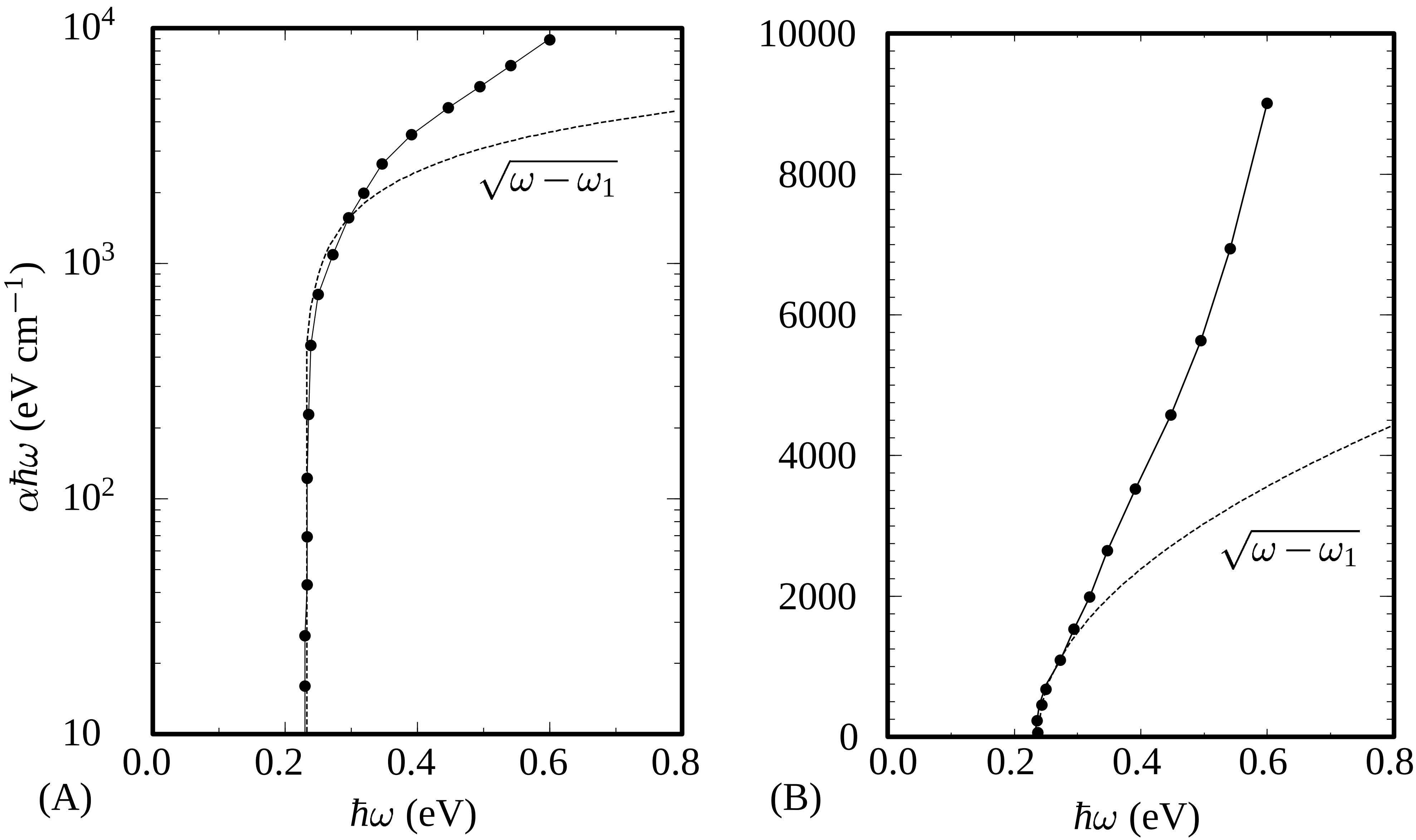
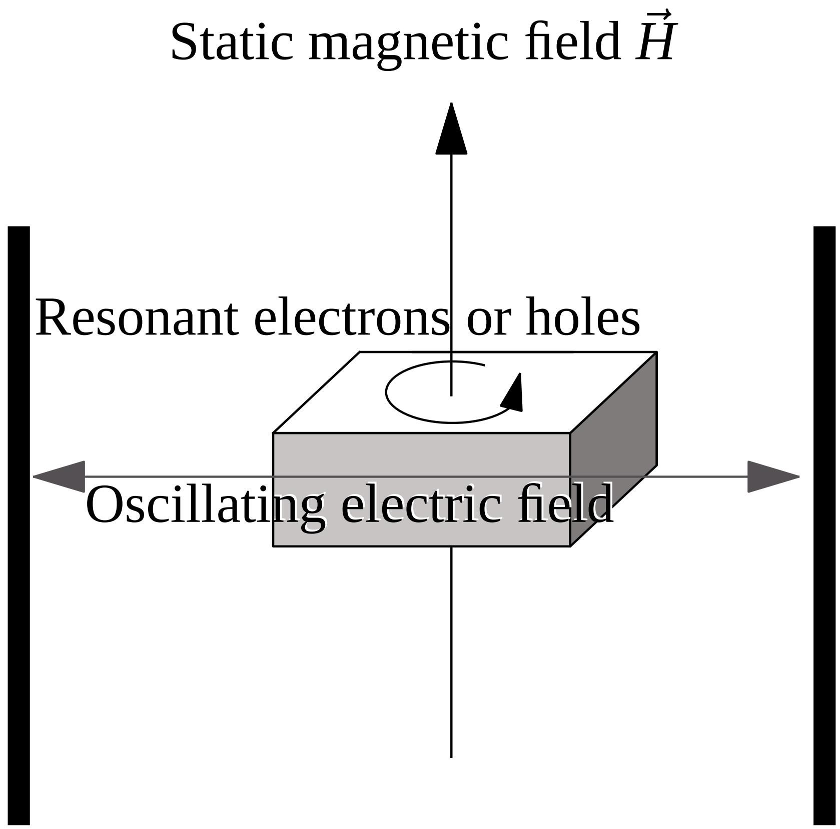


10^4
10^3
10^2
10
$\alpha\hbar\omega$ (eV cm^{-1})
$\sqrt{\omega - \omega_1}$
0.0
0.2
0.4
0.6
0.8
(A)
$\hbar\omega$ (eV)
10000
8000
6000
4000
2000
0
$\sqrt{\omega - \omega_1}$
0.0
0.2
0.4
0.6
0.8
(B)
$\hbar\omega$ (eV)

Definitions

- ☞ Cyclotron Resonance
- ☞ Direct and Indirect Optical Transitions
- ☞ Excitons
- ☞ Optoelectronics
- ☞ Lasers



Setting centripetal force equal to Lorenz force gives

$$\frac{m^\star v^2}{R} = ? \quad ? \tag{L1}$$

$$\omega_c = \frac{v}{R} = \frac{eB}{m^\star c} = 17.6 \frac{m}{m^\star} \left[\frac{B}{\text{kG}} \right] \text{GHz}. \tag{L2}$$

