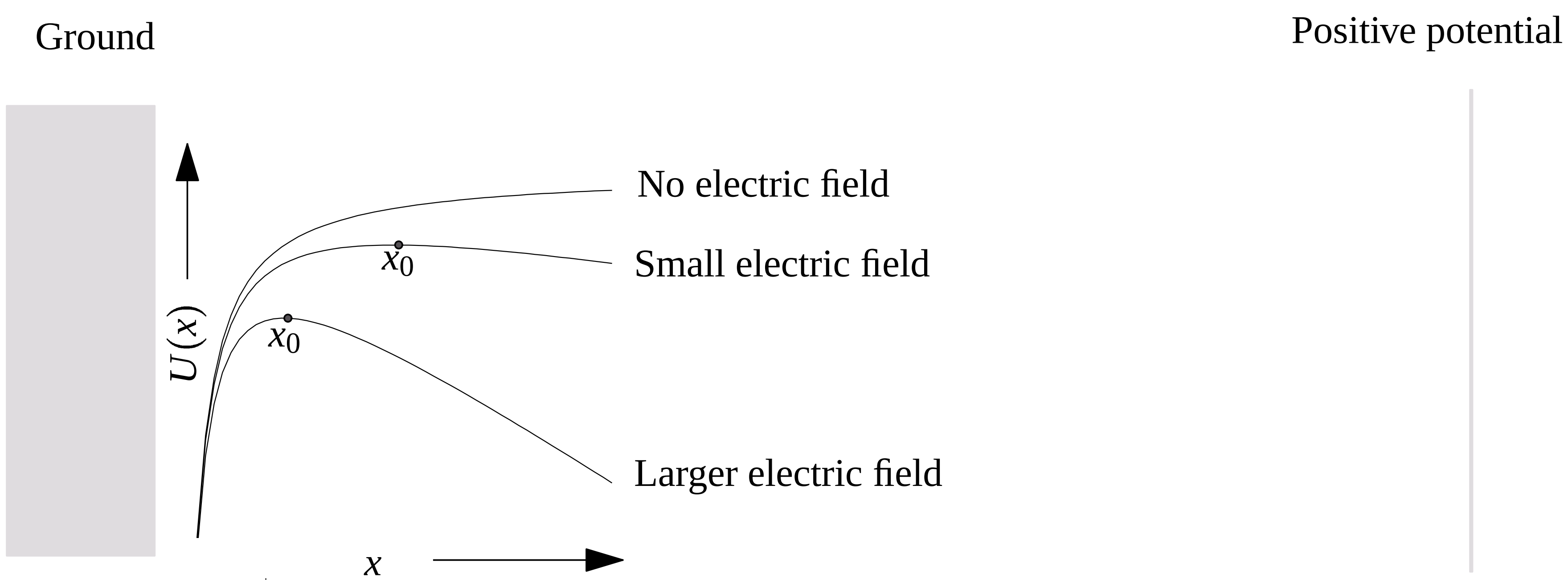


Figure 5: Schottky barrier

Richardson–Dushman Equation

$$f_{x\vec{k}} = \frac{1}{e^{\beta(\mathcal{E}^0_{\vec{k}}+U(x)-\mu)}+1}. \tag{L5}$$

$$f_{x\vec{k}} \approx e^{-\beta(\mathcal{E}^0_{\vec{k}}+U(x)+\phi)}. \tag{L6}$$

$$\begin{aligned} j &= -e\exp\{-\beta[\phi+U(x_0)]\}\int[d\vec{k}]\,\frac{\hbar k_x}{m}\theta(k_x)e^{-\beta\hbar^2k^2/2m} && \text{(L7)} \\ &= -\mathcal{A}T^2\exp\left\{-\beta\left[\phi-e\sqrt{e|E|}\right]\right\}, && \text{(L8)} \end{aligned}$$

where

$$\mathcal{A} = \frac{em}{2\pi^2\hbar^3}k_B^2 = 120.2\,\mathrm{A\,cm^{-2}\,K^{-2}}. \tag{L9}$$

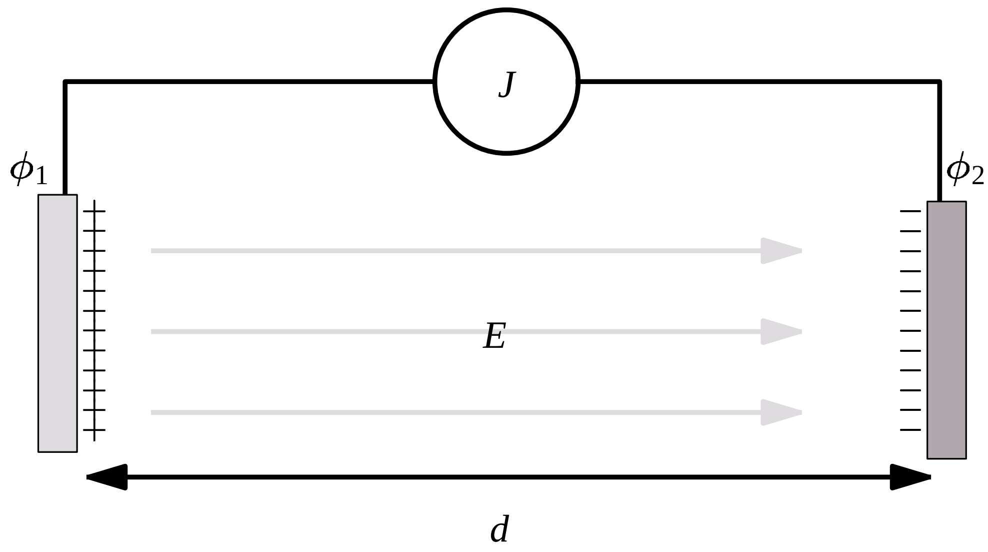


Figure 6: Contact potential of two metals

$$V = Ed = 4\pi\sigma d, \tag{L10}$$

$$\phi_2 - \phi_1 = 4\pi e\sigma d. \tag{L11}$$

Contact Potentials

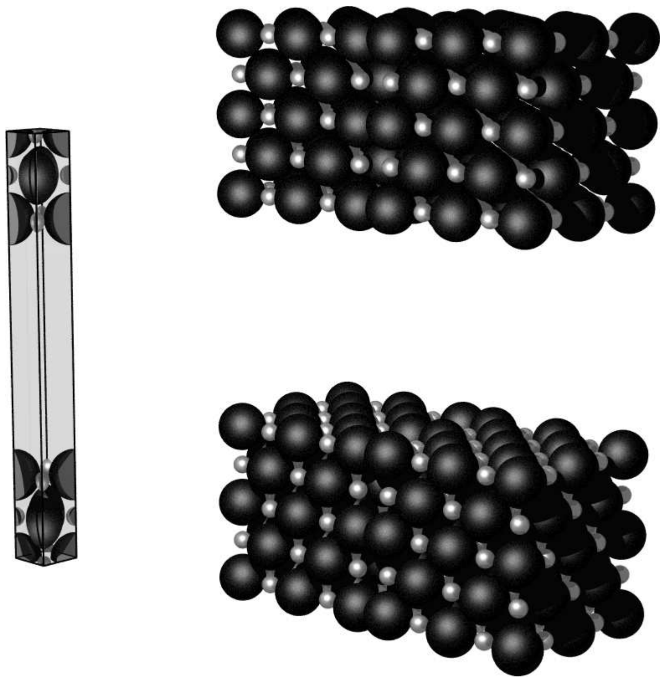


Figure 7: Periodic unit cell that produces surfaces

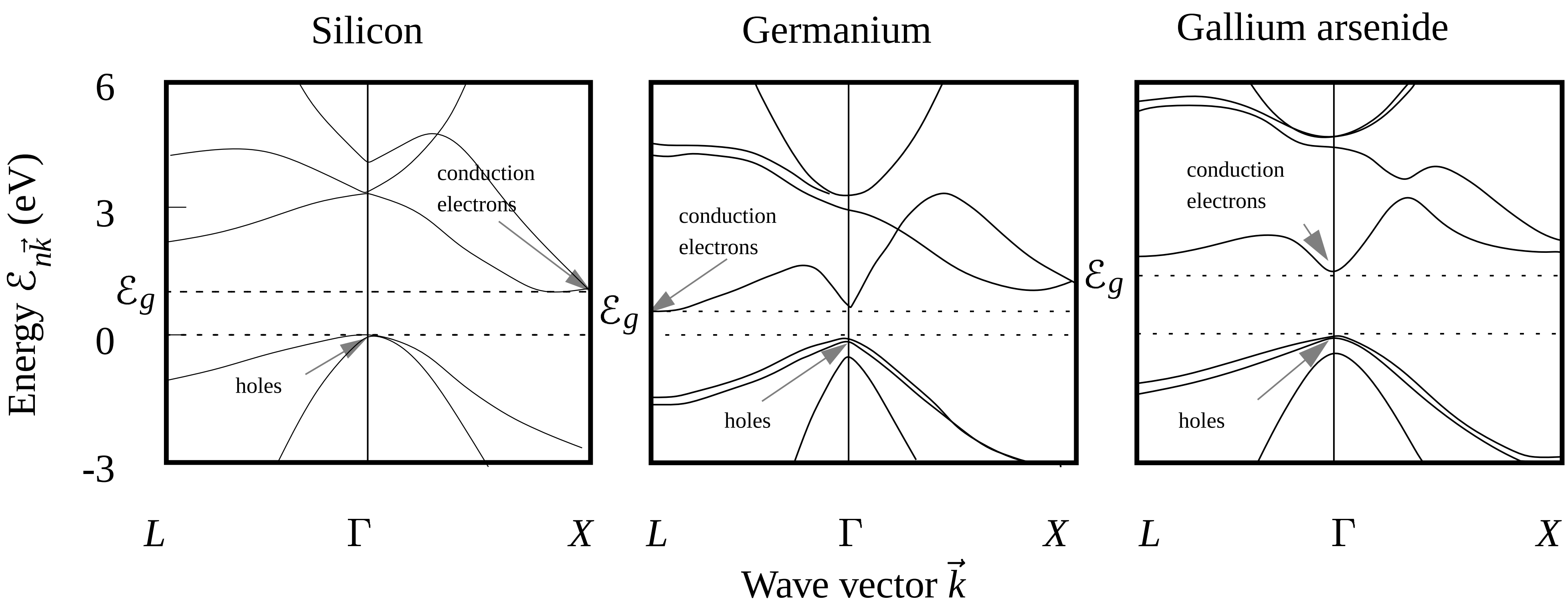


Figure 8: Essential features of band structures of silicon, germanium, and gallium arsenide.

