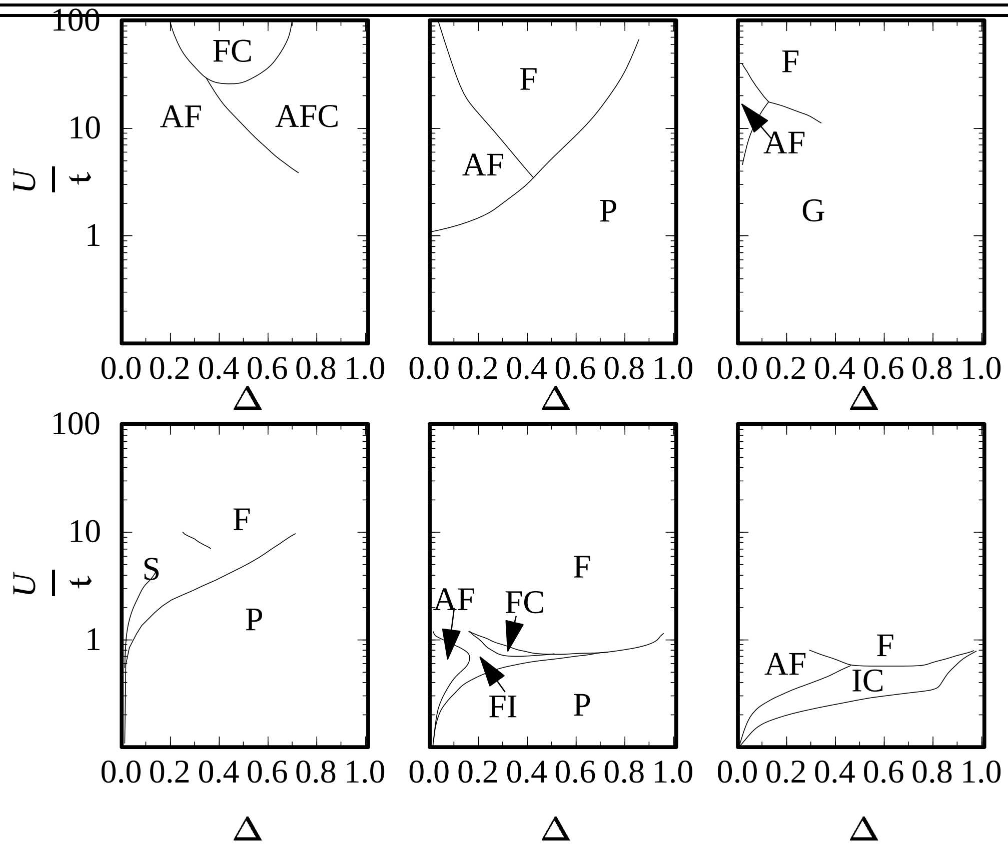
Quantum Mechanics of Interacting Magnetic Moments



F=Ferromagnetic

FC=Short-range ferromagnetic correlations

AF=Antiferromagnetic

AFC=Short-range antiferromagnetic correlations

P=Paramagnetic

FI=Ferrimagnetic

IC=Incommensurate, S=Spiral

G=Correlated state of Gutzwiller type

Definitions

- ☞ Heitler–London Calculation for Ferromagnetism
- ☞ Heisenberg Model of Ferromagnets
- ☞ Néel State
- ☞ Indirect Exchange
- ☞ Spin Waves
- ☞ Schwinger Bosons
- ☞ Holstein–Primakoff Transformation
- ☞ Stoner Model
- ☞ Anderson Model
- ☞ Kondo Effect and Scaling Theory
- ☞ Hubbard Model

$$\vec{B} = \vec{\nabla}\left[\vec{m}_1 \cdot \vec{\nabla}\frac{1}{r}\right] = \frac{3\hat{r}(\vec{m}_1 \cdot \hat{r}) - \vec{m}_1}{r^3}, \tag{L1}$$

$$\frac{\vec{m}_1 \cdot \vec{m}_2 - 3(\vec{m}_2 \cdot \hat{r}_{12})(\vec{m}_1 \cdot \hat{r}_{12})}{r_{12}^3}, \tag{L2}$$

$$\frac{1}{4}\frac{m_1}{\mu_B}\frac{m_2}{\mu_B}\left(\frac{2a_0}{r_{12}}\right)^3\frac{\mu_B^2}{a_0^3} = 0.9\cdot 10^{-4}\,\mathrm{eV}\cdot\frac{m_1}{\mu_B}\frac{m_2}{\mu_B}\left(\frac{2a_0}{r_{12}}\right)^3. \tag{L3}$$

Heitler–London Calculation

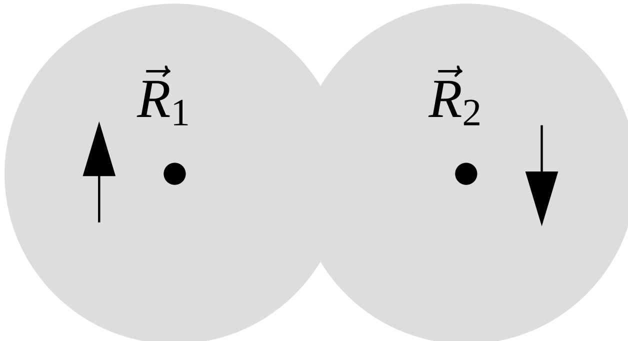


Figure 1: Setting for the calculation of Heitler and London (1927).

$$l \equiv \| \int d\vec{r}\, \phi_1^*(\vec{r})\phi_2(\vec{r}) \| \tag{L4}$$

Singlet:

$$\frac{1}{\sqrt{2}}\left(\chi_\uparrow(\sigma_1)\chi_\downarrow(\sigma_2) - \chi_\downarrow(\sigma_1)\chi_\uparrow(\sigma_2)\right), \tag{L5a}$$

Triplet:

$$\begin{aligned} &\chi_\uparrow(\sigma_1)\chi_\uparrow(\sigma_2) && S=1 \quad ;S_z=1 \\ &\frac{1}{\sqrt{2}}\left(\chi_\uparrow(\sigma_1)\chi_\downarrow(\sigma_2)+\chi_\downarrow(\sigma_1)\chi_\uparrow(\sigma_2)\right) && S=1 \quad ;S_z=0 \\ &\chi_\downarrow(\sigma_1)\chi_\downarrow(\sigma_2) && S=1 \quad ;S_z=-1. \end{aligned} \tag{L6a}$$

$$\phi_s(\vec{r}_1,\vec{r}_2) = \frac{1}{\sqrt{2+2l^2}}[\phi_1(\vec{r}_1)\phi_2(\vec{r}_2)+\phi_1(\vec{r}_2)\phi_2(\vec{r}_1)], \tag{L7a}$$

$$\phi_t(\vec{r}_1,\vec{r}_2) = \frac{1}{\sqrt{2-2l^2}}[\phi_1(\vec{r}_1)\phi_2(\vec{r}_2)-\phi_1(\vec{r}_2)\phi_2(\vec{r}_1)]. \tag{L7b}$$

$$\left[\frac{\hat{P}_1^2}{2m}-\frac{e^2}{|\vec{r}_1-\vec{R}_1|}\right]\phi_1(\vec{r}_1)=\mathcal{E}_0\phi_1(\vec{r}_1). \tag{L8}$$

$$\hat{\mathcal{H}} = \frac{\hat{P}_1^2}{2m} - \frac{e^2}{|\vec{r}_1 - \vec{R}_1|} + \frac{\hat{P}_2^2}{2m} - \frac{e^2}{|\vec{r}_2 - \vec{R}_2|} + \frac{e^2}{|\vec{r}_1 - \vec{r}_2|} + \frac{e^2}{|\vec{R}_1 - \vec{R}_2|} - \frac{e^2}{|\vec{r}_1 - \vec{R}_2|} - \frac{e^2}{|\vec{r}_2 - \vec{R}_1|}. \tag{L9}$$

$$\int d\vec{r}_1 d\vec{r}_2\, \phi_1^*(\vec{r}_1)\phi_2^*(\vec{r}_2)\hat{\mathcal{H}}\phi_1(\vec{r}_1)\phi_2(\vec{r}_2)$$

$$= \int d\vec{r}_1 d\vec{r}_2\, \phi_2^*(\vec{r}_1)\phi_1^*(\vec{r}_2)\hat{\mathcal{H}}\phi_2(\vec{r}_1)\phi_1(\vec{r}_2) \tag{L10}$$

$$= 2\mathcal{E}_0 + U, \tag{L11}$$

where

$$U = \int d\vec{r}_1 d\vec{r}_2 \begin{bmatrix} |\phi_1(\vec{r}_1)|^2 \\ |\phi_2(\vec{r}_2)|^2 \end{bmatrix} \left[\frac{e^2}{|\vec{r}_1 - \vec{r}_2|} + \frac{e^2}{|\vec{R}_1 - \vec{R}_2|} - \frac{e^2}{|\vec{r}_1 - \vec{R}_2|} - \frac{e^2}{|\vec{r}_2 - \vec{R}_1|}\right], \tag{L12}$$