Cyclotron Resonance

$$\dot{\vec{v}} + \frac{\vec{v}}{\tau} = -\frac{e\vec{E}}{m^\star} - \frac{e}{m^\star c}\vec{v} \times \vec{B}. \tag{L3}$$

$$\left(-i\omega + \frac{1}{\tau}\right)\vec{v} = -\frac{e\vec{E}}{m^\star} - \omega_c(\hat{x}v_y - \hat{y}v_x) \tag{L4}$$

$$\Rightarrow \left(-i\omega + \frac{1}{\tau}\right)\vec{v} = -\frac{e\vec{E}}{m^\star} - \omega_c \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \vec{v}. \tag{L5}$$

$$\vec{j} = -ne\vec{v} \equiv \sigma\vec{E}, \tag{L6}$$

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \tag{L7}$$

$$\sigma_{xx} = \frac{\sigma_0(1 - i\omega\tau)}{(1 - i\omega\tau)^2 + \omega_c^2\tau^2} \tag{L8a}$$

$$\sigma_{xy} = -\frac{\sigma_0 \tau \omega_c}{(1 - i\omega\tau)^2 + \omega_c^2 \tau^2} \tag{L8b}$$

and

$$\sigma_{zz} = \frac{\sigma_0}{1 - i\omega\tau}, \tag{L8c}$$

with

$$\sigma_0 = \frac{ne^2\tau}{m^\star}. \tag{L8d}$$

$$\mathrm{Re}[\sigma_{xx}] = \sigma_0 \frac{\omega_c^2\tau^2 + \omega^2\tau^2 + 1}{(\omega_c^2\tau^2 - \omega^2\tau^2 + 1)^2 + 4\omega^2\tau^2}. \tag{L9}$$

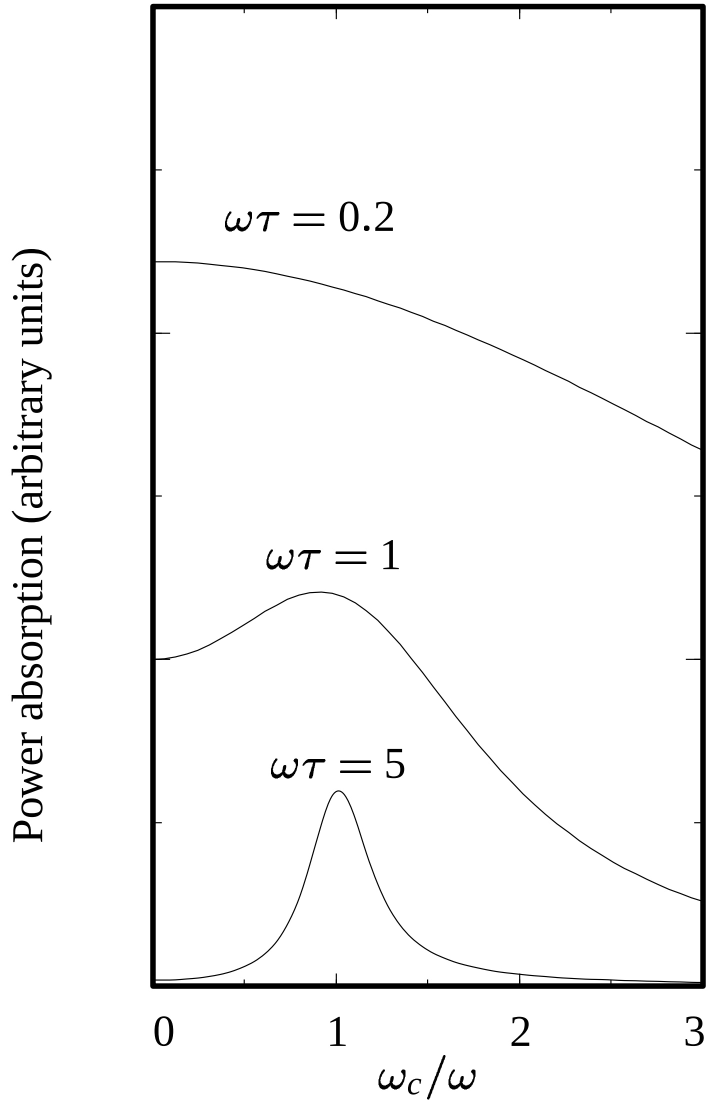


Figure 1: Cyclotron theory

Cyclotron Resonance

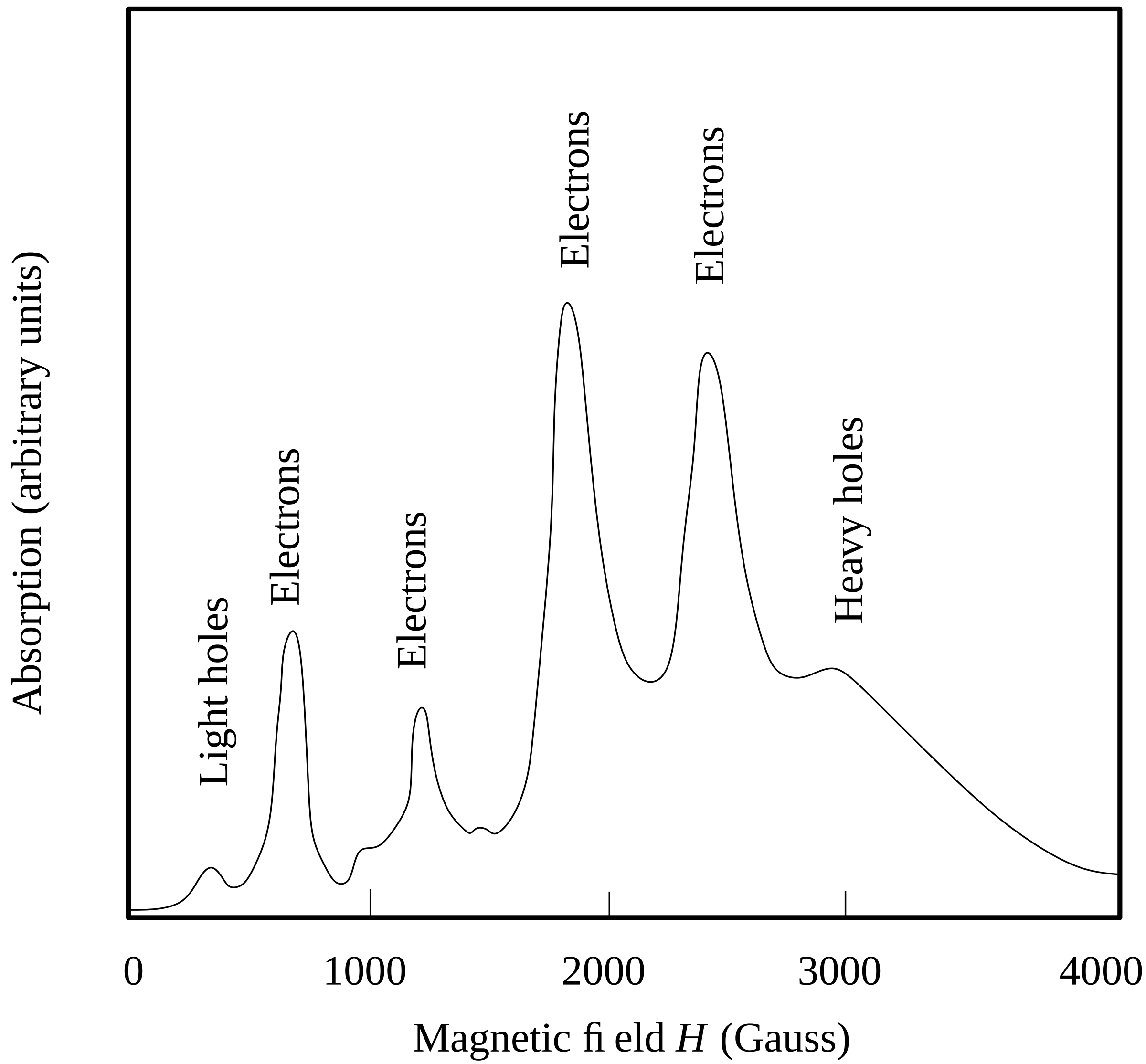


Figure 2: Cyclotron resonance in germanium. The magnetic field is oriented at 10° from the (110) plane and 30° from the [100] direction. [Source: Dexter et al. (1956)]