$$e^{-\beta \mathcal{E}_g/2} \sim 10^{-9}. \tag{L12}$$

Pure Semiconductors

Com-pound		$\mathcal{E}_g$ (eV)	$d\mathcal{E}_g/dT$ (eV/K)	n_i (cm^{-3})	ϵ^0	$m_n^\star$ (m)	$m_{ph}^\star$ (m)	$m_{pl}^\star$ (m)	μ_n (cm^2/V s)	μ_p (cm^2/V s)
Si	i	1.11	$-9.0\cdot 10^{-5}$	$1.02\cdot 10^{10}$	11.9	1.18	0.54	0.15	1350	480
Ge	i	0.74	$-3.7\cdot 10^{-4}$	$2.33\cdot 10^{13}$	16.5	0.55	0.3	0.04	3900	1800
GaAs	d	1.43	$-3.9\cdot 10^{-4}$	$2\cdot 10^{6}$	12.5	0.067	0.50	0.07	7900	450
SiC	i	2.2	$-5.8\cdot 10^{-4}$		9.7	0.82	1		900	50
AlAs	i	2.14	$-4\cdot 10^{-4}$	$2\cdot 10^{17}$	10.0	0.5	0.5	0.26	294	
AlSb	i	1.63	$-4\cdot 10^{-4}$		12.0	0.3	1	0.5	200	400
GaN	d	3.44	$-6.7\cdot 10^{-4}$	$2\cdot 10^{17}$	12.0	0.3	1		440	
GaSb	d	0.7	$-3.7\cdot 10^{-4}$	10^{14}	15.7	0.05	0.3	0.04	7700	1600
InP	d	1.34	$-2.9\cdot 10^{-4}$	$1.2\cdot 10^{8}$	15.2	0.073	0.6	0.12	5400	150
InAs	d	0.36	$-3.5\cdot 10^{-4}$	$1.3\cdot 10^{15}$	15.2	0.027	0.4	0.03	30 000	450
InSb	d	0.18	$-2.8\cdot 10^{-4}$	$2.0\cdot 10^{16}$	16.8	0.013	0.4	0.02	77 000	850

$$\mathcal{E}_{\vec{k}} = \mathcal{E}_c + \frac{\hbar^2}{2}\vec{k}^*\mathbf{M}^{-1}\vec{k} \qquad \text{(L13a)}$$

$$\mathcal{E}_{\vec{k}} = \mathcal{E}_v - \frac{\hbar^2}{2}\vec{k}^*\mathbf{M}^{-1}\vec{k}, \qquad \text{(L13b)}$$

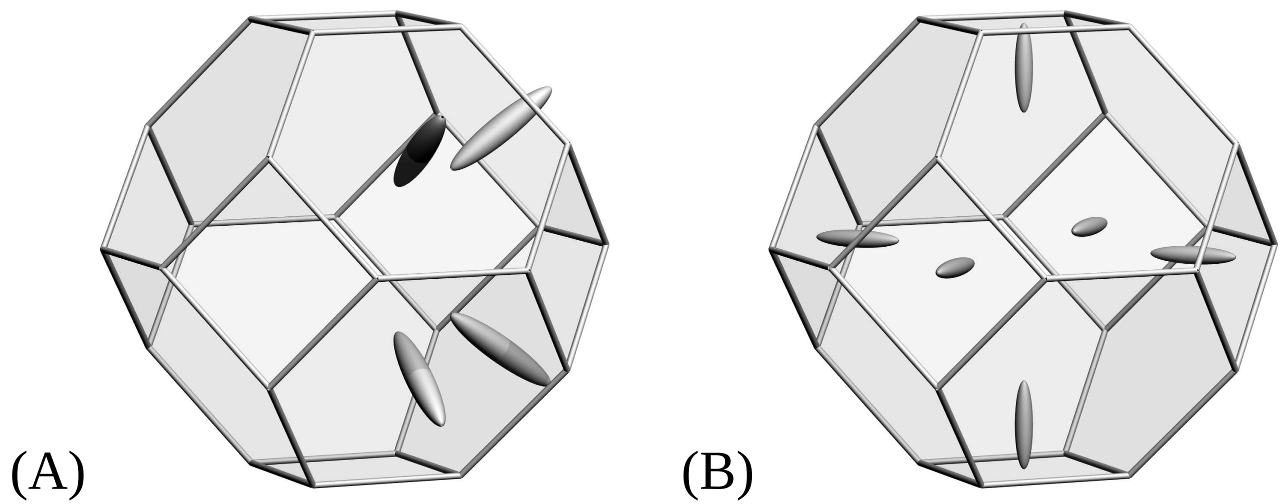


Figure 9: Semiconductor conduction band energy surfaces

$$n = \int_{\mathcal{E}_c}^{\infty} d\mathcal{E}\, D(\mathcal{E}) \frac{1}{e^{\beta(\mathcal{E}-\mu)}+1}, \tag{L14}$$

$$p = \int_{-\infty}^{\mathcal{E}_v} d\mathcal{E}\, D(\mathcal{E}) \left\{1 - \frac{1}{e^{\beta(\mathcal{E}-\mu)}+1}\right\} \tag{L15a}$$

$$= \int_{-\infty}^{\mathcal{E}_v} d\mathcal{E}\, D(\mathcal{E}) \frac{1}{e^{-\beta(\mathcal{E}-\mu)}+1}. \tag{L15b}$$

$$\mathcal{E}_c - \mu \gg k_B T \quad \text{and} \quad \mu - \mathcal{E}_v \gg k_B T. \tag{L16}$$

$$n = \mathcal{N}_c e^{-\beta(\mathcal{E}_c-\mu)}, \quad p = \mathcal{N}_v e^{-\beta(\mu-\mathcal{E}_v)} \tag{L17}$$

$$\mathcal{N}_c = \int_{\mathcal{E}_c}^{\infty} d\mathcal{E}\, D(\mathcal{E}) e^{-\beta(\mathcal{E}-\mathcal{E}_c)} \tag{L18a}$$

$$\mathcal{N}_v = \int_{-\infty}^{\mathcal{E}_v} d\mathcal{E}\, D(\mathcal{E}) e^{-\beta(\mathcal{E}_v-\mathcal{E})}. \tag{L18b}$$

Semiconductor in Equilibrium

$$D(\mathcal{E}) = \int [d\vec{k}]\,\delta\Big(\mathcal{E}-\mathcal{E}_c-\frac{1}{2}\hbar^2\vec{k}^*\mathbf{M}^{-1}\vec{k}\Big) \quad \text{(L19)}$$

$$= \int [d\vec{k}]\,\delta\Big(\mathcal{E}-\mathcal{E}_c-\frac{1}{2}\hbar^2\sum_l k_l^2/m_l\Big). \quad \text{(L20)}$$

$$m_n^\star = [m_1m_2m_3]^{1/3} \quad \text{and} \quad \vec{q} = (k_1/\sqrt{m_1}, k_2/\sqrt{m_2}, k_3/\sqrt{m_3}) \quad \text{(L21)}$$

$$D(\mathcal{E}) = 2\int m_n^{\star 3/2}\frac{d\vec{q}}{(2\pi)^3}\delta\Big(\mathcal{E}-\mathcal{E}_c-\frac{1}{2}\hbar^2q^2\Big) = \sqrt{2(\mathcal{E}-\mathcal{E}_c)}\frac{m_n^{\star 3/2}}{\hbar^3\pi^2}\mathcal{M}_c. \quad \text{(L22)}$$

$$\mathcal{N}_c = \frac{1}{4}\left(\frac{2m_n^\star k_BT}{\pi\hbar^2}\right)^{3/2}\mathcal{M}_c \quad \text{(L23)}$$

$$\mathcal{N}_v = \frac{1}{4}\left(\frac{2m_p^\star k_BT}{\pi\hbar^2}\right)^{3/2}. \quad \text{(L24)}$$

Mass action: $np = ?$ $?$. (L25)

$$a_* = \frac{\epsilon\hbar^2}{m^\star e^2} \quad \text{and} \quad \mathcal{E}_b = \frac{e^2}{2\epsilon a_*} = \frac{m^\star}{m}\frac{1}{\epsilon^2}\cdot 13.6\,\text{eV}. \qquad \text{(L26)}$$

Group V donors, $\mathcal{E}_c - \mathcal{E}_d$ (meV)										
Host	Eq. (L26)	N	P	As	Sb	Bi				
Si	113	140	45	53.7	42.7	70.6				
Ge	28		12.9	14.2	10.3	12.8				
Group III acceptors, $\mathcal{E}_a - \mathcal{E}_v$ (meV)										
Host	Eq. (L26)	B	In	Ga	Al	Tl				
Si	48	45	155	74	67	25				
Ge	15	9.73	12.0	11.3	10.8	13.5				
Donors, $\mathcal{E}_c - \mathcal{E}_d$ (meV)										
Host	Eq. (L26)	Pb	Se	Si	S	Ge	C			
GaAs	5.8	5.8	5.8	5.8	5.9	5.9	5.9			
Acceptors, $\mathcal{E}_a - \mathcal{E}_v$ (meV)										
Host	Eq. (L26)	Be	Mg	Zn	Cd	C	Si	Ge	Sn	Mn
GaAs	23	28	29	31	35	27	35	40	167	113
InP	21	31	31	46	57	41		210		270

$$n_i = \sqrt{\mathcal{N}_c\mathcal{N}_v} ? \qquad ? \qquad \text{(L27a)}$$

$$= 2.510\cdot 10^{19}\text{cm}^{-3}\left(\frac{m_n^\star m_p^\star}{m^2}\right)^{3/4}\mathcal{M}_c^{1/2}\left(\frac{T}{300\,\text{K}}\right)^{3/2} ? \qquad ?. \qquad \text{(L27b)}$$

Intrinsic Semiconductor

$$\mu_i = k_B T \ln \frac{n_i}{\mathcal{N}_c} + \mathcal{E}_c = \mathcal{E}_v + \frac{\mathcal{E}_g}{2} + \frac{3}{4} k_B T \ln(m_p^\star / m_n^\star) - \frac{1}{2} k_B T \ln \mathcal{M}_c. \tag{L28}$$

$$np = n_i^2 \tag{L29}$$

$$n = n_i e^{-\beta(\mu_i - \mu)}, \quad p = n_i e^{-\beta(\mu - \mu_i)}. \tag{L30}$$