and

$$\begin{aligned}
&\int d\vec{r}_1 d\vec{r}_2\, \phi_2^*(\vec{r}_1)\phi_1^*(\vec{r}_2)\hat{\mathcal{H}}\phi_1(\vec{r}_1)\phi_2(\vec{r}_2) \\
= &\int d\vec{r}_1 d\vec{r}_2\, \phi_1^*(\vec{r}_1)\phi_2^*(\vec{r}_2)\hat{\mathcal{H}}\phi_2(\vec{r}_1)\phi_1(\vec{r}_2) \qquad \text{(L13)} \\
= &\; 2\mathcal{E}_0 l^2 + V, \qquad \text{(L14)}
\end{aligned}$$

with

$$V = \int d\vec{r}_1 d\vec{r}_2 \left[\begin{array}{c} \phi_1^*(\vec{r}_1)\phi_2^*(\vec{r}_2) \\ \phi_2(\vec{r}_1)\phi_1(\vec{r}_2) \end{array} \right] \left[\frac{e^2}{|\vec{r}_1 - \vec{r}_2|} + \frac{e^2}{|\vec{R}_1 - \vec{R}_2|} - \frac{e^2}{|\vec{r}_1 - \vec{R}_1|} - \frac{e^2}{|\vec{r}_2 - \vec{R}_2|} \right]. \qquad \text{(L15)}$$

$$\mathcal{E}_s = \langle \phi_s | \hat{\mathcal{H}} | \phi_s \rangle = 2\frac{2\mathcal{E}_0 + U + 2l^2\mathcal{E}_0 + V}{2 + 2l^2} = 2\mathcal{E}_0 + \frac{U + V}{1 + l^2} \qquad \text{(L16a)}$$

and

$$\mathcal{E}_t = \langle\phi_t|\hat{\mathcal{H}}|\phi_t\rangle = 2\frac{2\mathcal{E}_0 + U - 2l^2\mathcal{E}_0 - V}{2-2l^2} = 2\mathcal{E}_0 + \frac{U-V}{1-l^2} \tag{L16b}$$

$$\mathcal{E}_t - \mathcal{E}_s = \frac{2l^2U - 2V}{1-l^4} \equiv -J. \tag{L17}$$

$$\mathcal{F}\{\phi\} = \frac{\int d\vec{r}_1 d\vec{r}_2 \, \frac{\hbar^2}{2m}|\nabla_1\phi|^2 + \frac{\hbar^2}{2m}|\nabla_2\phi|^2 + U(\vec{r}_1,\vec{r}_2)|\phi(\vec{r}_1,\vec{r}_2)|^2}{\int d\vec{r}_1 d\vec{r}_2 \, |\phi(\vec{r}_1,\vec{r}_2)|^2}. \tag{L18}$$

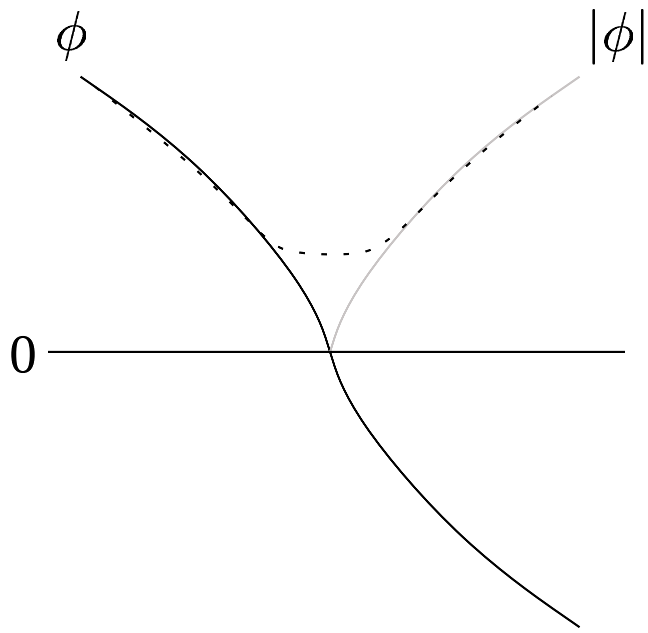


Figure 2: The energy of a wave function with a cusp is always lowered by smoothing out the cusp.

$$\hat{\mathcal{H}} = a + b\hat{S}_1 \cdot \hat{S}_2 \quad \text{(L19)}$$

$$= a + b\left(\hat{S}_1^z\hat{S}_2^z + \frac{1}{2}[\hat{S}_1^+\hat{S}_2^- + \hat{S}_2^+\hat{S}_1^-]\right). \quad \text{(L20)}$$

$$\hat{\mathcal{H}} = 2\mathcal{E}_0 + \frac{U-V}{1-l^2} + \left(\frac{1}{4} - \hat{S}_1 \cdot \hat{S}_2\right)J. \quad \text{(L21)}$$

Heisenberg Model

$$\hat{\mathcal{H}} = -\sum_{\langle ll' \rangle} J_{ll'} \hat{S}_l \cdot \hat{S}_{l'}. \tag{L22}$$

$$\sum_{l=1}^{N} \frac{\hat{P}_l^2}{2m} + \hat{U} = \hat{\mathcal{H}}_{\text{kinetic}} + \hat{\mathcal{H}}_{\text{int}}, \tag{L23}$$

$$\hat{\mathcal{H}}_{\text{int}} = \sum_{\substack{ll'l''l''' \\ \sigma\sigma'}} \langle \vec{R}_l \vec{R}_{l'} | \hat{U} | \vec{R}_{l''} \vec{R}_{l'''} \rangle \hat{c}^\dagger_{l\sigma} \hat{c}^\dagger_{l'\sigma'} \hat{c}_{l'''\sigma'} \hat{c}_{l''\sigma}. \tag{L24}$$

$$\begin{aligned} \hat{\mathcal{H}}_{\text{int}} &= \sum_{\substack{ll' \\ \sigma\sigma'}} \begin{array}{l} \langle \vec{R}_l \vec{R}_{l'} | \hat{U} | \vec{R}_l \vec{R}_{l'} \rangle \hat{c}^\dagger_{l\sigma} \hat{c}^\dagger_{l'\sigma'} \hat{c}_{l'\sigma'} \hat{c}_{l\sigma} \\ + \quad \langle \vec{R}_l \vec{R}_{l'} | \hat{U} | \vec{R}_{l'} \vec{R}_l \rangle \hat{c}^\dagger_{l\sigma} \hat{c}^\dagger_{l'\sigma'} \hat{c}_{l\sigma'} \hat{c}_{l'\sigma} \end{array} && \text{(L25)} \\ &= \sum_{\substack{ll' \\ \sigma\sigma'}} \begin{array}{l} \langle \vec{R}_l \vec{R}_{l'} | \hat{U} | \vec{R}_l \vec{R}_{l'} \rangle \hat{c}^\dagger_{l\sigma} \hat{c}^\dagger_{l'\sigma'} \hat{c}_{l'\sigma'} \hat{c}_{l\sigma} \\ + \quad \langle \vec{R}_l \vec{R}_{l'} | \hat{U} | \vec{R}_{l'} \vec{R}_l \rangle [\hat{n}_{l\sigma} \delta_{ll'} - \hat{c}^\dagger_{l\sigma} \hat{c}_{l\sigma'} \hat{c}^\dagger_{l'\sigma'} \hat{c}_{l'\sigma}]. \end{array} && \text{(L26)} \end{aligned}$$

Heisenberg Model

$$\hat{S}^z = \frac{1}{2}[\hat{c}^\dagger_\uparrow \hat{c}_\uparrow - \hat{c}^\dagger_\downarrow \hat{c}_\downarrow] = \frac{1}{2}[\hat{n}_\uparrow - \hat{n}_\downarrow] \tag{L27a}$$

$$\hat{S}^+ = \hat{c}^\dagger_\uparrow c_\downarrow; \;\; \hat{S}^- = \hat{c}^\dagger_\downarrow c_\uparrow. \tag{L27b}$$

$$\hat{n}_{l\uparrow}\hat{n}_{l'\uparrow} + \hat{n}_{l\downarrow}\hat{n}_{l'\downarrow} \tag{L28}$$

$$= \frac{1}{2}\left\{(\hat{n}_{l\uparrow} + \hat{n}_{l\downarrow})(\hat{n}_{l'\uparrow} + \hat{n}_{l'\downarrow}) + (\hat{n}_{l\uparrow} - \hat{n}_{l\downarrow})(\hat{n}_{l'\uparrow} - \hat{n}_{l'\downarrow})\right\} \tag{L29}$$