Electron Energy Surfaces

$$\mathcal{E} = \frac{\hbar^2}{2}\left[\frac{k_1^2}{m_1^\star} + \frac{k_2^2}{m_2^\star} + \frac{k_3^2}{m_3^\star}\right] \tag{L10}$$

$$0 = i\omega\vec{v} - \frac{e}{c}\begin{pmatrix} \frac{1}{m_1^\star} & 0 & 0 \\ 0 & \frac{1}{m_2^\star} & 0 \\ 0 & 0 & \frac{1}{m_1^\star} \end{pmatrix}\begin{pmatrix} 0 & B_3 & -B_2 \\ -B_3 & 0 & B_1 \\ B_2 & -B_1 & 0 \end{pmatrix}\vec{v} \tag{L11}$$

$$\Rightarrow \omega = \frac{e}{c}\sqrt{\sum_{\alpha=1}^{3}\frac{B_\alpha^2 m_\alpha^\star}{m_1^\star m_2^\star m_3^\star}}. \tag{L12}$$

Theory for absorption across energy gap

$$\mathrm{Im}[\epsilon_{\alpha\beta}] = \frac{4e^2\pi^2}{m^2\omega^2\mathcal{V}}\sum_{ll'}(f_l - f_{l'})\langle l|\hat{P}_\alpha|l'\rangle\langle l'|\hat{P}_\beta|l\rangle\delta(\mathcal{E}_{l'} - \mathcal{E}_l - \hbar\omega) \tag{L13}$$

$$= \left(\frac{2\pi e}{m\omega}\right)^2 \frac{1}{\mathcal{V}}\sum_{\vec{k}n_1n_2}\langle\vec{k}n_1|\hat{P}_\alpha|\vec{k}n_2\rangle\langle\vec{k}n_2|\hat{P}_\beta|\vec{k}n_1\rangle\delta(\mathcal{E}_{n_2\vec{k}} - \mathcal{E}_{n_1\vec{k}} - \hbar\omega) \tag{L14}$$

$$= \left(\frac{2\pi e}{m\omega}\right)^2 |P_{\alpha\beta}(\omega)|^2 D_{\mathrm{j}}(\hbar\omega), \tag{L15}$$

where

$$|P_{\alpha\beta}(\omega)|^2 \equiv \frac{\sum_{n_1n_2\vec{k}}\langle\vec{k}n_1|\hat{P}_\alpha|\vec{k}n_2\rangle\langle\vec{k}n_2|\hat{P}_\beta|\vec{k}n_1\rangle\delta(\mathcal{E}_{n_2\vec{k}} - \mathcal{E}_{n_1\vec{k}} - \hbar\omega)}{\sum_{n_1n_2\vec{k}}\delta(\mathcal{E}_{n_2\vec{k}} - \mathcal{E}_{n_1\vec{k}} - \hbar\omega)} \tag{L16}$$

and

$$D_{\mathrm{j}}(\hbar\omega) \equiv \frac{1}{\mathcal{V}}\sum_{n_1n_2\vec{k}}\delta(\mathcal{E}_{n_2\vec{k}} - \mathcal{E}_{n_1\vec{k}} - \hbar\omega). \tag{L17}$$

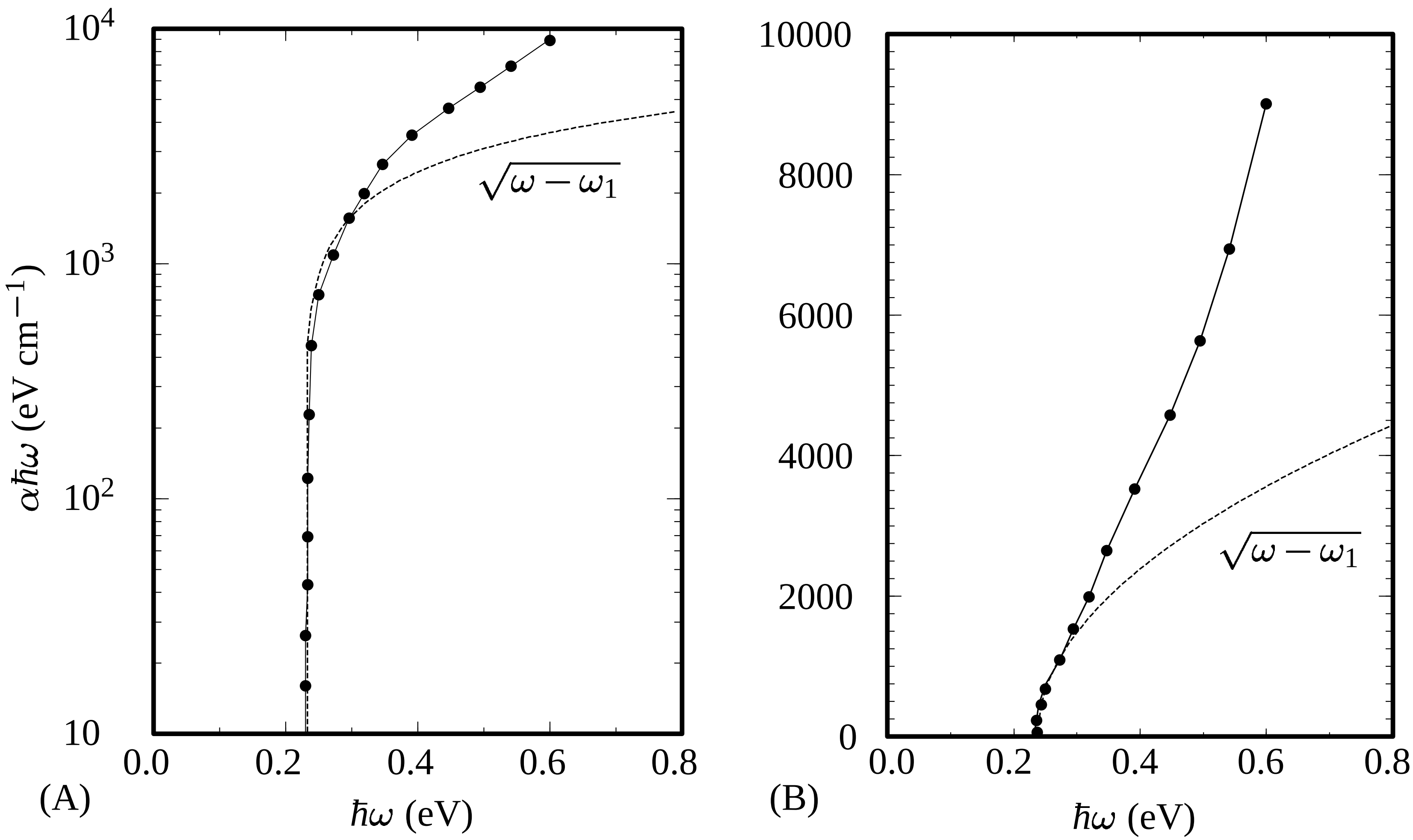


Figure 3: Measurement of absorption coefficient α times $\hbar\omega$, showing a van Hove singularity at onset of optical absorption in the direct gap semiconductor InSb. Data of Goebli and Fan and reported by Johnson (1967).