Extrinsic Semiconductor

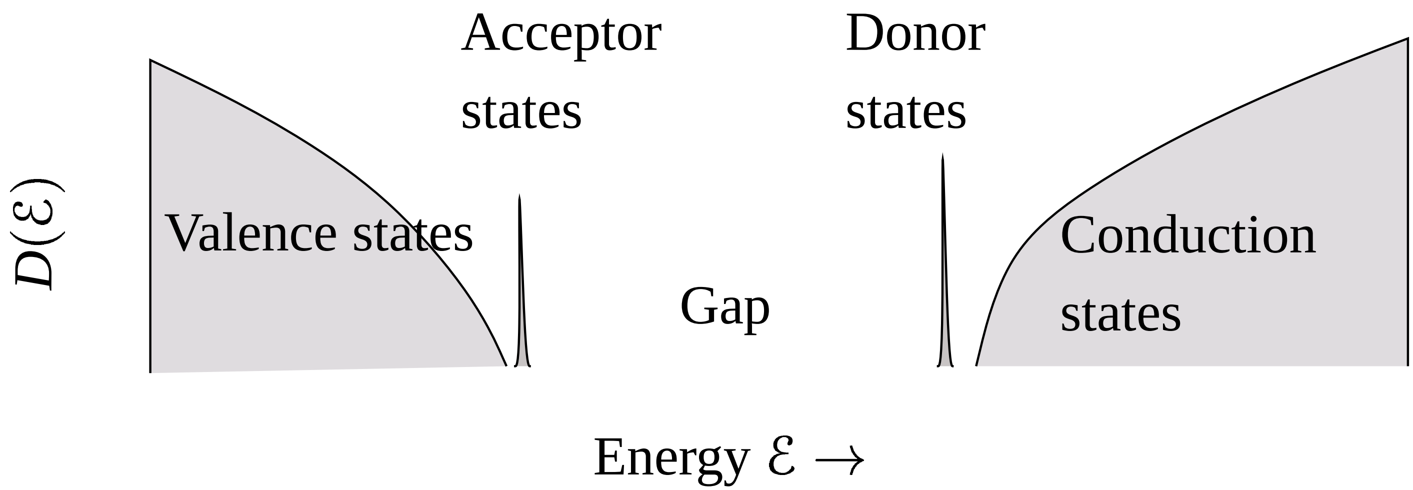


Figure 10: Densities of states with doping

$$f_d = \frac{0\times 1 + 1\times 2\times e^{-\beta(\mathcal{E}_d-\mu)}}{1+2\times e^{-\beta(\mathcal{E}_d-\mu)}} \quad \text{(L31)}$$

$$= \frac{1}{1+\frac{1}{2}e^{\beta(\mathcal{E}_d-\mu)}} \ll 1.. \quad \text{(L32)}$$

$$f_a = \frac{1}{\frac{1}{4}e^{\beta(\mu-\mathcal{E}_a)}+1} \ll 1. \quad \text{(L33)}$$

$$n_t + \mathcal{N}_d = \int_{\mathcal{E}_c} d\mathcal{E}\, D(\mathcal{E}) \frac{1}{1+e^{\beta(\mathcal{E}-\mu)}} + \int^{\mathcal{E}_v} d\mathcal{E}\, D(\mathcal{E}) \frac{1}{1+e^{\beta(\mathcal{E}-\mu)}} + \mathcal{N}_d f_d. \quad \text{(L34)}$$

$$\mathcal{N}_d = \int_{\mathcal{E}_c} d\mathcal{E}\, D(\mathcal{E}) \frac{1}{1+e^{\beta(\mathcal{E}-\mu)}} - \int^{\mathcal{E}_v} d\mathcal{E}\, D(\mathcal{E}) \frac{1}{1+e^{-\beta(\mathcal{E}-\mu)}} \quad \text{(L35)}$$

$$\Rightarrow \mathcal{N}_d = n - p = n_i e^{-\beta(\mu_i-\mu)} - n_i e^{-\beta(\mu-\mu_i)}. \quad \text{(L36)}$$

$$n - p = \mathcal{N}_d - \mathcal{N}_a. \quad \text{(L37)}$$

$$n = \frac{1}{2}[\mathcal{N}_d - \mathcal{N}_a] + \frac{1}{2}\left[(\mathcal{N}_d - \mathcal{N}_a)^2 + 4n_i^2\right]^{1/2} \quad \text{(L38a)}$$

$$p = \frac{1}{2}[\mathcal{N}_a - \mathcal{N}_d] + \frac{1}{2}\left[(\mathcal{N}_d - \mathcal{N}_a)^2 + 4n_i^2\right]^{1/2}. \quad \text{(L38b)}$$

$$n - p = 2n_i \sinh\beta(\mu-\mu_i) \Rightarrow \mu = \mu_i + k_B T \sinh^{-1}\left([\mathcal{N}_d - \mathcal{N}_a]/2n_i\right). \quad \text{(L39)}$$

$$n \approx ? \quad ? \tag{L40a}$$

$$p \approx ? \quad ?. \tag{L40b}$$

$$p \approx ? \quad ? \tag{L41a}$$

$$n \approx ? \quad ?. \tag{L41b}$$

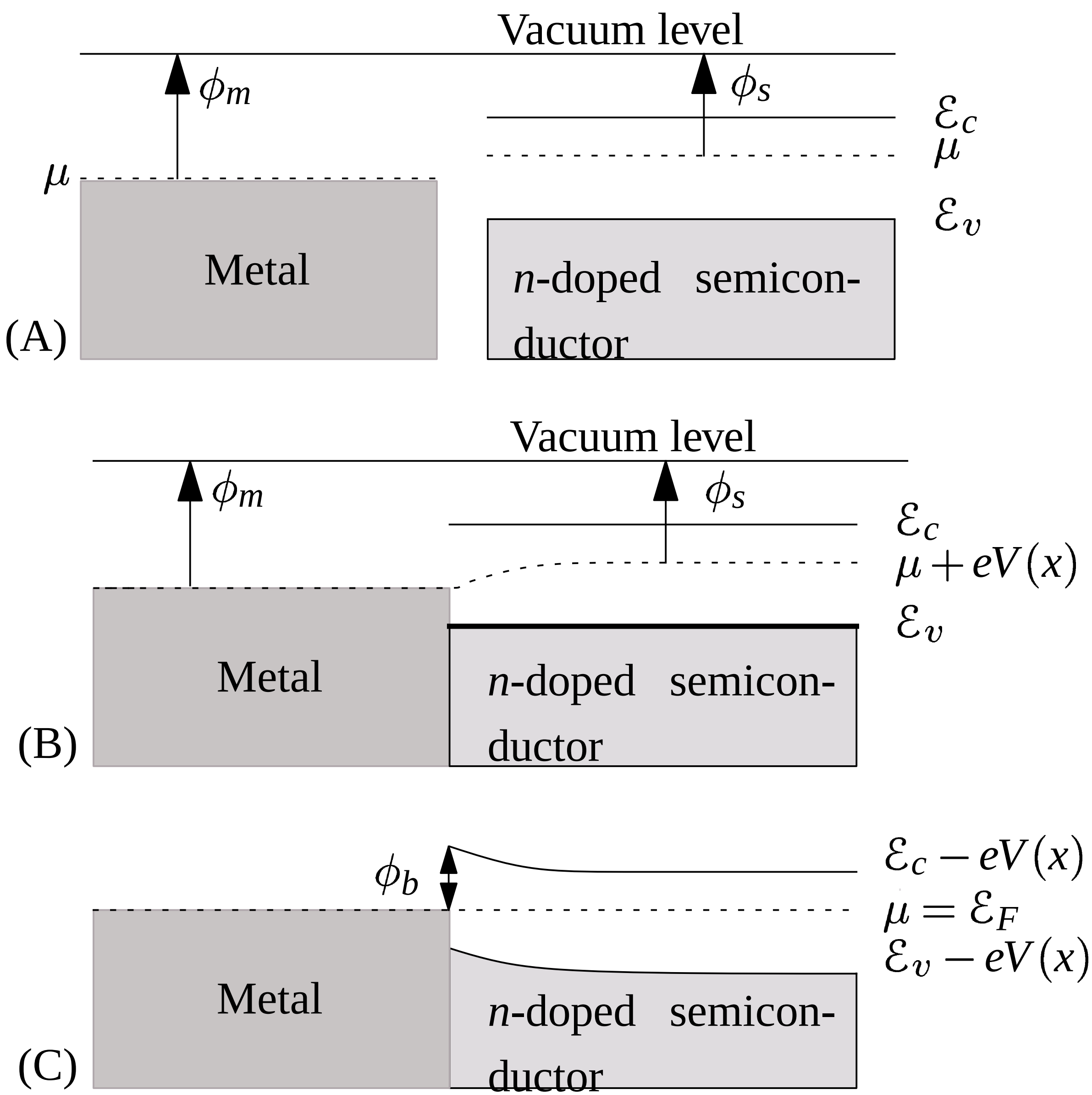


Figure 11: Schottky diode

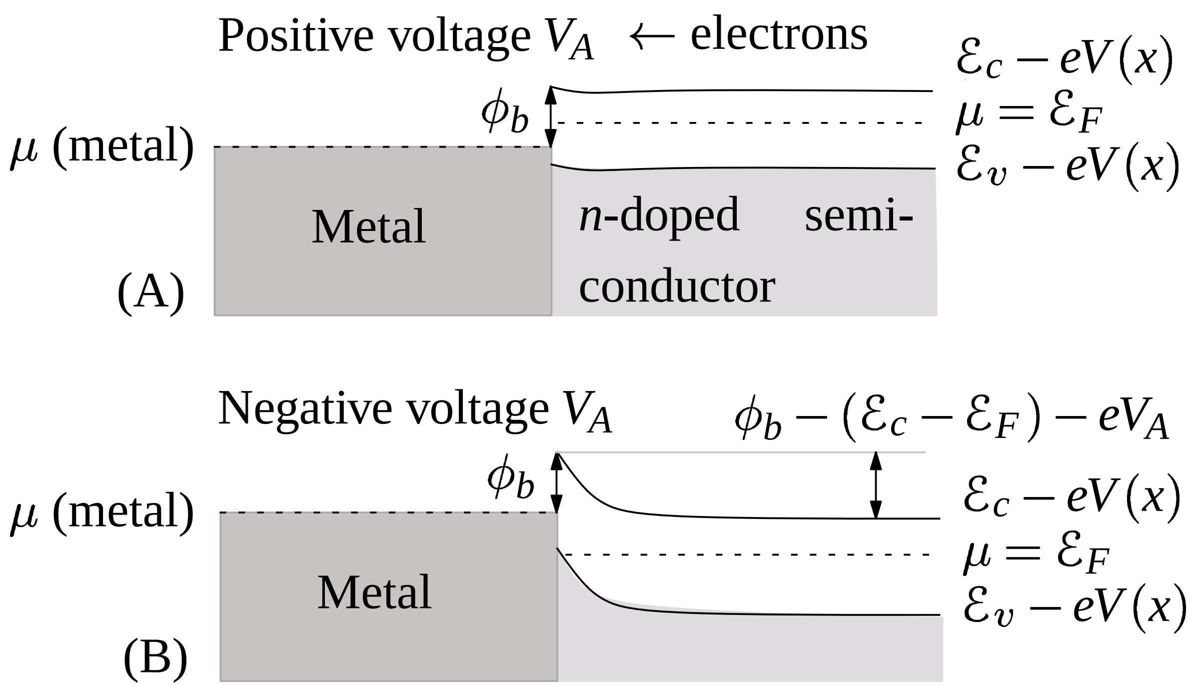


Figure 12: Biased Schottky diode

$$\frac{\hbar^2 k_x^2}{2m_n^\star} > \phi_b - (\mathcal{E}_c - \mu) - eV_A. \qquad \text{(L42)}$$

$$j_{s\rightarrow m} \quad = \quad \int [d\vec{k}]\, \theta\left(\frac{\hbar^2 k_x^2}{2m_n^\star} - [\phi_b - (\mathcal{E}_c - \mu) - eV_A]\right) \frac{e\hbar k_x}{m_n^\star}\, e^{-\beta(\hbar^2 k^2/2m_n^\star + \mathcal{E}_c - \mu)} \qquad \text{(L43)}$$

$$= \quad \frac{2}{(2\pi)^3} \frac{2m_n^\star \pi k_B T}{\hbar^2} \frac{e}{\hbar} \int_{\phi_b - \mathcal{E}_c + \mu - eV_A}^{\infty} d\left(\frac{\hbar^2 k_x^2}{2m_n^\star}\right) e^{-\beta(\hbar^2 k_x^2/2m_n^\star + \mathcal{E}_c - \mu)} \qquad \text{(L44)}$$

$$= \quad \frac{m_n^\star}{m} \mathcal{A} T^2 \exp\{-\beta[\phi_b - eV_A]\}. \qquad \text{(L45)}$$

$$j = \frac{m_n^\star}{m} \mathcal{A} T^2 \left[\exp\{-\beta[\phi_b - eV_A]\} - \exp\{-\beta\phi_b\}\right]. \qquad \text{(L46)}$$