$$= \frac{1}{2}\left\{1 + 4\hat{S}^z_l \cdot \hat{S}^z_{l'}\right\}. \tag{L30}$$

$$\hat{\mathcal{H}}_{\text{exch}} = -\langle \vec{R}_l \vec{R}_{l'} | \hat{U} | \vec{R}_{l'} \vec{R}_l \rangle \left\{ \begin{array}{ll} & \hat{n}_{l\uparrow}\hat{n}_{l'\uparrow} + \hat{c}^\dagger_{l\uparrow}\hat{c}_{l\downarrow}c^\dagger_{l'\downarrow}\hat{c}_{l'\uparrow} \\ + & \hat{c}^\dagger_{l\downarrow}\hat{c}_{l\uparrow}c^\dagger_{l'\uparrow}\hat{c}_{l'\downarrow} + \hat{n}_{l\downarrow}\hat{n}_{l'\downarrow} \end{array} \right\} \tag{L31}$$

$$= -2\langle \vec{R}_l \vec{R}_{l'} | \hat{U} | \vec{R}_{l'} \vec{R}_l \rangle \left\{ \frac{1}{4} + \hat{S}^z_l \hat{S}^z_{l'} + \frac{1}{2}[\hat{S}^+_l \hat{S}^-_{l'} + \hat{S}^-_l \hat{S}^+_{l'}] \right\} \tag{L32}$$

$$= \quad -2\langle \vec{R}_l \vec{R}_{l'} | \hat{U} | \vec{R}_{l'} \vec{R}_l \rangle \left\{ \frac{1}{4} + \hat{S}_l \cdot \hat{S}_{l'} \right\}, \tag{L33}$$

$$-4 \sum_{\langle ll' \rangle} \langle \vec{R}_l \vec{R}_{l'} | \hat{U} | \vec{R}_{l'} \vec{R}_l \rangle \hat{S}_l \cdot \hat{S}_{l'} \tag{L34}$$

Ground State

$$\langle \uparrow\uparrow\uparrow \ldots | \hat{\mathcal{H}} | \uparrow\uparrow\uparrow \ldots \rangle = -\sum_{\langle ll' \rangle} \frac{J_{ll'}}{4}. \tag{L35}$$

Spin Waves

$$\hat{S}^{\alpha} = \frac{1}{2}\sum_{ll'} a_l^{\dagger}\sigma_{ll'}^{\alpha}a_{l'}. \qquad \text{(L36)}$$

$$\hat{S}^z = \frac{1}{2}\left(\hat{a}_1^{\dagger}\hat{a}_1 - \hat{a}_2^{\dagger}\hat{a}_2\right) \qquad \text{(L37a)}$$

$$\hat{S}^x = \frac{1}{2}\left(\hat{a}_1^{\dagger}\hat{a}_2 + \hat{a}_2^{\dagger}\hat{a}_1\right) \qquad \text{(L37b)}$$

$$\hat{S}^y = i\frac{1}{2}\left(\hat{a}_2^{\dagger}\hat{a}_1 - \hat{a}_1^{\dagger}\hat{a}_2\right). \qquad \text{(L37c)}$$

$$\begin{aligned}\left[\hat{S}^x, \hat{S}^y\right] &= i\frac{1}{4}\left[\hat{a}_1^{\dagger}\hat{a}_2 + \hat{a}_2^{\dagger}\hat{a}_1, \hat{a}_2^{\dagger}\hat{a}_1 - \hat{a}_1^{\dagger}\hat{a}_2\right] \\ &= i\hat{S}^z. \end{aligned} \qquad \text{(L38)}$$

$$\hat{S}^{+} = \hat{a}_1^{\dagger}\hat{a}_2;\ \hat{S}^{-} = \hat{a}_2^{\dagger}\hat{a}_1. \qquad \text{(L39)}$$

$$\frac{1}{2}\left(\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2\right) = S. \tag{L40}$$

$$\hat{a}_2^\dagger \hat{a}_2 = 2S - \hat{a}_1^\dagger \hat{a}_1 \tag{L41}$$

$$\hat{a}_2 = \sqrt{2S - \hat{a}_1^\dagger \hat{a}_1}. \tag{L42}$$

$$\hat{S}^+ = a_1^\dagger \sqrt{2S - \hat{a}_1^\dagger \hat{a}_1} \tag{L43a}$$

$$\hat{S}^- = \sqrt{2S - \hat{a}_1^\dagger \hat{a}_1}\,\hat{a}_1 \tag{L43b}$$

$$\hat{S}^z = (\hat{a}_1^\dagger \hat{a}_1 - S). \tag{L43c}$$

$$[\hat{S}^+, \hat{S}^-] = 2\hat{S}^z. \tag{L44}$$

$$\hat{S}_l \cdot \hat{S}_{l'} = \frac{1}{2}\left(\hat{S}_l^+\hat{S}_{l'}^- + \hat{S}_{l'}^+\hat{S}_l^-\right) + \hat{S}_l^z\hat{S}_{l'}^z \tag{L45}$$

$$\begin{aligned} &= \frac{1}{2}\hat{a}_l^\dagger\sqrt{2S-\hat{a}_l^\dagger\hat{a}_l}\sqrt{2S-\hat{a}_{l'}^\dagger\hat{a}_{l'}}\hat{a}_{l'} \\ &+ \frac{1}{2}\hat{a}_{l'}^\dagger\sqrt{2S-\hat{a}_{l'}^\dagger\hat{a}_{l'}}\sqrt{2S-\hat{a}_l^\dagger\hat{a}_l}\hat{a}_l + (S-\hat{a}_l^\dagger\hat{a}_l)(S-\hat{a}_{l'}^\dagger\hat{a}_{l'}). \end{aligned} \tag{L46}$$

$$\hat{a}_l = \sqrt{S}b_l + \left(\hat{a}_l - \sqrt{S}b_l\right), \tag{L47}$$

$$\hat{\mathcal{H}} = -\sum_{ll'} J_{ll'}S^2 \left[\begin{array}{r} \frac{1}{2}\left(b_l b_{l'}^* + b_{l'}b_l^*\right)\sqrt{2-|b_l|^2}\sqrt{2-|b_{l'}|^2} \\ +\left(1-|b_l|^2\right)\left(1-|b_{l'}|^2\right) \end{array} \right]. \tag{L48}$$

$$\mathcal{E}_0 = -JNzS^2\left(|b|^2(2-|b|^2) + (1-|b|^2)^2\right) = -JNzS^2, \tag{L49}$$

$$\hat{\mathcal{H}} \approx -NJzS^2 - 2J\sum_{\langle ll'\rangle} S\left(\hat{a}_l^\dagger\hat{a}_{l'} + \hat{a}_{l'}^\dagger\hat{a}_l - \hat{a}_l^\dagger\hat{a}_l - \hat{a}_{l'}^\dagger\hat{a}_{l'}\right). \tag{L50}$$

$$\hat{a}_l = \frac{1}{\sqrt{N}} \sum_{\vec{k}} \hat{a}_k e^{-i\vec{k}\cdot\vec{r}_l} \tag{L52}$$

$$\hat{\mathcal{H}} = -NJzS^2 - 2JS \sum_{\vec{k}} \sum_{\vec{\delta}} \left[\cos\left(\vec{k}\cdot\vec{\delta}\right) - 1\right] \hat{a}^{\dagger}_{\vec{k}} \hat{a}_{\vec{k}} \tag{L53}$$

$$= -NJzS^2 + \sum_{\vec{k}} \hbar\omega_{\vec{k}} \hat{n}_{\vec{k}}, \tag{L54}$$

where

$$\hbar\omega = 2SJ \sum_{\vec{\delta}} \left(1 - \cos(\vec{\delta}\cdot\vec{k})\right), \tag{L55}$$

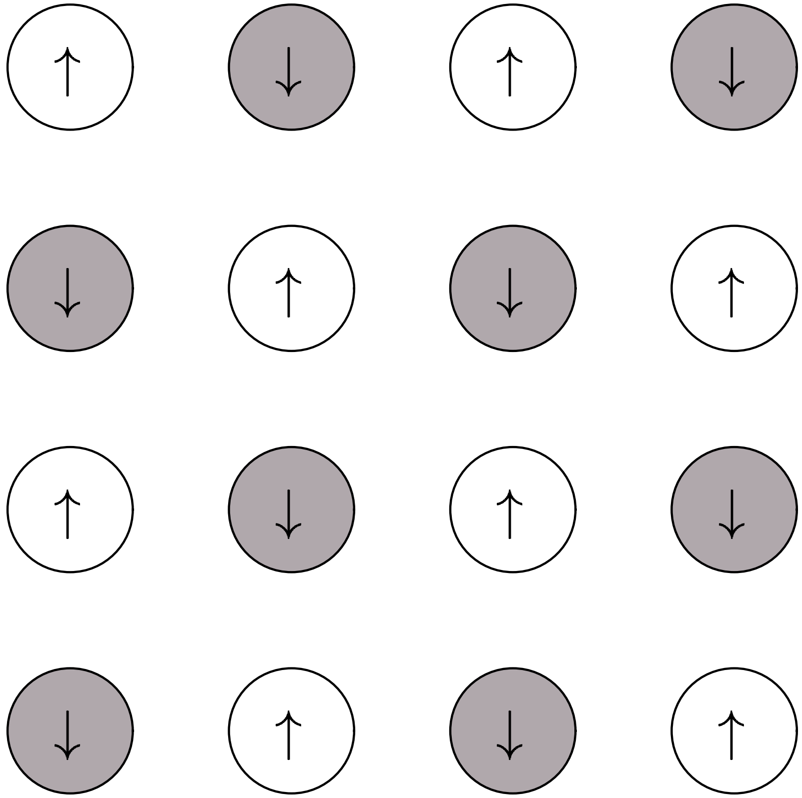


Figure 3: The Néel state.