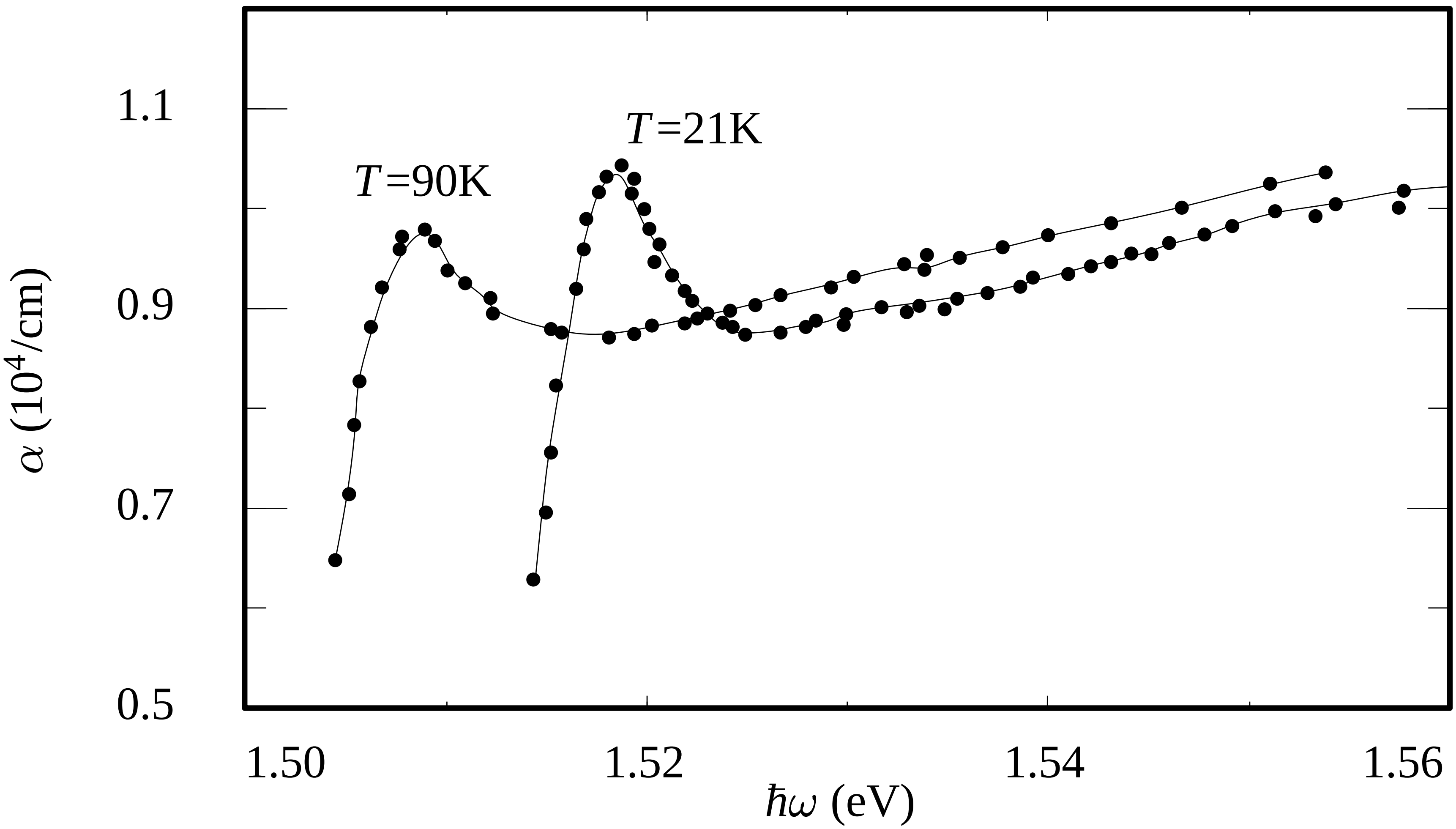


Figure 4: Absorption coefficient α in gallium arsenide, showing absorption due to excitons. [Source: Sturge (1962), p. 771.]

Indirect Transitions

$$\hbar\omega = \mathcal{E}_c - \mathcal{E}_v \pm \hbar\omega_{\text{ph}}(\vec{\delta k}). \tag{L18}$$

$$\kappa \propto \sum_{\vec{k}_c \vec{k}_v} \delta\left(\mathcal{E}_c(\vec{k}_c) - \mathcal{E}_v(\vec{k}_v) - \hbar\omega \pm \hbar\omega_{\text{ph}}(\vec{\delta k})\right) \tag{L19}$$

$$= \int d\mathcal{E}_c \int d\mathcal{E}_v \, D_{\text{c}}(\mathcal{E}_c) D_v(\mathcal{E}_v) \delta\left(\mathcal{E}_c - \mathcal{E}_v - \hbar\omega \pm \hbar\omega_{\text{ph}}\right) \tag{L20}$$

$$\propto \int_{\mathcal{E}_g} d\mathcal{E}_c \int^0 d\mathcal{E}_v \sqrt{\mathcal{E}_c - \mathcal{E}_g} \sqrt{-\mathcal{E}_v} \, \delta\left(\mathcal{E}_c - \mathcal{E}_v - \hbar\omega \pm \hbar\omega_{\text{ph}}\right) \tag{L21}$$

$$= \int_{\mathcal{E}_g}^{\hbar\omega \mp \hbar\omega_{\text{ph}}} d\mathcal{E}_c \sqrt{\mathcal{E}_c - \mathcal{E}_g} \sqrt{\hbar\omega - \mathcal{E}_c \mp \hbar\omega_{\text{ph}}} \tag{L22}$$

$$= (\hbar\omega \mp \hbar\omega_{\text{ph}} - \mathcal{E}_g)^2 \int_0^1 dy \sqrt{y}\sqrt{1-y}. \tag{L23}$$

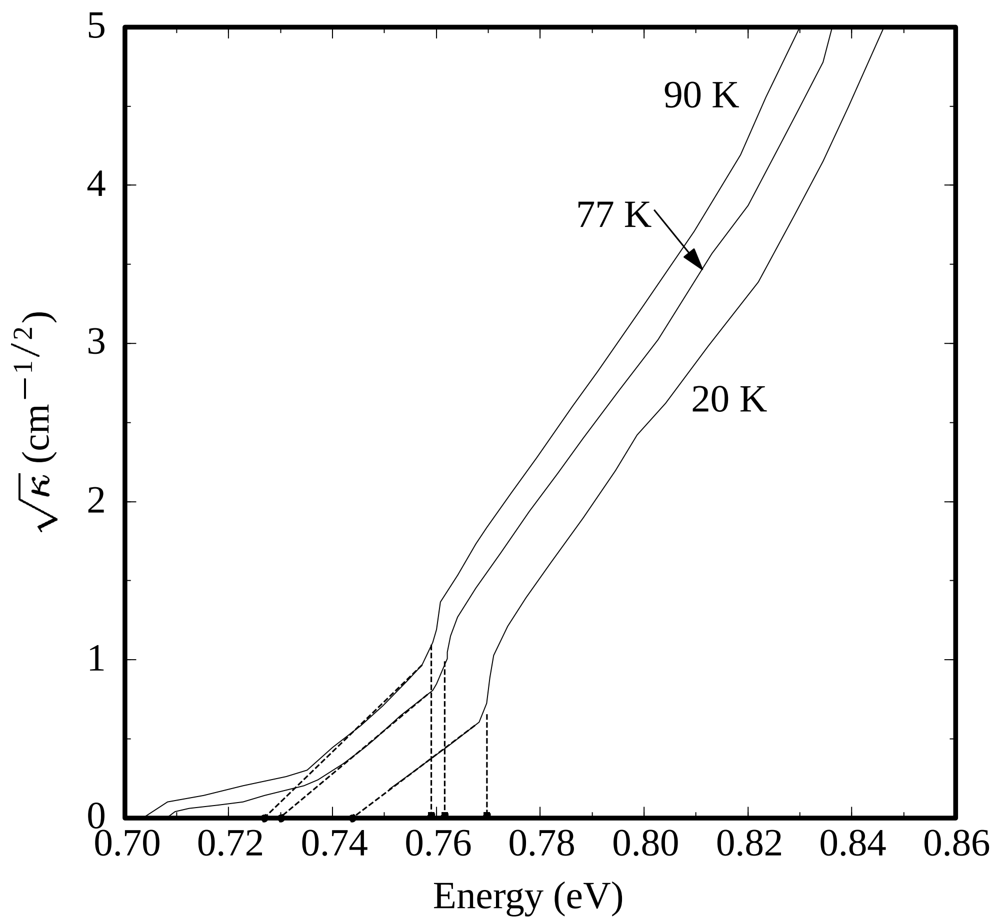


Figure 5: Onset of optical absorption in germanium.. [Source: Macfarlane et al. (1957)]