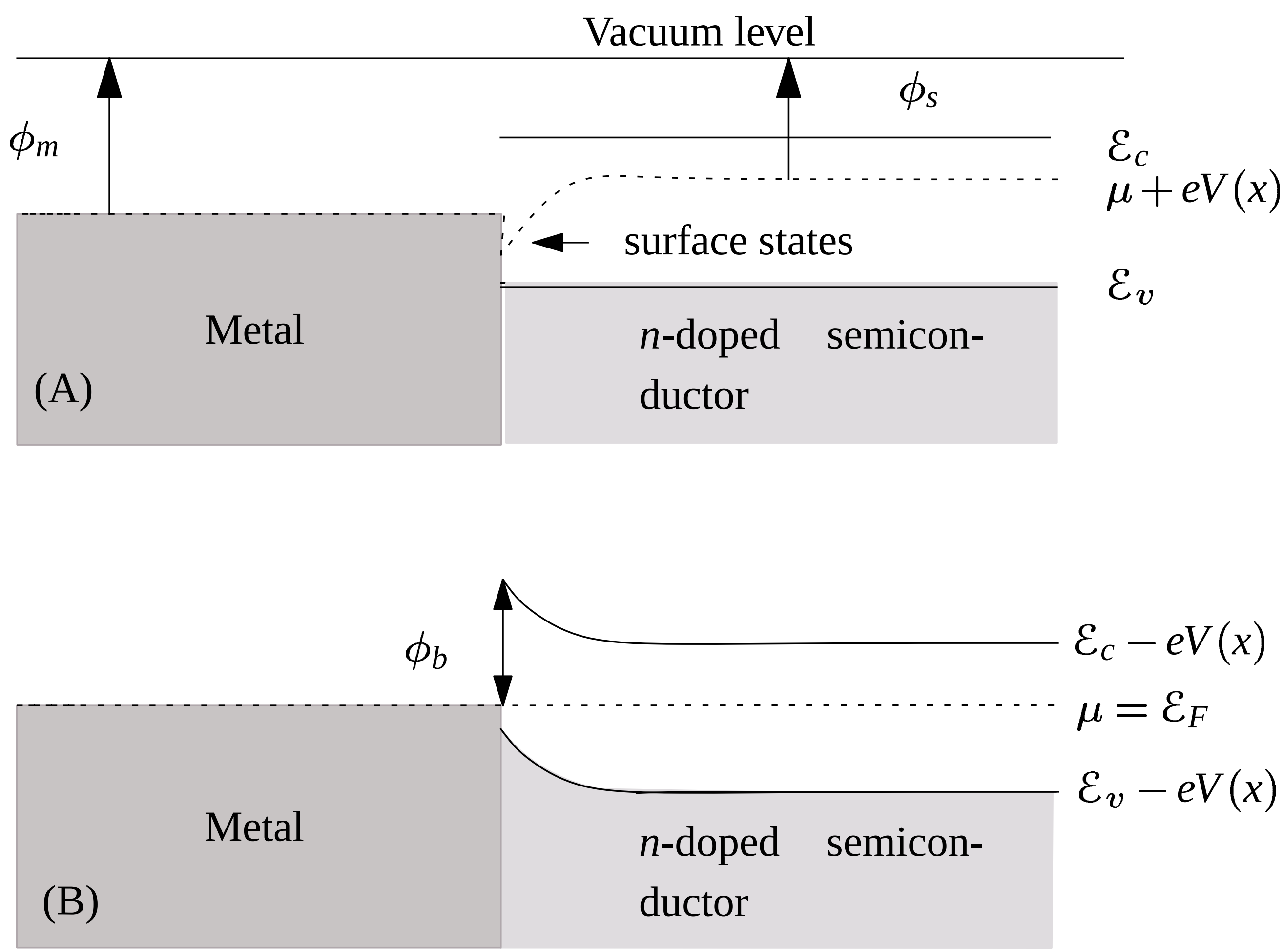


Figure 13: Effect of surface states on metal–semiconductor junction.

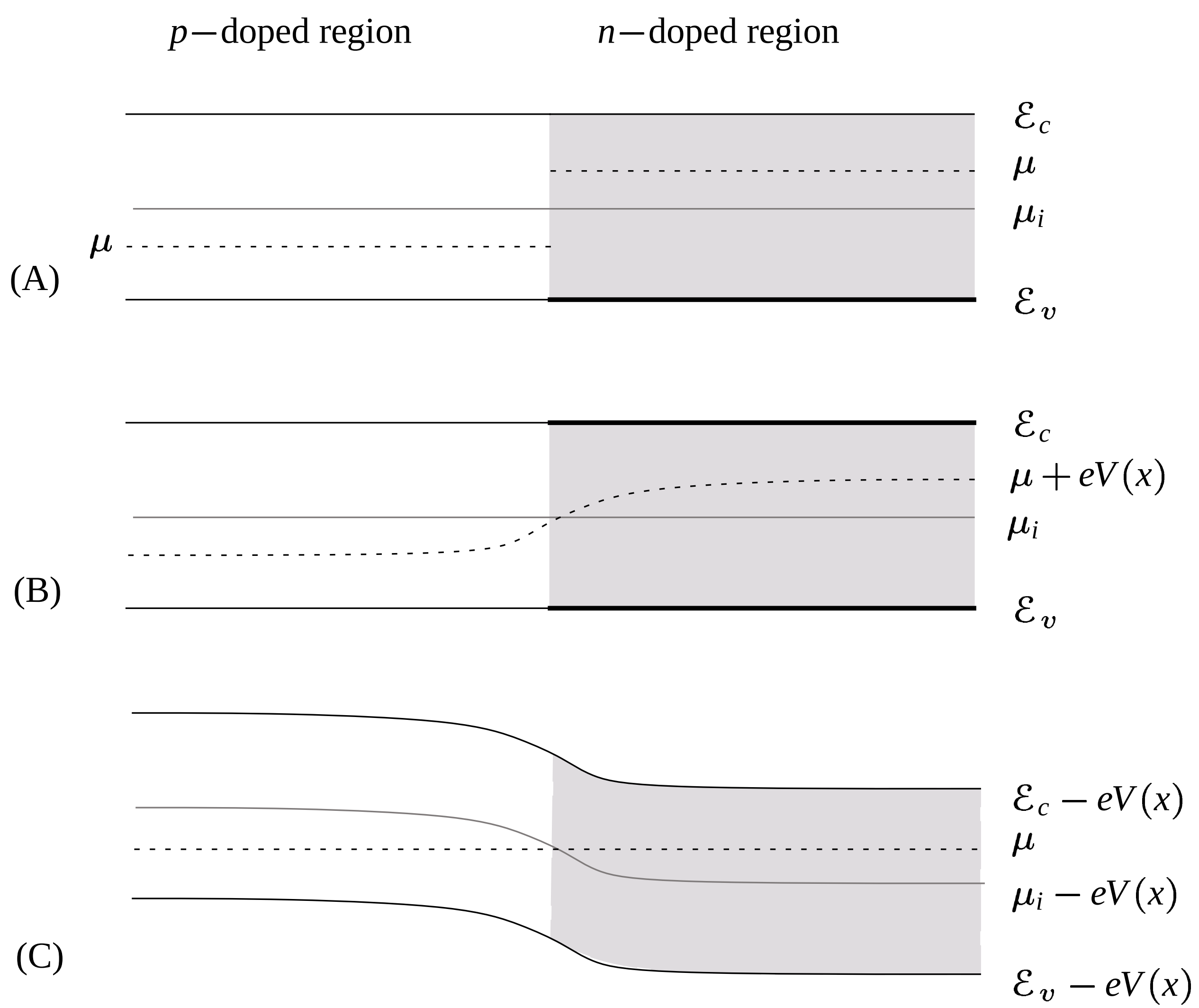


Figure 14: Band bending across semiconductor junction

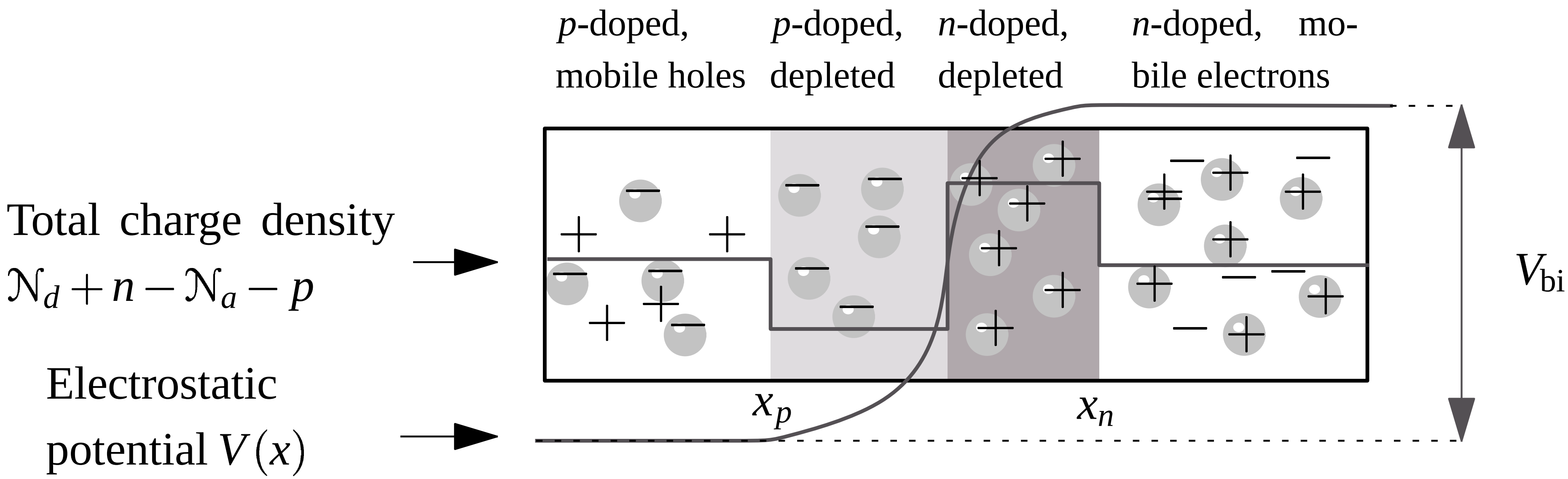


Figure 15: Illustration of the redistribution of mobile charges near a p–n junction.