$$S^{\pm}_{l'} \rightarrow S^{\mp}_{l'} \quad S^z_{l'} \rightarrow -S^z_{l'}. \tag{L56}$$

$$\hat{\mathcal{H}} = 2|J|\sum_{\langle ll'\rangle}\tfrac{1}{2}\left[\hat{S}_l^+\hat{S}_{l'}^+ + \hat{S}_l^-\hat{S}_{l'}^-\right] - \hat{S}_l^z\hat{S}_{l'}^z \tag{L57}$$

$$= 2|J|\sum_{\langle ll'\rangle}\begin{array}{l} \tfrac{1}{2}\hat{a}_l^\dagger\sqrt{2S-\hat{a}_l^\dagger\hat{a}_l}\,\hat{a}_{l'}^\dagger\sqrt{2S-\hat{a}_{l'}^\dagger\hat{a}_{l'}} \\ + \ \tfrac{1}{2}\sqrt{2S-\hat{a}_l^\dagger\hat{a}_l}\,\hat{a}_l\sqrt{2S-\hat{a}_{l'}^\dagger\hat{a}_{l'}}\,\hat{a}_{l'} - (\hat{a}_l^\dagger\hat{a}_l - S)(\hat{a}_{l'}^\dagger\hat{a}_{l'} - S). \end{array} \tag{L58}$$

$$\hat{\mathcal{H}} \approx -Nz|J|S^2\left[\left(1-b^2\right)^2 - b^2\left(2-b^2\right)\right], \tag{L59}$$

$$b = 0. \tag{L60}$$

$$\hat{\mathcal{H}} \approx 2|J|\sum_{\langle ll'\rangle}\left[-S^2 + S\left\{\hat{a}_l^\dagger\hat{a}_l + \hat{a}_{l'}^\dagger\hat{a}_{l'} + \hat{a}_l^\dagger\hat{a}_{l'}^\dagger + \hat{a}_l\hat{a}_{l'}\right\}\right]. \tag{L61}$$

$$\hat{a}_{\vec{k}} = \frac{1}{\sqrt{N}}\sum_l e^{i\vec{k}\cdot\vec{R}_l}\hat{a}_l \tag{L62}$$

$$\hat{\mathcal{H}} = -|J|NzS^2 + |J|S\sum_{\vec{k}\vec{\delta}}\left[\left(\hat{a}^\dagger_{\vec{k}}\hat{a}^\dagger_{-\vec{k}} + \hat{a}_{\vec{k}}\hat{a}_{-\vec{k}}\right)\cos(\vec{k}\cdot\vec{\delta}) + 2\hat{a}^\dagger_{\vec{k}}\hat{a}_{\vec{k}}\right]. \tag{L63}$$

$$\hat{a}_{\vec{k}} = \cosh\alpha_{\vec{k}}\,\hat{\gamma}_{\vec{k}} + \sinh\alpha_{\vec{k}}\,\hat{\gamma}^\dagger_{-\vec{k}}, \tag{L64}$$

$$\tanh 2\alpha_{\vec{k}} = -\frac{1}{z}\sum_{\vec{\delta}}\cos(\vec{k}\cdot\vec{\delta}). \tag{L65}$$

$$\hat{\mathcal{H}} = -Nz|J|S(S+1) + 2|J|zS\sum_{\vec{k}}\left(\hat{\gamma}^\dagger_{\vec{k}}\hat{\gamma}_{\vec{k}} + \frac{1}{2}\right)\sqrt{1-\tanh^2 2\alpha_{\vec{k}}}. \tag{L66}$$

$$-NS^2|J|z\left(1+\frac{\Gamma}{zS}\right). \tag{L67}$$

$$\mathcal{E}_{\vec{k}} = 2|J|S\sqrt{z^2 - (\sum_{\vec{\delta}}\cos\vec{k}\cdot\vec{\delta})^2}. \tag{L68}$$

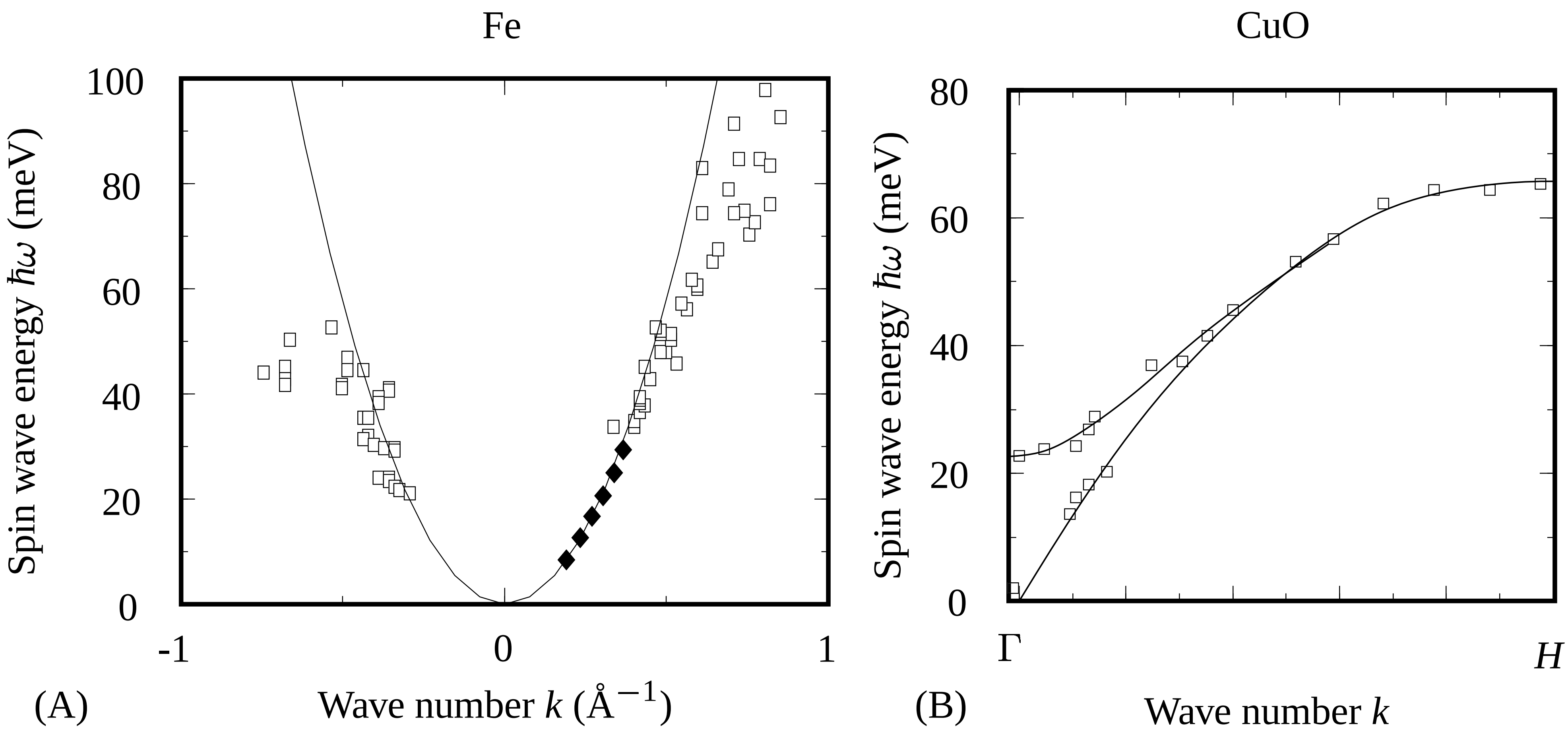


Figure 4: (A) Dispersion relation for ferromagnetic magnons in iron. [Yethiraj et al. (1991), and Lynn (1975),.] (B) Dispersion relation for antiferromagnetic magnons in CuO. [Aïn et al. (1989).]