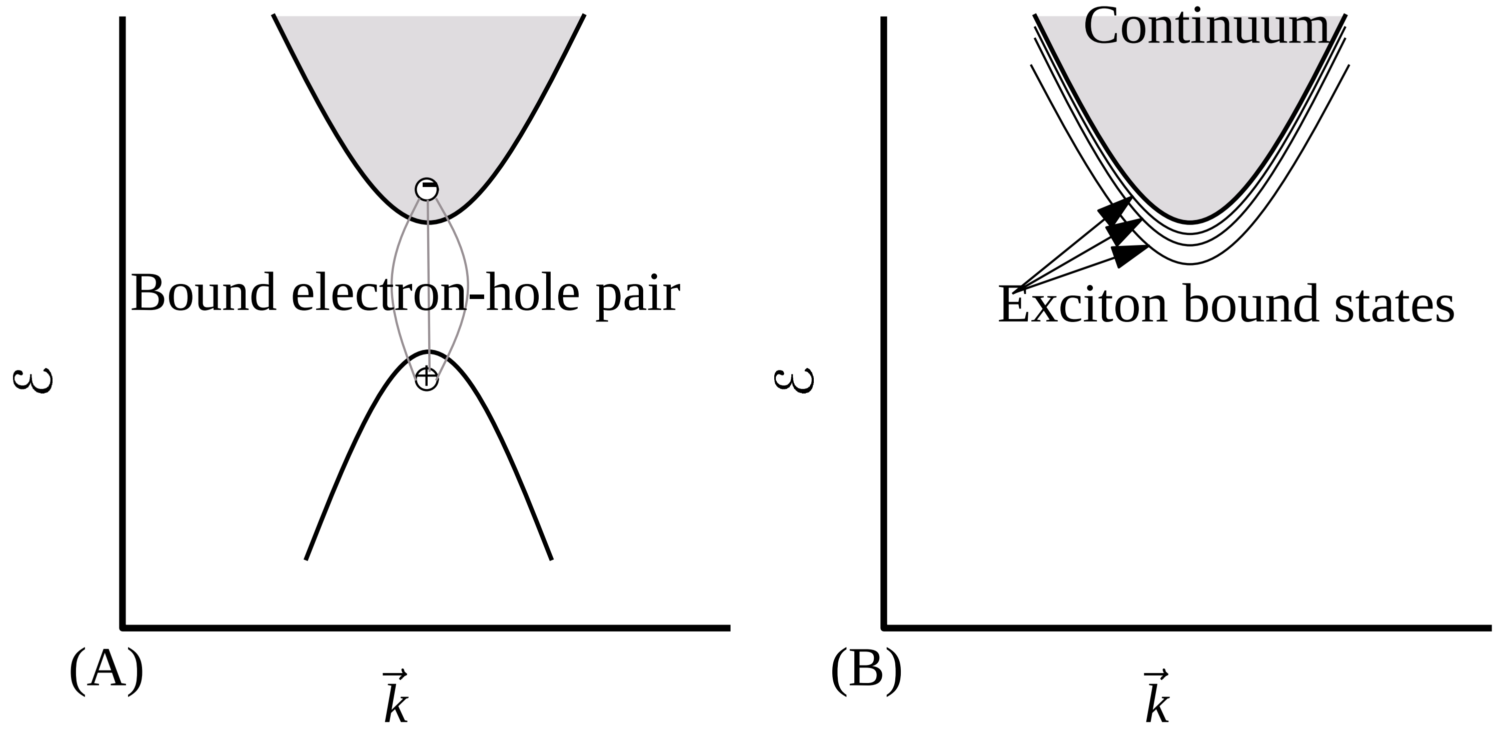


Figure 6: Schematic view of energy levels resulting from exciton formation.

$$\left[\frac{-\hbar^2}{2m_n^\star}\nabla^2_{\vec{r}_n}+\frac{-\hbar^2}{2m_p^\star}\nabla^2_{\vec{r}_p}-\frac{e^2}{\epsilon^0|\vec{r}_n-\vec{r}_n|}-\mathcal{E}\right]\Psi(\vec{r}_n,\vec{r}_p)=0. \tag{L24}$$

$$\vec{R} = \frac{m_n^\star\vec{r}_n+m_p^\star\vec{r}_p}{m_n^\star+m_p^\star} \tag{L25}$$

$$\vec{r} = \vec{r}_n-\vec{r}_p \tag{L26}$$

to give

$$0 = \left[\frac{-\hbar^2}{2(m_n^\star+m_p^\star)}\nabla^2_{\vec{R}}-\mathcal{E}_{\rm cm}\right]\Psi_{\rm cm}(\vec{R}) \tag{L27}$$

$$0 = \left[\frac{-\hbar^2}{2\mu}\nabla^2_{\vec{r}}-\frac{e^2}{\epsilon^0 r}-\mathcal{E}_{\rm b}\right]\Psi_{\rm b}(\vec{r}), \tag{L28}$$

with the reduced mass μ given by

$$\mu = \frac{m_n^\star m_p^\star}{m_n^\star+m_p^\star}. \tag{L29}$$

Mott–Wannier Excitons

$$\mathcal{E}_l = -\frac{\mu e^4}{2\hbar^2 \epsilon^{0\,2} l^2} = -\frac{\mu}{m\epsilon^{0\,2} l^2} \cdot 13.6\,\mathrm{eV} \tag{L30}$$

$$a_0^* = \frac{\epsilon^0 \hbar^2}{e^2 \mu} = \frac{m\epsilon^0}{\mu} \cdot 0.529\,\text{Å}. \tag{L31}$$