$$n(x) = n_i e^{\beta(\mu + eV(x) - \mu_i)} \tag{L47a}$$

$$p(x) = n_i e^{\beta(\mu_i - eV(x) - \mu)}. \tag{L47b}$$

$$n(\infty)p(-\infty) = \mathcal{N}_d\mathcal{N}_a = n_i^2 e^{\beta(eV(\infty) - eV(-\infty))} \tag{L48}$$

$$\Rightarrow eV_{\rm bi} \equiv e[V(\infty) - V(-\infty)] \tag{L49}$$

$$= \quad k_B T \ln \frac{\mathcal{N}_d \mathcal{N}_a}{n_i^2} = \mathcal{E}_g + k_B T \ln[\frac{\mathcal{N}_d \mathcal{N}_a}{\mathcal{N}_c \mathcal{N}_v}], \tag{L50}$$

$$e n_{\text{ions}} = e[\mathcal{N}_d(x) - \mathcal{N}_a(x)]. \tag{L51}$$

$$\frac{\partial^2 V}{\partial x^2} = -4\pi e[\mathcal{N}_d(x) - n(x) - \mathcal{N}_a(x) + p(x)]/\epsilon^0, \tag{L52}$$

$$\mathcal{N}_a(x) \quad = \quad \mathcal{N}_a \theta(-x) \tag{L53a}$$

$$\mathcal{N}_d(x) \quad = \quad \mathcal{N}_d \theta(x). \tag{L53b}$$

$$V(x) = \begin{cases} V(-\infty) & \text{for } x < x_p \\ V(-\infty) + 2\pi e \frac{\mathcal{N}_a}{\epsilon^0}(x - x_p)^2 & \text{for } 0 > x > x_p \\ V(\infty) \quad - 2\pi e \frac{\mathcal{N}_d}{\epsilon^0}(x - x_n)^2 & \text{for } 0 < x < x_n \\ V(\infty) & \text{for } x > x_n. \end{cases} \tag{L54}$$

$$V(-\infty) + 2\pi e \frac{\mathcal{N}_a}{\epsilon^0} x_p^2 = V(\infty) - 2\pi e \frac{\mathcal{N}_d}{\epsilon^0} x_n^2, \;\; \mathcal{N}_d x_n = -\mathcal{N}_a x_p. \tag{L55}$$

$$x_n = \sqrt{\frac{\epsilon^0 \mathcal{N}_a V_{\rm bi}}{2\pi e \mathcal{N}_d [\mathcal{N}_a + \mathcal{N}_d]}} \tag{L56a}$$

$$x_p = -\sqrt{\frac{\epsilon^0 \mathcal{N}_d V_{\rm bi}}{2\pi e \mathcal{N}_a [\mathcal{N}_a + \mathcal{N}_d]}}, \tag{L56b}$$

$$J \propto e^{\beta e V_A} - 1, \tag{L57}$$

$$\frac{\partial g}{\partial t} = -\frac{\partial}{\partial \vec{r}} \cdot \dot{\vec{r}} g - \frac{\partial}{\partial \vec{k}} \cdot \dot{\vec{k}} g + \frac{f-g}{\tau}. \qquad \text{(L58)}$$

$$n = \int [d\vec{k}]\, g_{\vec{r}\vec{k}}, \qquad \text{(L59)}$$

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial \vec{r}} \cdot \langle \dot{\vec{r}} \rangle n + \frac{n^{(0)} - n}{\tau_n}, \qquad \text{(L60)}$$

$$\langle \dot{\vec{r}} \rangle = \frac{1}{n} \int [d\vec{k}]\, g_{\vec{r}\vec{k}} \vec{v}_{\vec{k}} \qquad \text{(L61)}$$

$$= \frac{1}{n} \int [d\vec{k}] \left[f - \tau \vec{v}_{\vec{k}} \cdot \left\{ e\vec{E} \frac{\partial f}{\partial \mu} + \frac{\partial f}{\partial \vec{r}} \right\} \right] \vec{v}_{\vec{k}} \qquad \text{(L62)}$$

$$\approx \frac{1}{n} \int [d\vec{k}] \left[-\tau \vec{v}_{\vec{k}} \cdot \left\{ e\vec{E} \beta g + \frac{\partial g}{\partial \vec{r}} \right\} \right] \vec{v}_{\vec{k}} \qquad \text{(L63)}$$

$$= -\mu_n \vec{E} - \frac{\mathcal{D}_n}{n} \frac{\partial n}{\partial \vec{r}} \qquad \text{(L64)}$$

$$\mu_n = \frac{e}{3} \beta \left\langle \tau v_{\vec{k}}^2 \right\rangle \qquad \text{(L65)}$$

$$\mathcal{D}_n = \frac{1}{3}\left\langle \tau v_{\vec{k}}^2 \right\rangle = \frac{k_B T \mu_n}{e}. \tag{L66}$$

$$\vec{j}_n = e\mu_n n\vec{E} + e\mathcal{D}_n\vec{\nabla} n \tag{L67a}$$

$$\vec{j}_p = e\mu_p p\vec{E} - e\mathcal{D}_p\vec{\nabla} p, \tag{L67b}$$

$$\frac{\partial n}{\partial t} = \frac{1}{e}\vec{\nabla}\cdot\vec{j}_n + \frac{n^{(0)} - n}{\tau_n} \tag{L68a}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{e}\vec{\nabla}\cdot\vec{j}_p + \frac{p^{(0)} - p}{\tau_p}, \tag{L68b}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{4\pi e(p - n + n_{\text{ions}})}{\epsilon^0}. \tag{L69}$$

Detailed Theory of Rectification

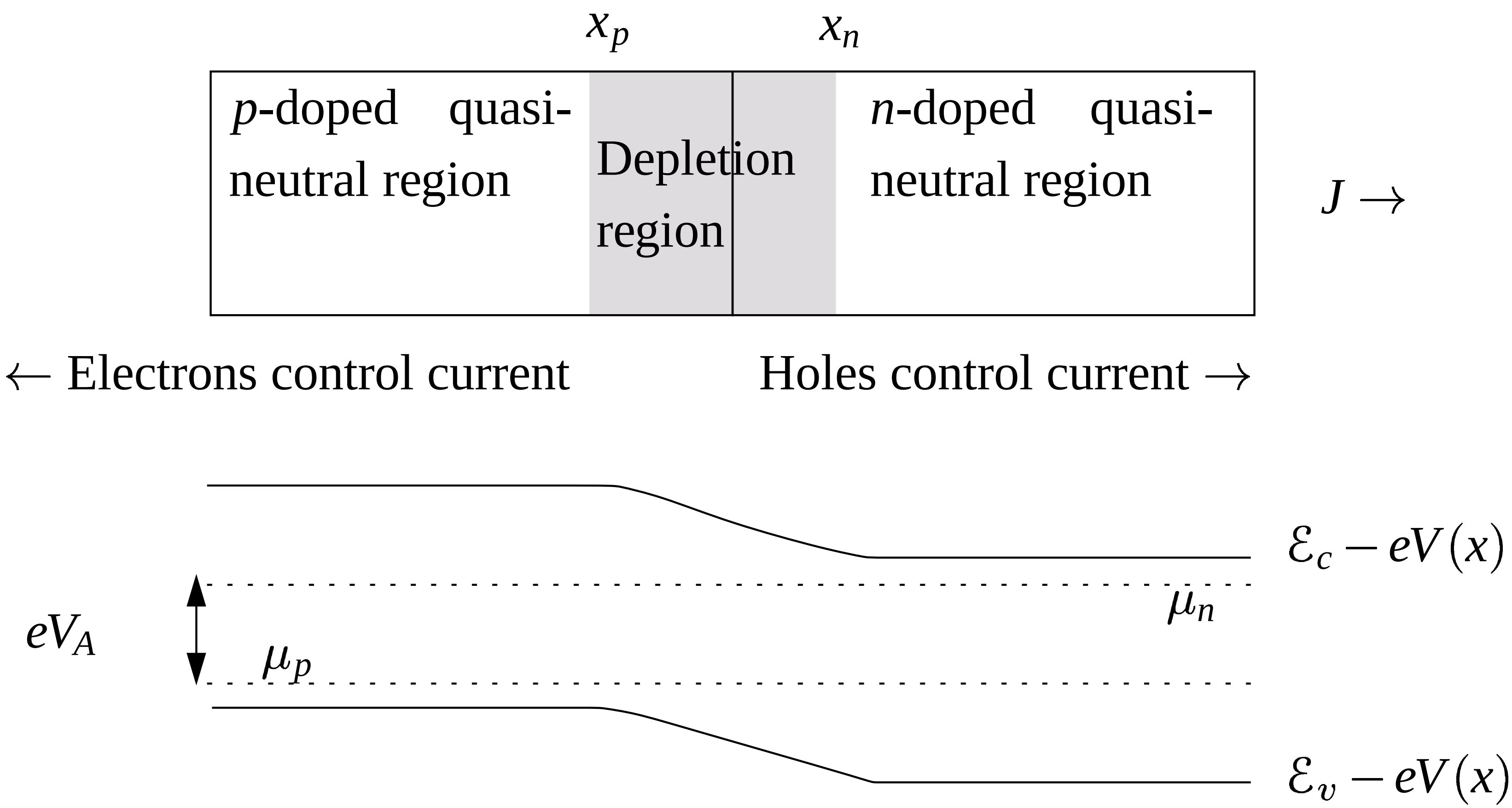


Figure 16: p–n junction in forward bias

$$j_n = e\mathcal{D}_n \vec{n}' \quad \text{(L70a)}$$

$$j_p = -e\mathcal{D}_p \vec{p}' \quad \text{(L70b)}$$

Detailed Theory of Rectification

$$n(x) = \mathcal{N}_d e^{\beta e[V(x)-V(x_n)]}\left[1+\frac{j_n}{e\mathcal{N}_d\mathcal{D}_n}\int_{x_n}^{x} dx'\, e^{-\beta e[V(x')-V(x_n)]}\right] \tag{L71a}$$

$$p(x) = \mathcal{N}_a e^{-\beta e[V(x)-V(x_p)]}\left[1-\frac{j_p}{e\mathcal{N}_a\mathcal{D}_p}\int_{x_p}^{x} dx'\, e^{\beta e[V(x')-V(x_p)]}\right]. \tag{L71b}$$