$$\mathcal{E} = \int_0^{\mathcal{E}_F-\Delta} d\mathcal{E}'\,D(\mathcal{E}')\mathcal{E}' + \frac{1}{2}\int_{\mathcal{E}_F-\Delta}^{\mathcal{E}_F+\Delta} d\mathcal{E}'\,D(\mathcal{E}')\mathcal{E}' - \frac{1}{2}nJ\,\langle S\rangle^2, \tag{L69}$$

$$\langle S\rangle = \frac{1}{2n}\int_{\mathcal{E}_F-\Delta}^{\mathcal{E}_F+\Delta} d\mathcal{E}'\,\frac{1}{2}D(\mathcal{E}') = \frac{1}{2n}D(\mathcal{E}_F)\Delta. \tag{L70}$$

$$\left.\frac{\partial\mathcal{E}}{\partial\Delta}\right|_{\mathcal{E}_F} = \Delta D(\mathcal{E}_F) - \frac{J}{4n}D(\mathcal{E}_F)^2\Delta, \tag{L71}$$

$$\left.\frac{\partial\mathcal{E}}{\partial\Delta}\right|_{\mathcal{E}_F} = 0 \Rightarrow \frac{J}{n}D(\mathcal{E}_F) = 4. \tag{L72}$$

$$\mathcal{E} = \int_0^{\mathcal{E}_F-\Delta_1} d\mathcal{E}'\,D(\mathcal{E}')\mathcal{E}' + \frac{1}{2}\int_{\mathcal{E}_F-\Delta_1}^{\mathcal{E}_F+\Delta_2} d\mathcal{E}'\,D(\mathcal{E}')\mathcal{E}' - \frac{1}{2}Jn\,\langle S\rangle^2, \tag{L73}$$

$$\frac{\partial\Delta_2}{\partial\Delta_1} = \frac{D(\mathcal{E}_F-\Delta_1)}{D(\mathcal{E}_F+\Delta_2)}, \tag{L74}$$

$$\frac{\partial \mathcal{E}}{\partial \Delta_1} \leq 0 \qquad \text{(L75)}$$

$$\Rightarrow \Delta_1 + \Delta_2 \leq \frac{J}{4n} \int_{\mathcal{E}_F - \Delta_1}^{\mathcal{E}_F + \Delta_2} d\mathcal{E}' D(\mathcal{E}'). \qquad \text{(L76)}$$

$$\mathcal{E} = \mathcal{E}_\uparrow + \mathcal{E}_\downarrow \tag{L77}$$

where

$$\mathcal{E}_\uparrow = N_\uparrow \left[\frac{3}{5} \mathcal{E}_{F\uparrow} - \frac{3}{4} \frac{e^2 k_{F\uparrow}}{\pi} \right], \tag{L78}$$

$$\mathcal{E}_{F\uparrow} = \frac{\hbar^2 k_{F\uparrow}^2}{2m}, \text{ and } \frac{4\pi}{3} \frac{1}{(2\pi)^3} k_{F\uparrow}^3 = \frac{N_\uparrow}{\mathcal{V}}. \tag{L79}$$

$$\mathcal{E}_{\text{polarized}} = N \left[\frac{3}{5} \frac{\hbar^2}{2m} \left(6\pi^2 n\right)^{2/3} - \frac{3}{4\pi} e^2 \left(6\pi^2 n\right)^{1/3} \right], \tag{L80}$$

$$\mathcal{E}_{\text{unpolarized}} = N \left[\frac{3}{5} \frac{\hbar^2}{2m} \left(3\pi^2 n\right)^{2/3} - \frac{3}{4\pi} e^2 \left(3\pi^2 n\right)^{1/3} \right]. \tag{L81}$$

$$\frac{2\pi\hbar^2}{5m} \left(\frac{1}{2^{1/3}} + 1 \right) < e^2 (6\pi^2 n)^{-1/3} \tag{L82}$$

Calculations Within Band Theory

$$\Rightarrow \quad \frac{r_W}{a_0} > \frac{2\pi}{5}\left(\frac{1}{2^{1/3}}+1\right)\left(\frac{9\pi}{2}\right)^{1/3} = 5.45. \qquad \text{(L83)}$$

Element:	Sc	Ti	V	Cr	Mn	Fe	Co	Ni
Calculated m/μ_B (bcc):	0	0	0	0	0.70	2.15	1.68	0.38
Experimental m/μ_B (bcc):				0		2.12		
Calculated m/μ_B (fcc):	0	0	0	0	0	0	1.56	0.60
Experimental m/μ_B (fcc):							1.61	0.61

Kondo Effect

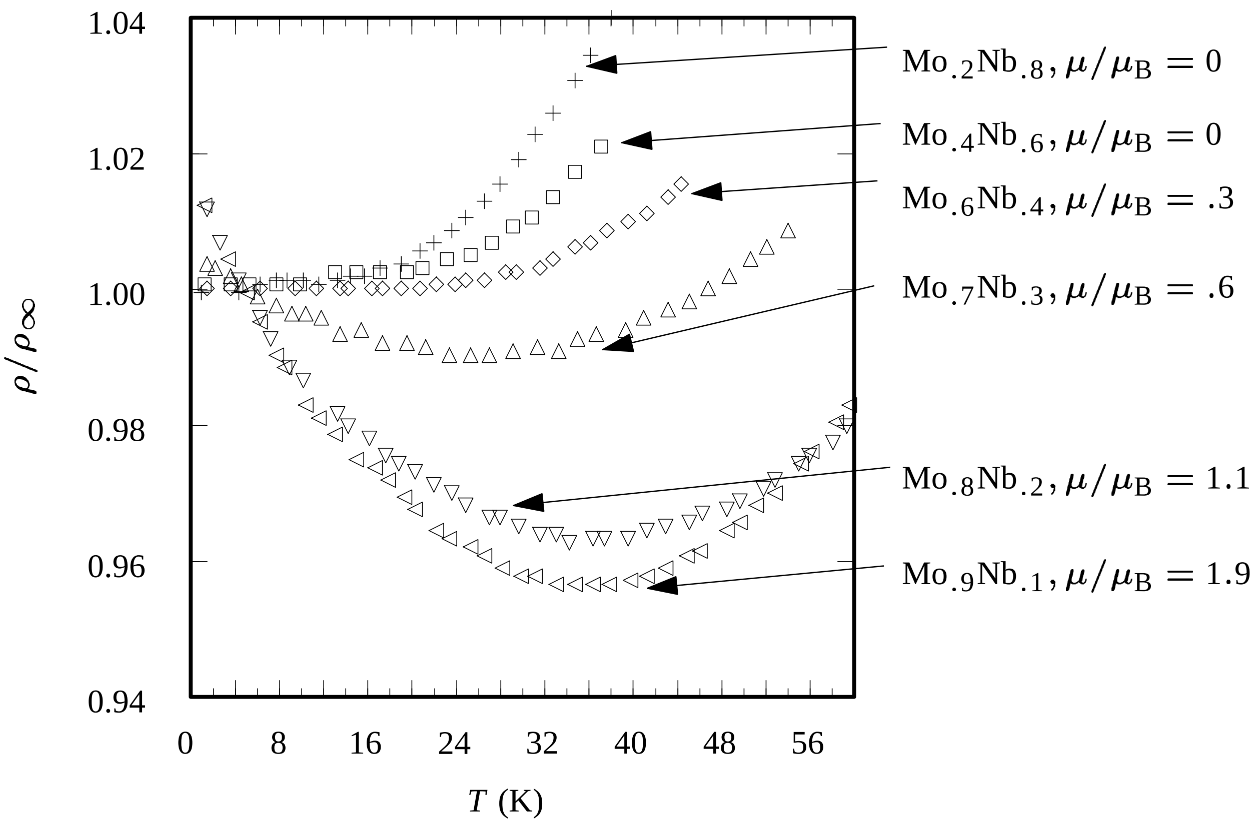


Figure 5: Resistivity data for Mo_xNb_{1-x} alloys. [Source: Sarachik et al. (1964).]

Anderson Model

$$\hat{\mathcal{H}} = \epsilon_0[\hat{n}_{0\uparrow} + \hat{n}_{0\downarrow}] + U\hat{n}_{0\uparrow}\hat{n}_{0\downarrow} + \sum_{\vec{k}\sigma}[\epsilon_{\vec{k}}\hat{c}^\dagger_{\vec{k}\sigma}\hat{c}_{\vec{k}\sigma} + v_{\vec{k}}\hat{c}^\dagger_{0\sigma}\hat{c}_{\vec{k}\sigma} + v^*_{\vec{k}}\hat{c}^\dagger_{\vec{k}\sigma}\hat{c}_{0\sigma}]. \tag{L84}$$

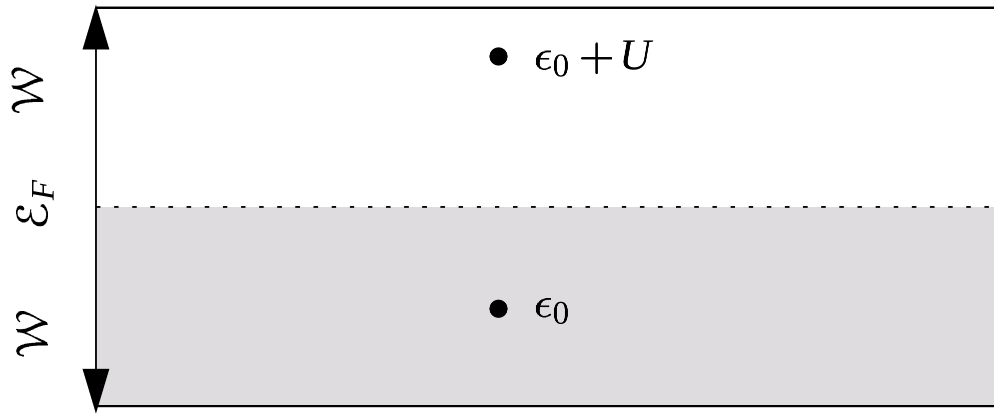


Figure 6: Conduction electrons placed in contact with an impurity site.