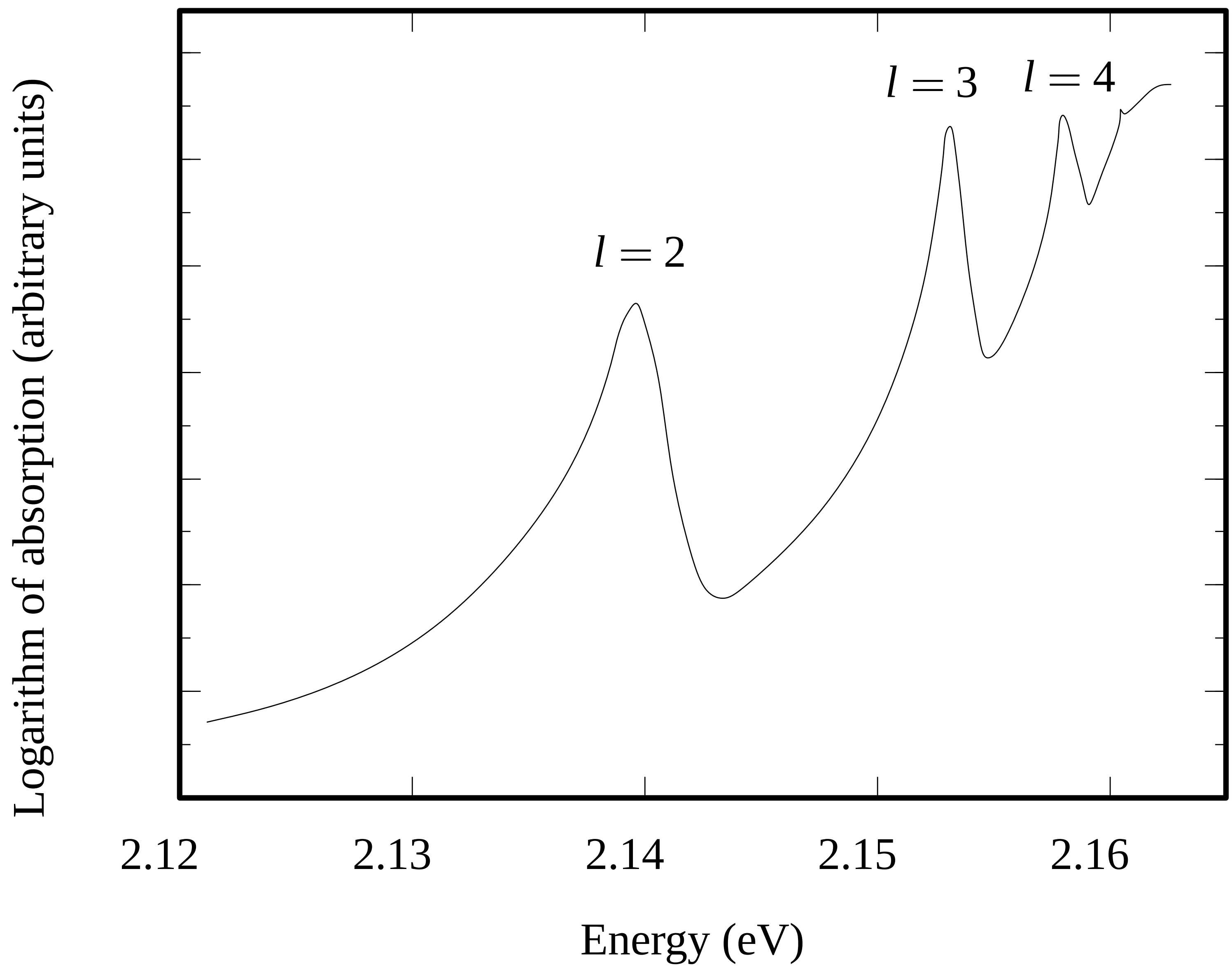


Figure 7: Absorption in Cu_2O. [Experiments of Baumeister (1961), p 361.]

$$j = -I\mathcal{G}(L_p + L_n) \tag{L32}$$

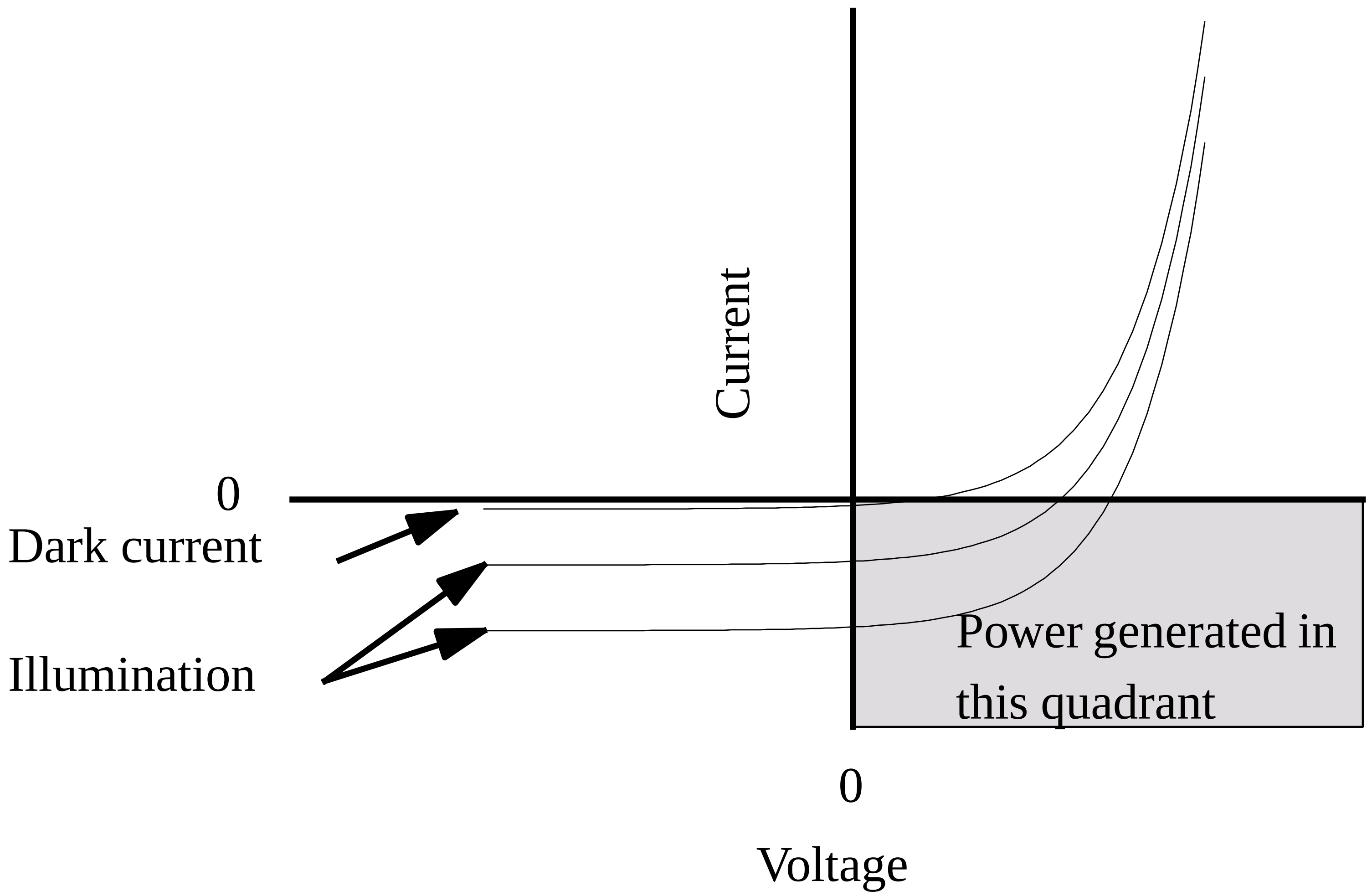


Figure 8: The current–voltage characteristic for a solar cell

$$R_{\rm sp} = A_{21} f_2(1 - f_1) \tag{L33}$$

$$R_{12} = B_{12} f_1(1 - f_2) N_{\mathcal{E}_{12}} D_{\rm ph}(\mathcal{E}_{12}) \tag{L34}$$

$$R_{21} \quad = \quad B_{21} f_2(1 - f_1) N_{\mathcal{E}_{12}} D_{\rm ph}(\mathcal{E}_{12}) + A_{21} f_2(1 - f_1). \tag{L35}$$

$$\frac{f_2(1 - f_1)}{f_1(1 - f_2)} = e^{-\beta\mathcal{E}_{12}}, \tag{L36}$$

$$R_{12} \quad = \quad R_{21} \tag{L37}$$

$$\Rightarrow B_{12} N_{\mathcal{E}_{12}} D_{\rm ph}(\mathcal{E}_{12}) \quad = \quad e^{-\beta\mathcal{E}_{12}} \left[B_{21} N_{\mathcal{E}_{12}} D_{\rm ph}(\mathcal{E}_{12}) + A_{21} \right] \tag{L38}$$