$$\frac{n_i^2}{\mathcal{N}_a\mathcal{N}_d}\frac{x_p-x_n}{L_n}e^{\beta e V_A} \approx 10^{-10}e^{\beta e V_A}. \tag{L72}$$

$$n(x) = \mathcal{N}_d e^{\beta e[V(x)-V(x_n)]} \tag{L73a}$$

$$p(x) = \mathcal{N}_a e^{-\beta e[V(x)-V(x_p)]} \tag{L73b}$$

$$\Rightarrow n(x_p) = \mathcal{N}_d e^{\beta e[V_A-V_{\text{bi}}]} = \frac{n_i^2}{\mathcal{N}_a}e^{\beta e V_A} \tag{L73c}$$

$$p(x_n) = \mathcal{N}_a e^{\beta e[V_A-V_{\text{bi}}]} = \frac{n_i^2}{\mathcal{N}_d}e^{\beta e V_A}. \tag{L73d}$$

$$0 = \mathcal{D}_p \frac{d^2 p}{dx^2} - \frac{p - p^{(0)}}{\tau_p} \quad \text{(L74a)}$$

$$0 = \mathcal{D}_n \frac{d^2 n}{dx^2} - \frac{n - n^{(0)}}{\tau_n}, \quad \text{(L74b)}$$

$$p - p^{(0)} = [p(x_n) - p^{(0)}] e^{-(x - x_n)/L_p} \quad \text{(L75a)}$$

$$n - n^{(0)} = [n(x_p) - n^{(0)}] e^{(x - x_p)/L_n} \quad \text{(L75b)}$$

$$L_n = \sqrt{\mathcal{D}_n \tau_n} \quad \text{and} \quad L_p = \sqrt{\mathcal{D}_p \tau_p} \quad \text{(L76)}$$

$$j_n = e \frac{\mathcal{D}_n}{L_n} [n(x_p) - n^{(0)}] \quad \text{(L77a)}$$

$$= e \frac{\mathcal{D}_n}{L_n} \frac{n_i^2}{\mathcal{N}_a} [e^{\beta e V_A} - 1] \quad \text{(L77b)}$$

Detailed Theory of Rectification

$$j_p = e\frac{\mathcal{D}_p}{L_p}[p(x_n) - p^{(0)}], \quad \text{(L77c)}$$

$$= e\frac{\mathcal{D}_p}{L_p}\frac{n_i^2}{\mathcal{N}_d}[e^{\beta eV_A} - 1], \quad \text{(L77d)}$$

$$j = en_i^2[e^{\beta eV_A} - 1]\left[\frac{\mathcal{D}_n}{L_n\mathcal{N}_a} + \frac{\mathcal{D}_p}{L_d\mathcal{N}_d}\right]. \quad \text{(L78)}$$

Transistor

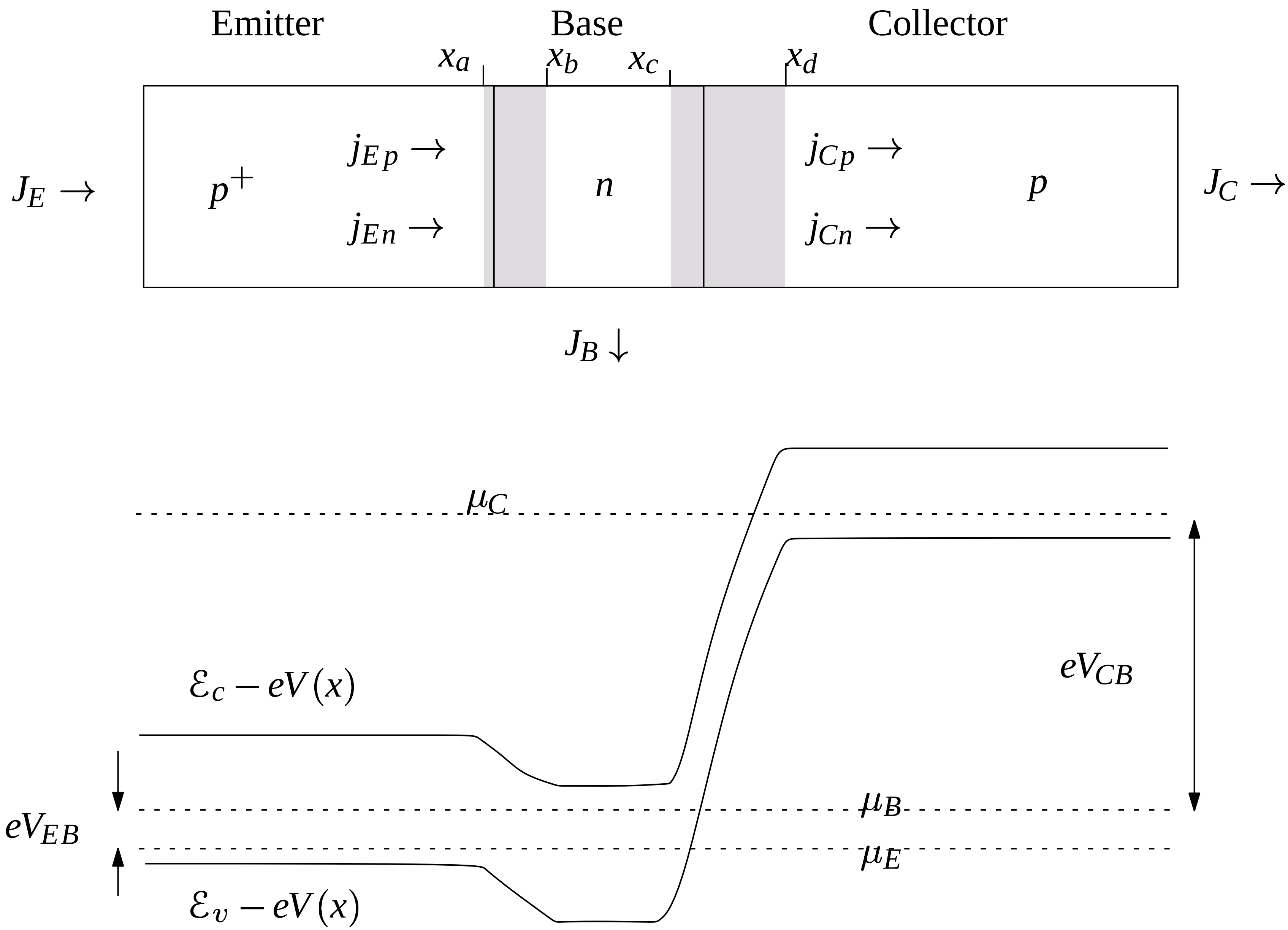


Figure 17: The binary junction transistor, made from two back-to-back p–n junctions.

$$n_E(x_a) = \frac{n_i^2}{\mathcal{N}_E} e^{\beta e V_{EB}} \tag{L79a}$$

$$p_B(x_b) = \frac{n_i^2}{\mathcal{N}_B} e^{\beta e V_{EB}} \tag{L79b}$$

$$p_B(x_c) = \frac{n_i^2}{\mathcal{N}_B} e^{\beta e V_{CB}} \tag{L79c}$$

$$n_C(x_d) = \frac{n_i^2}{\mathcal{N}_C} e^{\beta e V_{CB}}. \tag{L79d}$$

$$j_{En} = e\mathfrak{D}_E n_E'(x_a) \tag{L80a}$$

$$j_{Ep} = -e\mathfrak{D}_B p_B'(x_b) \tag{L80b}$$

$$j_{Cp} = -e\mathfrak{D}_B p_B'(x_c) \tag{L80c}$$

$$j_{Cn} = e\mathfrak{D}_C n_C'(x_d). \tag{L80d}$$

$$J_E = J_{\mathrm{FO}}(e^{\beta e V_{\mathrm{EB}}} - 1) - \alpha_{\mathrm{R}} J_{\mathrm{RO}}(e^{\beta e V_{\mathrm{CB}}} - 1) \tag{L81a}$$

$$J_C = \alpha_F J_{FO}(e^{\beta e V_{EB}} - 1) - J_{RO}(e^{\beta e V_{CB}} - 1) \quad \text{(L81b)}$$

with

$$J_{FO} = eA\left(\frac{\mathcal{D}_E}{L_E}\frac{n_i^2}{\mathcal{N}_E} + \frac{\mathcal{D}_B}{L_B}\frac{n_i^2}{\mathcal{N}_B}\coth(\frac{x_c - x_b}{L_B})\right) \quad \text{(L81c)}$$

$$J_{RO} = eA\left(\frac{\mathcal{D}_C}{L_C}\frac{n_i^2}{\mathcal{N}_C} + \frac{\mathcal{D}_B}{L_B}\frac{n_i^2}{\mathcal{N}_B}\coth(\frac{x_c - x_b}{L_B})\right) \quad \text{(L81d)}$$

$$\alpha_F J_{FO} = \alpha_R J_{RO} = eA\frac{\mathcal{D}_B}{L_B}\frac{n_i^2}{\mathcal{N}_B}\text{cosech}(\frac{x_c - x_b}{L_B}). \quad \text{(L81e)}$$