$$\hat{P}_0 = (1 - \hat{n}_{0\downarrow})(1 - \hat{n}_{0\uparrow}), \tag{L85}$$

$$|\psi_0\rangle = \hat{P}_0|\psi\rangle,\ |\psi_1\rangle = \hat{P}_1|\psi\rangle, \text{and} |\psi_2\rangle = \hat{P}_2|\psi\rangle. \tag{L86}$$

$$\hat{\mathcal{H}}_{ll'} = \hat{P}_l\hat{\mathcal{H}}\hat{P}_{l'} \tag{L87}$$

so $\hat{\mathcal{H}}|\psi\rangle = \mathcal{E}|\psi\rangle$ can be rewritten as

$$\begin{pmatrix} \hat{\mathcal{H}}_{00} & \hat{\mathcal{H}}_{01} & 0 \\ \hat{\mathcal{H}}_{10} & \hat{\mathcal{H}}_{11} & \hat{\mathcal{H}}_{12} \\ 0 & \hat{\mathcal{H}}_{21} & \hat{\mathcal{H}}_{22} \end{pmatrix} \begin{pmatrix} |\psi_0\rangle \\ |\psi_1\rangle \\ |\psi_2\rangle \end{pmatrix} = \mathcal{E} \begin{pmatrix} |\psi_0\rangle \\ |\psi_1\rangle \\ |\psi_2\rangle \end{pmatrix} \tag{L88}$$

$$\hat{\mathcal{H}}_{00}|\psi_0\rangle + \hat{\mathcal{H}}_{01}|\psi_1\rangle = \mathcal{E}|\psi_0\rangle \tag{L89}$$

$$\Rightarrow \qquad |\psi_0\rangle = \left(\mathcal{E} - \hat{\mathcal{H}}_{00}\right)^{-1} \hat{\mathcal{H}}_{01}|\psi_1\rangle \tag{L90}$$

$$\text{and} \qquad |\psi_2\rangle = \left(\mathcal{E} - \hat{\mathcal{H}}_{22}\right)^{-1} \hat{\mathcal{H}}_{21}|\psi_1\rangle; \tag{L91}$$

$$\left\{\hat{\mathcal{H}}_{10}\left(\mathcal{E} - \hat{\mathcal{H}}_{00}\right)^{-1}\hat{\mathcal{H}}_{01} + \left(\hat{\mathcal{H}}_{11} - \mathcal{E}\right) + \hat{\mathcal{H}}_{12}\left(\mathcal{E} - \hat{\mathcal{H}}_{22}\right)^{-1}\hat{\mathcal{H}}_{21}\right\}|\psi_1\rangle = 0. \tag{L92}$$

$$\sum_{\vec{k}\sigma} v_{\vec{k}} \hat{c}^{\dagger}_{0\sigma} \hat{c}_{\vec{k}\sigma}. \tag{L93}$$

$$\hat{\mathcal{H}}_{10} = \sum_{\vec{k}\sigma} v_{\vec{k}} \hat{c}^{\dagger}_{0\sigma} \hat{c}_{\vec{k}\sigma} \hat{P}_0 \quad \text{(L94)}$$

$$= \sum_{\vec{k}\sigma} v_{\vec{k}} \hat{c}^{\dagger}_{0\sigma} \hat{c}_{\vec{k}\sigma} (1 - \hat{n}_{0\downarrow})(1 - \hat{n}_{0\uparrow}) \quad \text{(L95)}$$

$$= \sum_{\vec{k}\sigma} v_{\vec{k}} \hat{c}^{\dagger}_{0\sigma} \hat{c}_{\vec{k}\sigma} (1 - \hat{n}_{0,-\sigma}). \quad \text{(L96)}$$

$$\hat{\mathcal{H}}_{01} = \hat{\mathcal{H}}^{*}_{10} = \sum_{\vec{k}\sigma} (1 - \hat{n}_{0,-\sigma}) v^{*}_{\vec{k}} \hat{c}^{\dagger}_{\vec{k}\sigma} \hat{c}_{0\sigma}. \quad \text{(L97)}$$

$$\hat{\mathcal{H}}_{11} = \hat{P}_1 [\epsilon_0 + \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} \hat{c}^{\dagger}_{\vec{k}\sigma} \hat{c}_{\vec{k}\sigma}]; \; \hat{\mathcal{H}}_{00} = \hat{P}_0 \sum_{\vec{k}\sigma} \epsilon_{\vec{k}} \hat{c}^{\dagger}_{\vec{k}\sigma} \hat{c}_{\vec{k}\sigma} \quad \text{(L98)}$$

and

$$\hat{\mathcal{H}}_{21} = \hat{\mathcal{H}}^{*}_{12} = \sum_{\vec{k}\sigma} v_{\vec{k}} \hat{c}^{\dagger}_{0\sigma} \hat{c}_{\vec{k}\sigma} \hat{n}_{0,-\sigma}. \quad \text{(L99)}$$

Anderson Model

$$\hat{\mathcal{H}}_{10}\left(\mathcal{E}-\hat{\mathcal{H}}_{00}\right)^{-1}\hat{\mathcal{H}}_{01}|\psi_1\rangle \tag{L100}$$

$$= \hat{\mathcal{H}}_{10}\sum_{\vec{k}\sigma}(1-\hat{n}_{0,-\sigma})v^*_{\vec{k}}\hat{c}^\dagger_{\vec{k}\sigma}\hat{c}_{0\sigma}\left(\mathcal{E}-[\hat{\mathcal{H}}_{11}-\epsilon_0+\epsilon_{\vec{k}}]\right)^{-1}|\psi_1\rangle. \tag{L101}$$

$$\frac{\hat{\mathcal{H}}_{10}}{\epsilon_0-\mathcal{E}_F}\sum_{\vec{k}\sigma}v^*_{\vec{k}}(1-\hat{n}_{0,-\sigma})\hat{c}^\dagger_{\vec{k}\sigma}\hat{c}_{0\sigma}|\psi_1\rangle \tag{L102}$$

$$= \sum_{\vec{k}\vec{k}'\sigma\sigma'}\frac{v_{\vec{k}'}v^*_{\vec{k}}}{\epsilon_0-\mathcal{E}_F}\hat{c}^\dagger_{0\sigma'}\hat{c}_{\vec{k}'\sigma'}(1-\hat{n}_{0,-\sigma'})(1-\hat{n}_{0,-\sigma})\hat{c}^\dagger_{\vec{k}\sigma}\hat{c}_{0\sigma}|\psi_1\rangle \tag{L103}$$

$$= \sum_{\vec{k}\vec{k}'\sigma\sigma'}\frac{v_{\vec{k}'}v^*_{\vec{k}}}{\mathcal{E}_F-\epsilon_0}\hat{c}^\dagger_{0\sigma'}\hat{c}^\dagger_{\vec{k}\sigma}\hat{c}_{\vec{k}'\sigma'}\hat{c}_{0\sigma}|\psi_1\rangle. \tag{L104}$$

$$\hat{n}_{0\uparrow}\hat{c}^\dagger_{\vec{k}\uparrow}\hat{c}_{\vec{k}'\uparrow}+\hat{n}_{0\downarrow}\hat{c}^\dagger_{\vec{k}\downarrow}\hat{c}_{\vec{k}'\downarrow} \tag{L105}$$

$$
\begin{aligned}
&= \frac{1}{2}(\hat{n}_{0\uparrow} - \hat{n}_{0\downarrow})(\hat{c}^{\dagger}_{\vec{k}\uparrow}\hat{c}_{\vec{k}'\uparrow} - \hat{c}^{\dagger}_{\vec{k}\downarrow}\hat{c}_{\vec{k}'\downarrow}) \\
&\quad + \frac{1}{2}(\hat{n}_{0\uparrow} + \hat{n}_{0\downarrow})(\hat{c}^{\dagger}_{\vec{k}\uparrow}\hat{c}_{\vec{k}'\uparrow} + \hat{c}^{\dagger}_{\vec{k}\downarrow}\hat{c}_{\vec{k}'\downarrow}) && \text{(L106)} \\
&= \hat{S}^z(\hat{c}^{\dagger}_{\vec{k}\uparrow}\hat{c}_{\vec{k}'\uparrow} - \hat{c}^{\dagger}_{\vec{k}\downarrow}\hat{c}_{\vec{k}'\downarrow}) + \frac{1}{2}\sum_{\sigma}\hat{c}^{\dagger}_{\vec{k}\sigma}\hat{c}_{\vec{k}'\sigma}. && \text{(L107)}
\end{aligned}
$$