$$\Rightarrow D_{\rm ph}(\mathcal{E}_{12}) B_{12} - A_{21} \quad = \quad e^{-\beta\mathcal{E}_{12}} \left[D_{\rm ph}(\mathcal{E}_{12}) B_{21} - A_{21} \right] \tag{L39}$$

$$\Rightarrow B_{12} \quad = \quad B_{21} \;\text{ and }\; A_{21} = D_{\text{ph}}(\mathcal{E}_{12})B_{21}. \tag{L40}$$

$$R_{21} = B_{21} f_2(1 - f_1)(N_{\mathcal{E}_{12}} + 1)D_{\text{ph}}(\mathcal{E}_{12}). \tag{L41}$$

$$R_{12} - R_{21} \quad = \quad B_{21}[(f_1 - f_2)N_{\mathcal{E}_{12}} - f_2(1 - f_1)]D_{\text{ph}}(\mathcal{E}_{12}). \tag{L42}$$

$$\text{Re}[\sigma_{\alpha\beta}(\omega)] = \frac{e^2\pi}{\hbar\omega m^2\mathcal{V}}(f_1 - f_2)F_{12}(\omega)\langle 1|\hat{P}_\alpha|2\rangle\langle 2|\hat{P}_\beta|1\rangle. \tag{L43}$$

$$g(\omega) = \frac{N}{\mathcal{V}}\frac{4\pi^2c^2}{\omega\bar{n}}\left(\frac{e^2}{\hbar c}\right)F_{12}(\omega)(f_2 - f_1)\frac{\sum_\beta |\langle 1|\hat{P}_\beta|2\rangle|^2}{3m^2c^2}. \tag{L44}$$

$$D_{\mathrm{ph}}(\omega) = \frac{\bar{n}^3\omega^2}{\pi^2 c^3}. \tag{L45}$$

$$R_{21} - R_{12} = \frac{\partial}{\partial t} N_{\mathcal{E}_{12}} = -N \frac{\mathcal{E}_{12}}{\hbar} \bar{n} \left(\frac{e^2}{\hbar c} \right) 4(f_1 - f_2) \sum_{\beta} \frac{|\langle 1|\hat{P}_\beta|2\rangle|^2}{3m^2c^2} N_{\mathcal{E}_{12}}. \tag{L46}$$

$$\Re \exp[x(g-\alpha)] > 1. \tag{L47}$$

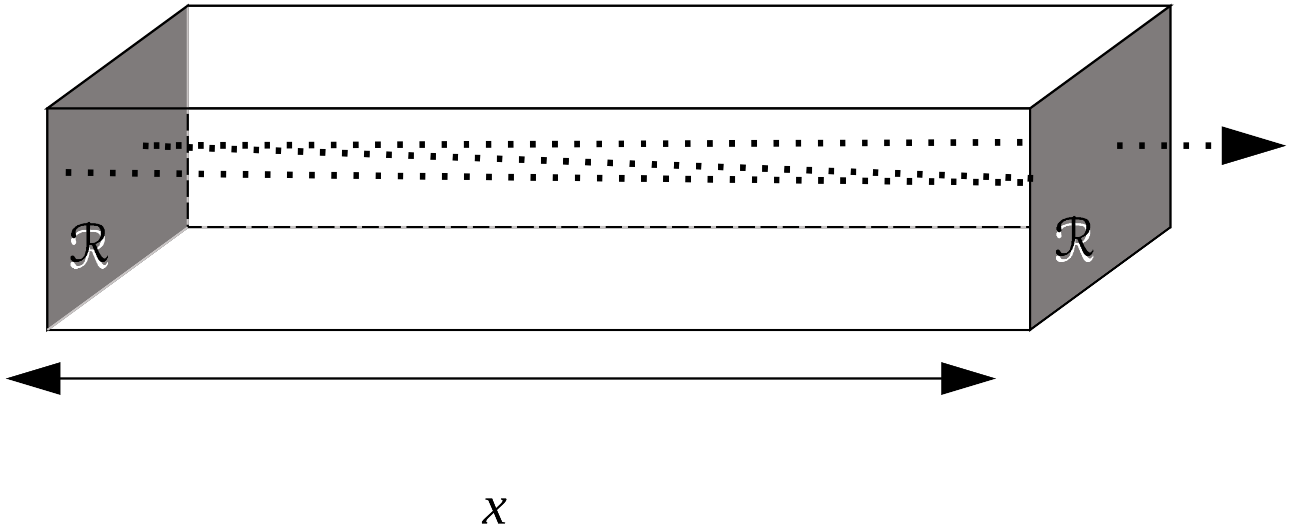


Figure 9: Light in a laser cavity reflects several times back and forth from mirrored ends of reflection coefficient $\mathcal{R}$ so as to stimulate more light emission before exiting.