Heterostructures

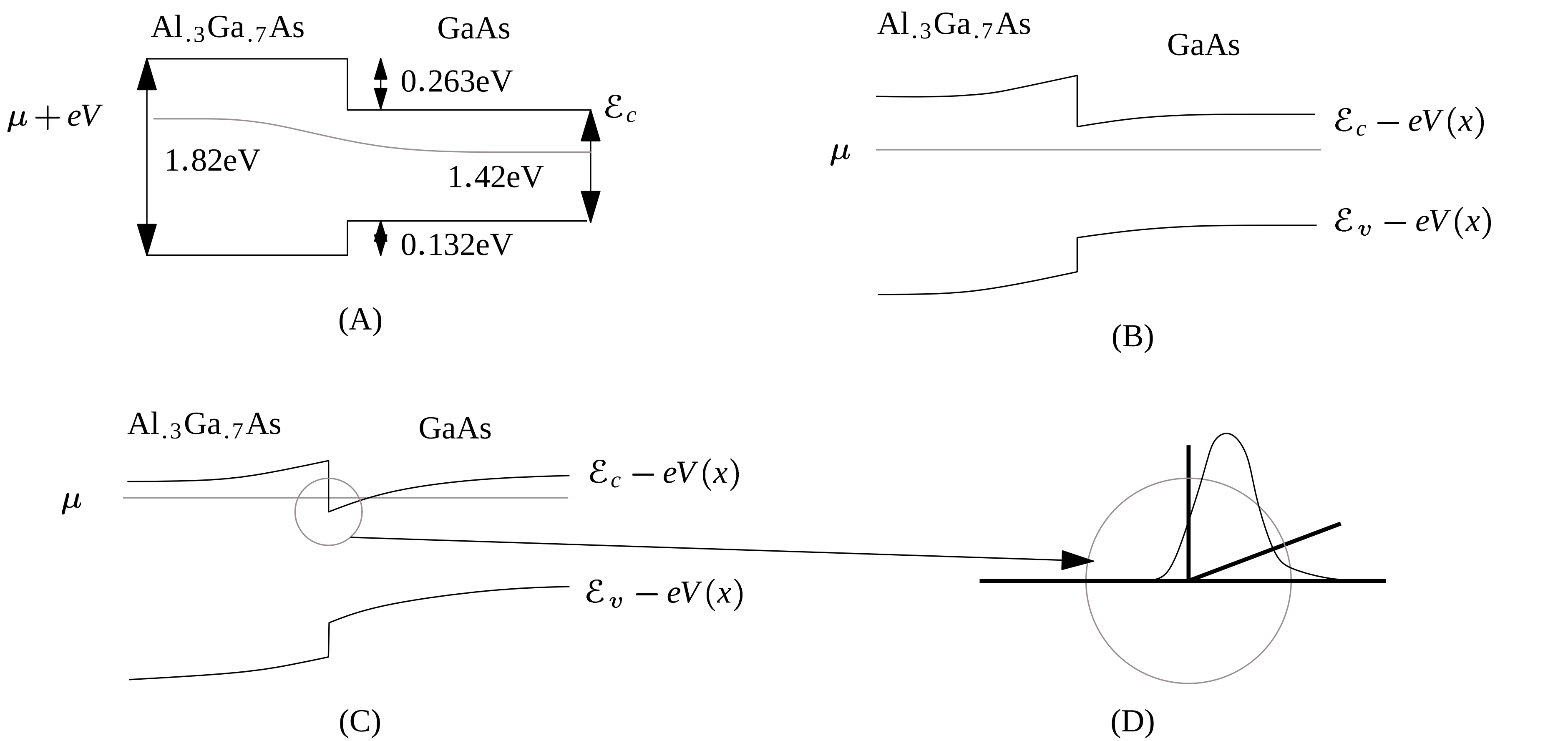


Figure 18: Junction between two semiconductors with different band gaps

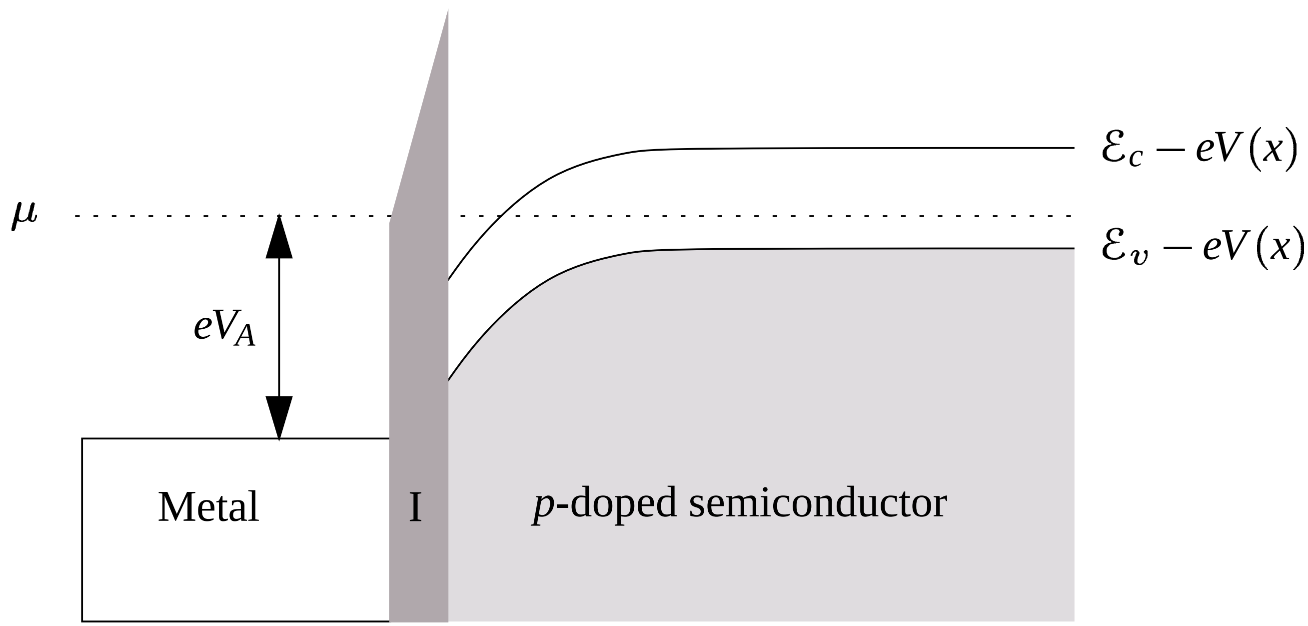


Figure 19: Metal–oxide–silicon junction

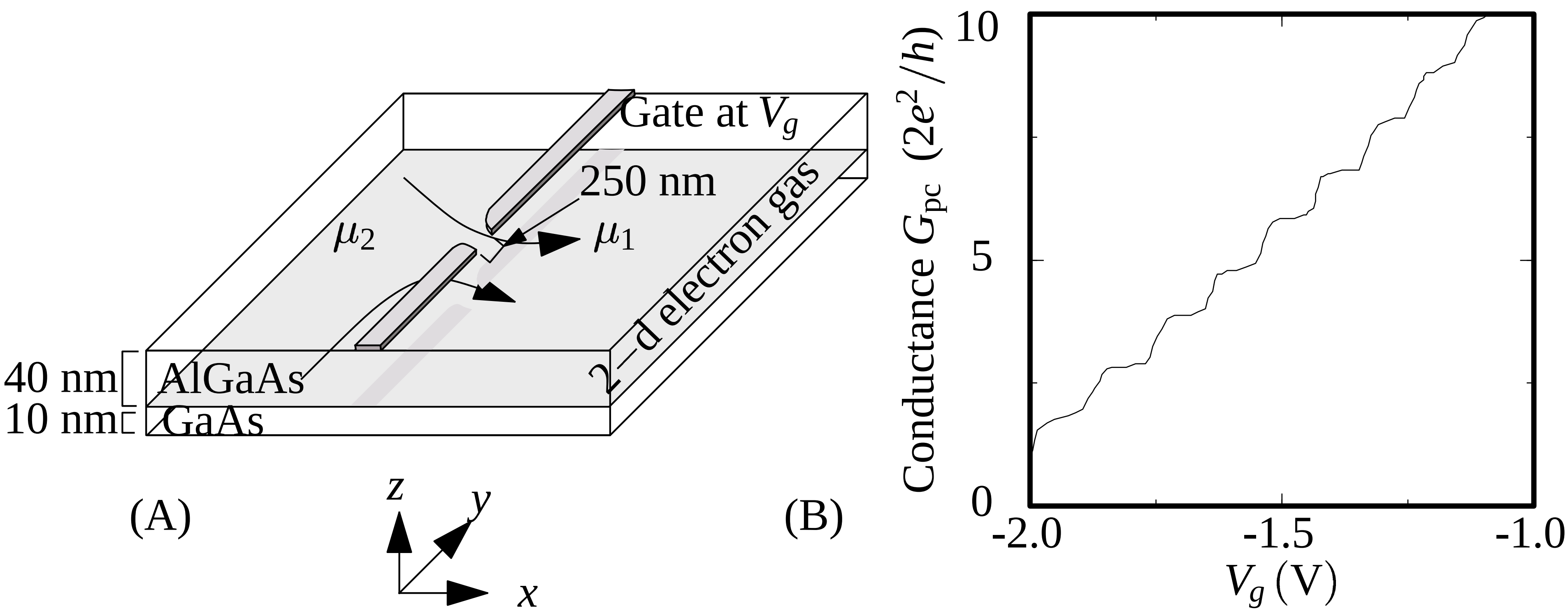


Figure 20: Quantum point contact. Data of van Wees et al. (1988)

Quantum Point Contact

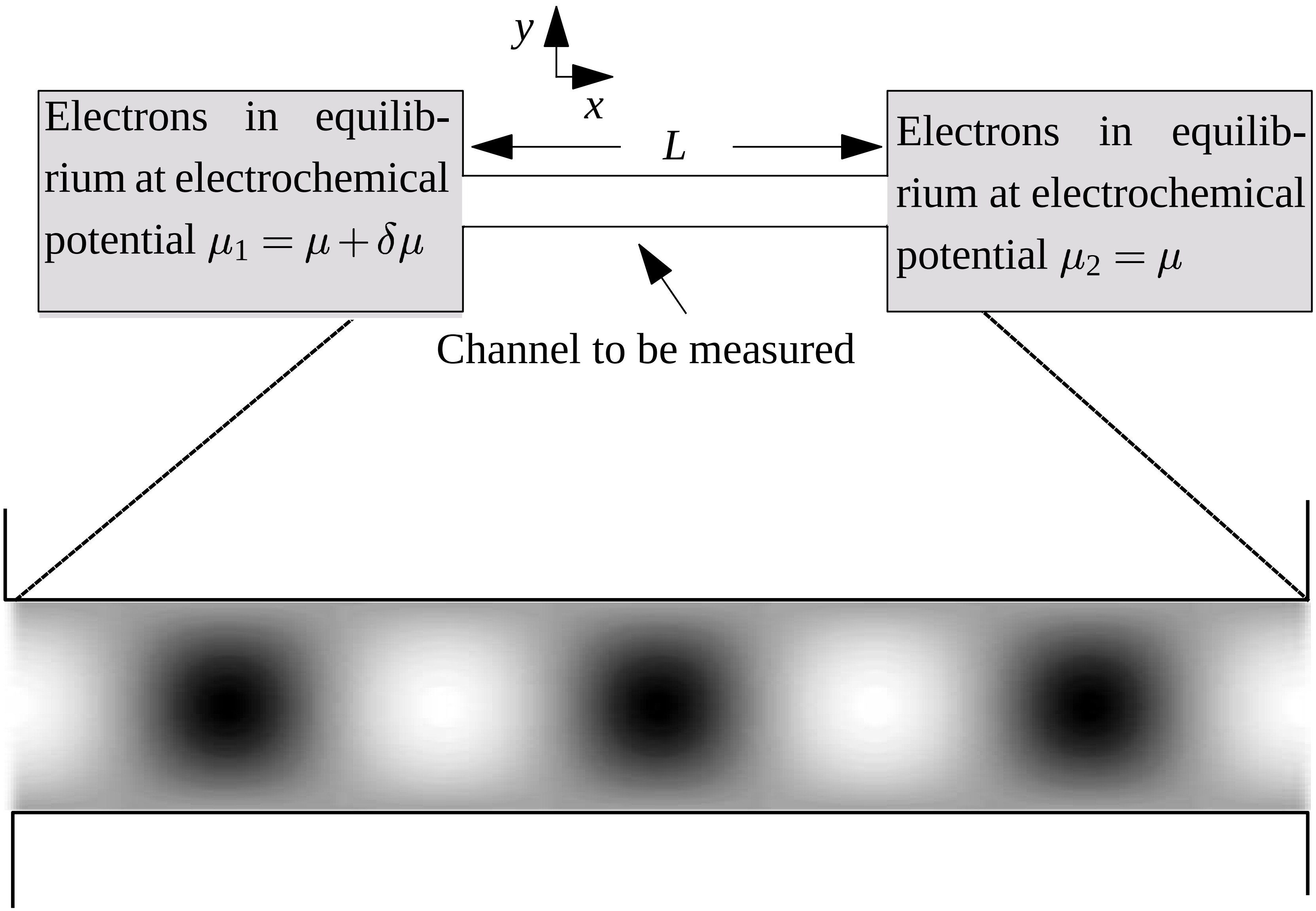


Figure 21: Setting for Landauer argument

Quantum Point Contact

$$\mathcal{E}_{lk_x} = \mathcal{E}_l^y + \frac{\hbar^2 k_x^2}{2m}. \tag{L82}$$

$$J = \frac{1}{L}\sum_{lk_x} -ev_{lk_x}[f_2(\mathcal{E}_{lk_x}) - f_1(\mathcal{E}_{lk_x})] \tag{L83}$$