$$
\sum_{\vec{k}\vec{k}'} \frac{v_{\vec{k}'}v^*_{\vec{k}}}{\mathcal{E}_F - \epsilon_0}\left[\hat{S}^+\hat{c}^{\dagger}_{\vec{k}\downarrow}\hat{c}_{\vec{k}'\uparrow} + \hat{S}^-\hat{c}^{\dagger}_{\vec{k}\uparrow}\hat{c}_{\vec{k}'\downarrow} + \hat{S}^z(\hat{c}^{\dagger}_{\vec{k}\uparrow}\hat{c}_{\vec{k}'\uparrow} - \hat{c}^{\dagger}_{\vec{k}\downarrow}\hat{c}_{\vec{k}'\downarrow}) + \frac{1}{2}\sum_{\sigma}\hat{c}^{\dagger}_{\vec{k}\sigma}\hat{c}_{\vec{k}'\sigma}\right]|\psi_1\rangle. \tag{L108}
$$

$$
\begin{aligned}
\hat{\mathcal{H}}_{\text{eff}} &= \hat{\mathcal{H}}_{11} + \sum_{\vec{k}\vec{k}'} J_{\vec{k}\vec{k}'}\left[\hat{S}^+\hat{c}^{\dagger}_{\vec{k}\downarrow}\hat{c}_{\vec{k}'\uparrow} + \hat{S}^-\hat{c}^{\dagger}_{\vec{k}\uparrow}\hat{c}_{\vec{k}'\downarrow} + \hat{S}^z(\hat{c}^{\dagger}_{\vec{k}\uparrow}\hat{c}_{\vec{k}'\uparrow} - \hat{c}^{\dagger}_{\vec{k}\downarrow}\hat{c}_{\vec{k}'\downarrow})\right] \\
&\quad + K_{\vec{k}\vec{k}'}\sum_{\sigma}\hat{c}^{\dagger}_{\vec{k}\sigma}\hat{c}_{\vec{k}'\sigma} && \text{(L109a)} \\
J_{\vec{k}\vec{k}'} &= v_{\vec{k}'}v^*_{\vec{k}}\left[\frac{1}{\mathcal{E}_F - \epsilon_0} + \frac{1}{U + \epsilon_0 - \mathcal{E}_F}\right]. && \text{(L109b)}
\end{aligned}
$$

$$\hat{\mathcal{H}} = \sum_{\substack{\vec{k}\sigma \\ \epsilon_{\vec{k}} < \mathcal{W}}} \epsilon_{\vec{k}} \hat{c}^{\dagger}_{\vec{k}\sigma} \hat{c}_{\vec{k}\sigma} + \sum_{\vec{k}\vec{k}'} J \left\{ \hat{S}^{-} \hat{c}^{\dagger}_{\vec{k}\uparrow} \hat{c}_{\vec{k}'\downarrow} + \hat{S}^{+} \hat{c}^{\dagger}_{\vec{k}\downarrow} \hat{c}_{\vec{k}'\uparrow} + \hat{S}^{z} \left[\hat{c}^{\dagger}_{\vec{k}\uparrow} \hat{c}_{\vec{k}'\uparrow} - \hat{c}^{\dagger}_{\vec{k}\downarrow} \hat{c}_{\vec{k}'\downarrow} \right] \right\}. \quad \text{(L110)}$$

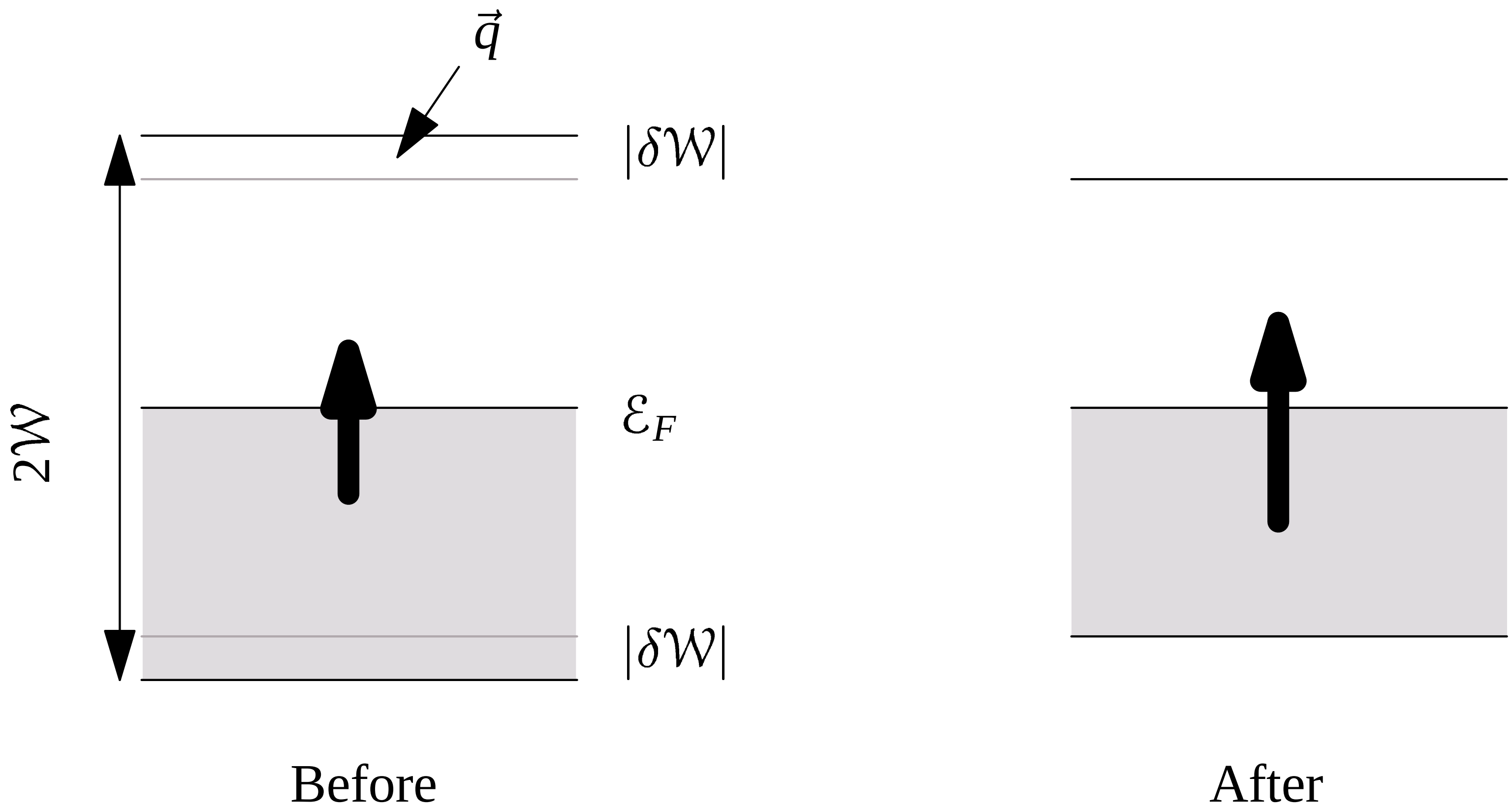


Figure 7: A small band of states, of width $|\delta\mathcal{W}|$ is eliminated from the upper and lower band edges.

$$\hat{\mathcal{H}}_{12} = J\sum_{\vec{k}\vec{q}} \hat{S}^{-}\hat{c}^{\dagger}_{\vec{k}\uparrow}\hat{c}_{\vec{q}\downarrow} + \hat{S}^{+}\hat{c}^{\dagger}_{\vec{k}\downarrow}\hat{c}_{\vec{q}\uparrow} + \hat{S}^{z}\left[\hat{c}^{\dagger}_{\vec{k}\uparrow}\hat{c}_{\vec{q}\uparrow} - \hat{c}^{\dagger}_{\vec{k}\downarrow}\hat{c}_{\vec{q}\downarrow}\right], \tag{L111a}$$

$$\hat{\mathcal{H}}_{21} = J\sum_{\vec{k}'\vec{q}'} \hat{S}^{-}\hat{c}^{\dagger}_{\vec{q}'\uparrow}\hat{c}_{\vec{k}\downarrow} + \hat{S}^{+}\hat{c}^{\dagger}_{\vec{q}'\downarrow}\hat{c}_{\vec{k}'\uparrow} + \hat{S}^{z}\left[\hat{c}^{\dagger}_{\vec{q}'\uparrow}\hat{c}_{\vec{k}'\uparrow} - \hat{c}^{\dagger}_{\vec{q}'\downarrow}\hat{c}_{\vec{k}'\downarrow}\right]. \tag{L111b}$$

$$\hat{\mathcal{H}}_{12}(\mathcal{E}-\hat{\mathcal{H}}_{22})^{-1}\hat{\mathcal{H}}_{21}|\psi_1\rangle \tag{L112}$$