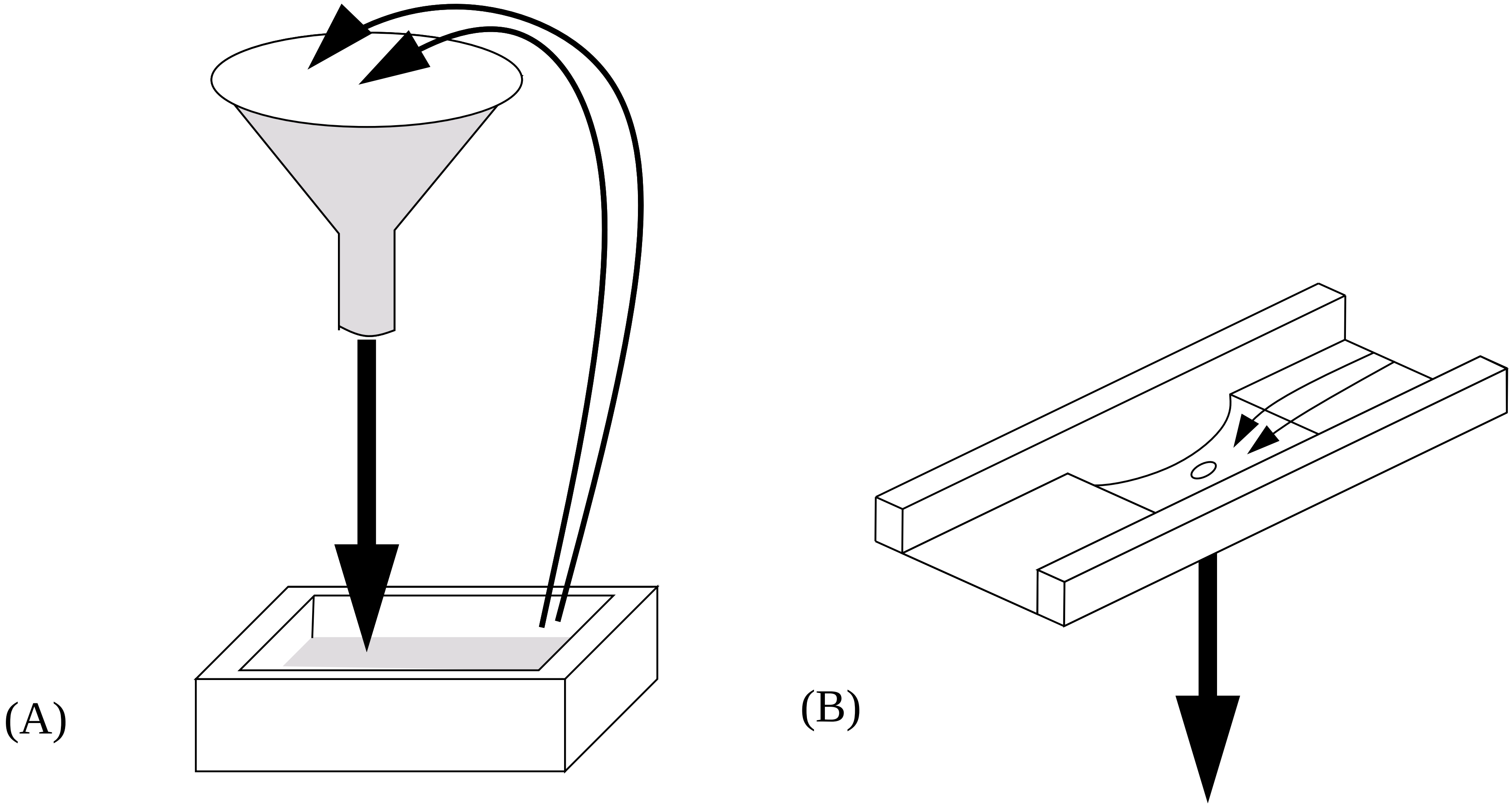


Figure 10:

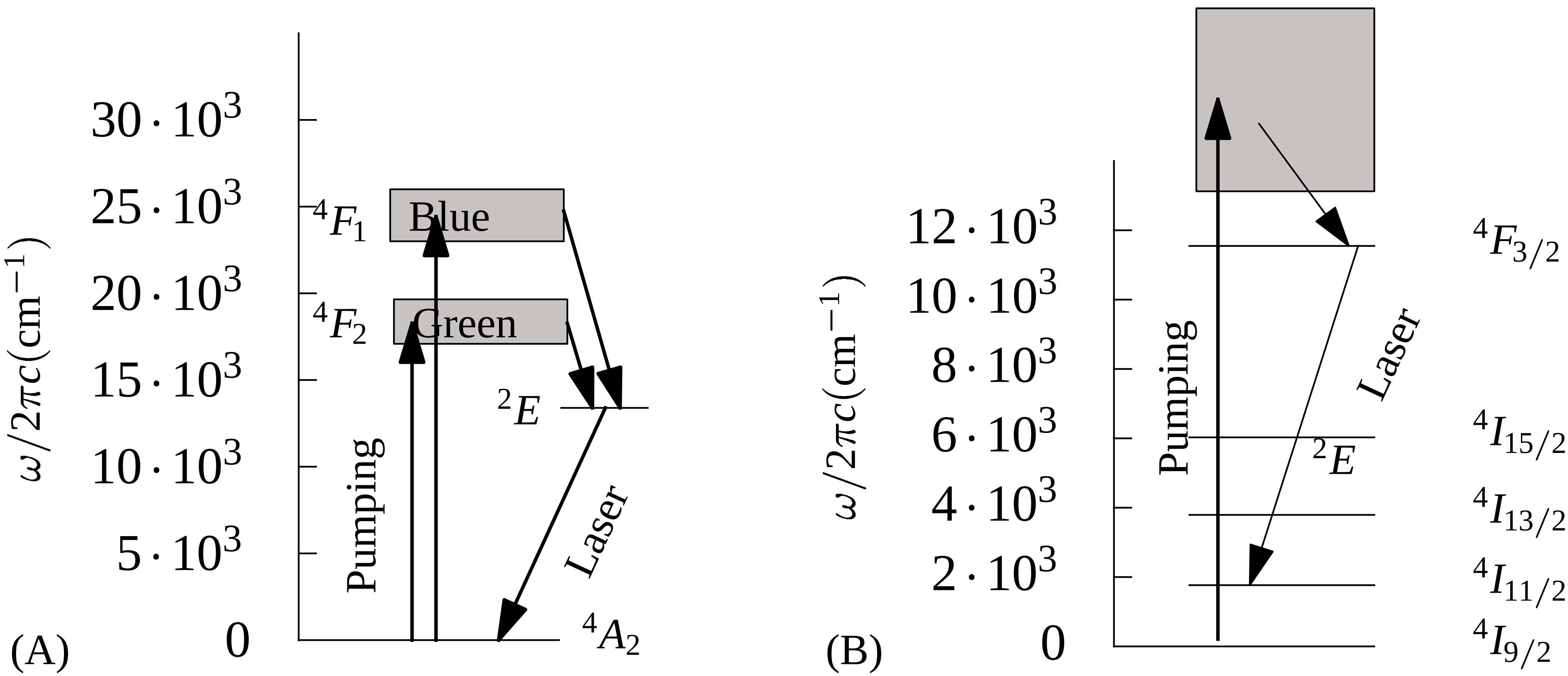


Figure 11: (A) Energy levels of $Cr^{3}+$ in Al_2O_3 (ruby). (B) Energy levels of Nd in $Y_3Al_5O_{12}$ (Nd:YAG).

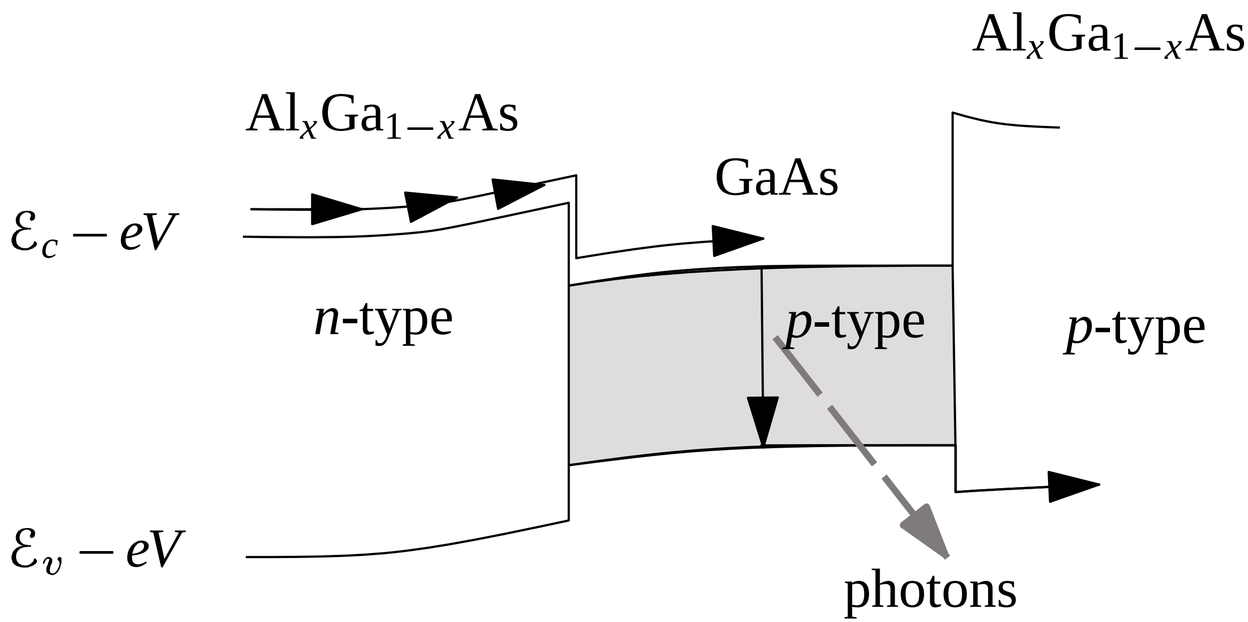


Figure 12: Double heterojunction structure

Active Areas

- ☞ Porous Silicon
- ☞ Negative μ dielectrics
- ☞ Materials to manipulate light as semiconductors manipulate electrons.