$$= -e\sum_l \int dk_x D_{k_x} \frac{\partial \mathcal{E}_{lk_x}}{\partial \hbar k_x}[\theta(\mu+\delta\mu-\mathcal{E}_{lk_x}) - \theta(\mu-\mathcal{E}_{lk_x})] \tag{L84}$$

$$= -e\frac{2}{2\pi\hbar}\sum_l \int_{\mathcal{E}_l^y}^{\infty} d\mathcal{E}\,[\theta(\mu+\delta\mu-\mathcal{E}) - \theta(\mu-\mathcal{E})] \tag{L85}$$

$$= -e\frac{2}{2\pi\hbar}\delta\mu\sum_l \theta(\mu-\mathcal{E}_l^y) \tag{L86}$$

$$= \frac{2Ne^2}{h}V \tag{L87}$$

$$\Rightarrow G_{\mathrm{pc}} = \frac{2Ne^2}{h}. \tag{L88}$$

$$V = J\left(R + \frac{1}{G_{\mathrm{pc}}}\right) \tag{L89}$$

$$\Rightarrow G_{\mathrm{pc}} = \frac{J}{V - JR}. \tag{L90}$$

Quantum Dot

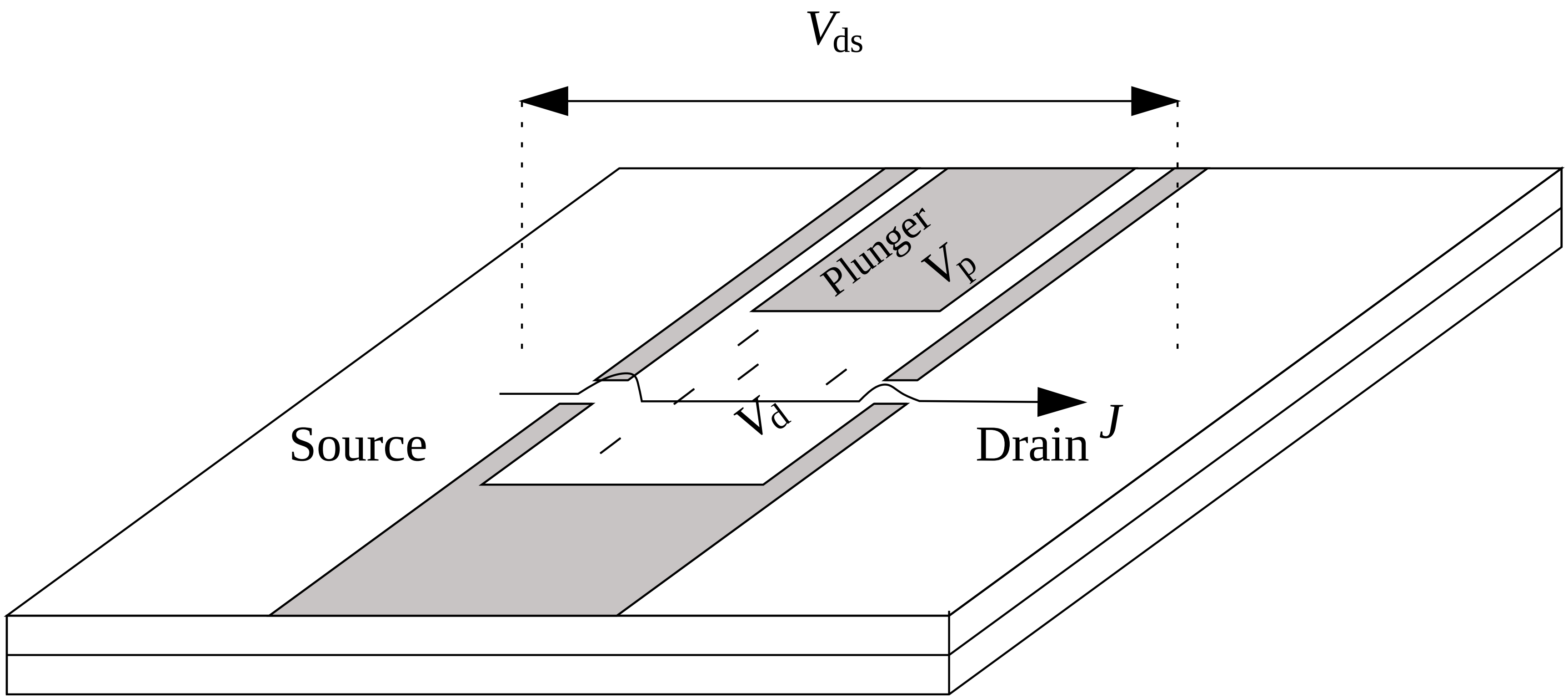


Figure 22: Quantum dot.

$$\frac{\hbar^2 k^2}{2m} = 1.5 \cdot 10^{-6} \frac{\text{eV}}{d^2/[\mu\text{m}]^2}. \tag{L91}$$

$$\frac{e^2}{d} = 1.4 \cdot 10^{-3} \frac{\text{eV}}{d/[\mu\text{m}]}. \tag{L92}$$

$$Q_{\rm d} = C_{\rm d}V_{\rm d} - C_{\rm dp}V_{\rm p}, \tag{L93}$$

$$Q_{\rm p} = -C_{\rm pd}V_{\rm d} + C_{\rm p}V_{\rm p}. \tag{L94}$$

$$C_{\rm d} = C_{\rm dp} = C_{\rm pd}. \tag{L95}$$

$$U_{\rm electrostatic} = \frac{1}{2}[Q_{\rm d}V_{\rm d} + Q_{\rm p}V_{\rm p}] + [Q_{\rm reservoir} - Q_{\rm p}]V_{\rm p}. \tag{L96}$$

$$= \frac{Q_{\rm d}^2}{2C_{\rm d}} + V_{\rm p}Q_{\rm d} + \ldots. \tag{L97}$$

$$N \equiv \frac{Q_{\rm d}}{-e} = \frac{C_{\rm d}V_{\rm p}}{e}. \tag{L98}$$

$$N = 0.625\frac{C_{\rm d}}{100\,{\rm aF}}\frac{V_{\rm p}}{10^{-3}\,{\rm V}}, \tag{L99}$$

Quantum Dot

$$V_p = [N + 1/2]\frac{e}{C_d}. \quad \text{(L100)}$$

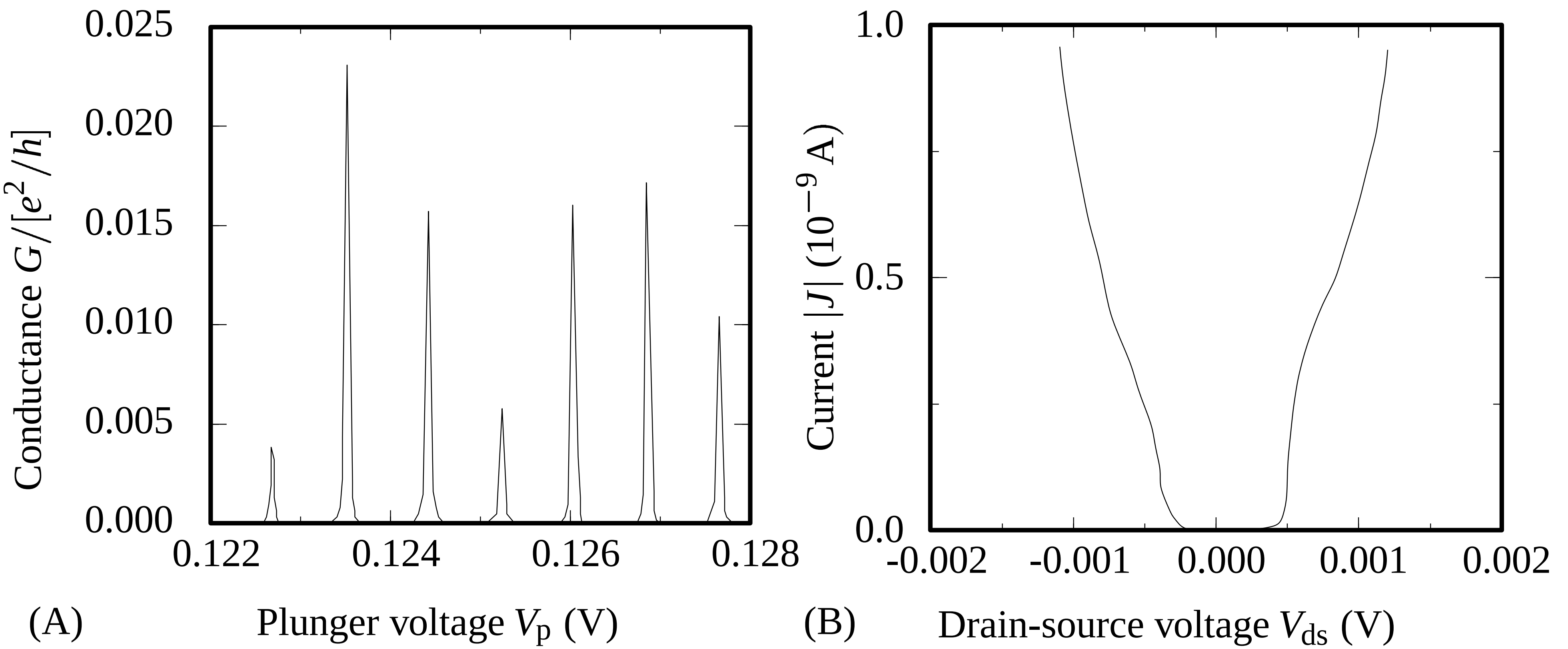


Figure 23: Conductance of quantum dot; Meirav and Foxman (1996)