$$\approx \hat{\mathcal{H}}_{12}\hat{\mathcal{H}}_{21}(\mathcal{E}-\hat{\mathcal{H}}_{22}-[\mathcal{W}-\mathcal{E}_F])^{-1}|\psi_1\rangle \tag{L113}$$

$$\approx \hat{\mathcal{H}}_{12}\hat{\mathcal{H}}_{21}(-\mathcal{W})^{-1}|\psi_1\rangle. \tag{L114}$$

$$\hat{\mathcal{H}}_{12}\hat{\mathcal{H}}_{21} = J^2 D(\mathcal{W})[-\delta\mathcal{W}]\sum_{\vec{k}\vec{k}'}\frac{3}{4}\sum_{\sigma}\hat{c}^{\dagger}_{\vec{k}\sigma}\hat{c}_{\vec{k}'\sigma} - \begin{Bmatrix}\hat{S}^{-}\hat{c}^{\dagger}_{\vec{k}\uparrow}\hat{c}_{\vec{k}'\downarrow} + \hat{S}^{+}\hat{c}^{\dagger}_{\vec{k}\downarrow}\hat{c}_{\vec{k}'\uparrow} \\ +\hat{S}^{z}\left[\hat{c}^{\dagger}_{\vec{k}\uparrow}\hat{c}_{\vec{k}'\uparrow} - \hat{c}^{\dagger}_{\vec{k}\downarrow}\hat{c}_{\vec{k}'\downarrow}\right]\end{Bmatrix}. \tag{L115}$$

$$J+\delta J = J - 2\frac{J^2}{\mathcal{W}}D(\mathcal{W})\delta\mathcal{W}, \tag{L116}$$

$$\frac{3}{2}J^2 D(\mathcal{W})\frac{\delta\mathcal{W}}{\mathcal{W}}\sum_{\vec{k}\vec{k}'\sigma}\hat{c}^\dagger_{\vec{k}\sigma}\hat{c}_{\vec{k}'\sigma}. \tag{L117}$$

$$\frac{dJ}{d\mathcal{W}} = -2\frac{J^2}{\mathcal{W}}D(\mathcal{W}). \tag{L118}$$

$$\mathcal{W}\exp\left[-\frac{1}{2D_0 J}\right] = \text{constant} \equiv k_B T_K, \tag{L119}$$

$$\rho = \mathcal{F}\left(\frac{T}{T_K}\right). \tag{L120}$$

$$\mathcal{F}(x) \quad = \quad \left[\frac{1}{\ln(x)}\right]^2 \tag{L121}$$

Scaling Theory

$$\Rightarrow \mathcal{F}\left(\frac{T}{T_K}\right) = \rho = \left[\frac{2D_0J}{1+2D_0J\ln(k_BT/\mathcal{W})}\right]^2 \quad \text{(L122)}$$

$$\sim 4D_0^2J^2\left(1-4D_0J\ln(k_BT/\mathcal{W})\right). \quad \text{(L123)}$$

$$\rho \sim \mathcal{A}T^5 - \mathcal{B}n_{\text{mi}}\ln(k_BT/\mathcal{W}), \quad \text{(L124)}$$

$$\frac{d\rho}{dT} = 0 \Rightarrow T_{\text{min}} = \left(\frac{\mathcal{B}n_{\text{mi}}}{5\mathcal{A}}\right)^{1/5}. \quad \text{(L125)}$$

$$\mathcal{F}\left(\frac{T}{T_K}\right) = \left[\frac{1}{\cosh^{-1}(T/T_K)}\right]^2, \quad \text{(L126)}$$

$$C_\mathcal{V} \propto n\frac{T}{T_K} = n\frac{k_BT}{\mathcal{W}}\exp\left[\frac{1}{2D_0J}\right]. \quad \text{(L127)}$$

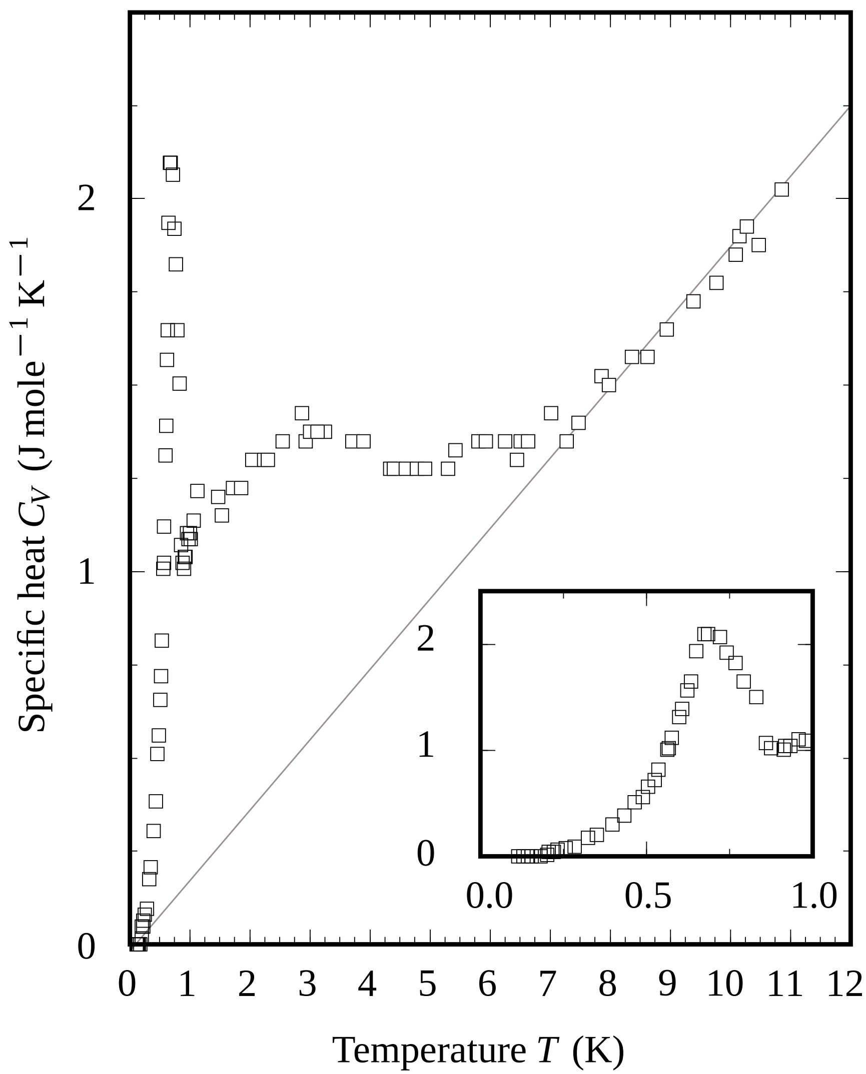


Figure 8: Low-temperature specific heat of the heavy fermion compound UBe_{13}. [Source Ott et al. (1983, 1984).]

Hubbard Model

$$\hat{\mathcal{H}} = \sum_{\substack{\langle ll' \rangle \\ \sigma}} -\mathfrak{t}\left[\hat{c}^\dagger_{l\sigma}\hat{c}_{l'\sigma} + \hat{c}^\dagger_{l'\sigma}\hat{c}_{l\sigma}\right] + U\sum_l \hat{c}^\dagger_{l\uparrow}\hat{c}_{l\uparrow}\hat{c}^\dagger_{l\downarrow}\hat{c}_{l\downarrow}, \tag{L128}$$

Mean-Field Solution

$$\hat{\mathcal{H}} = \sum_{\substack{\langle ll' \rangle \\ \sigma}} -\mathfrak{t}\left[\hat{c}^\dagger_{l\sigma}\hat{c}_{l'\sigma} + \hat{c}^\dagger_{l'\sigma}\hat{c}_{l\sigma}\right] + U\sum_l \hat{n}_{l\uparrow}\hat{n}_{l\downarrow}. \tag{L129}$$

$$\hat{n}_{l\sigma} = n_\sigma + (\hat{n}_{l\sigma} - n_\sigma)\,. \tag{L130}$$

$$\hat{\mathcal{H}} \approx \sum_{\substack{\langle ll' \rangle \\ \sigma}} -\mathfrak{t}\left[\hat{c}^\dagger_{l\sigma}\hat{c}_{l'\sigma} + \hat{c}^\dagger_{l'\sigma}\hat{c}_{l\sigma}\right] + U\sum_l \hat{n}_{l\uparrow}n_\downarrow + n_\uparrow\hat{n}_{l\downarrow} - n_\uparrow n_\downarrow. \tag{L131}$$

$$\sum_{\vec{k}\vec{\delta}\sigma} -\mathfrak{t}\hat{c}^\dagger_{\vec{k}\sigma}\hat{c}_{\vec{k}\sigma}\cos\vec{\delta}\cdot\vec{k} + U\sum_{\vec{k}} \hat{n}_{\vec{k}\uparrow}n_\downarrow + n_\uparrow\hat{n}_{\vec{k}\downarrow} - n_\uparrow n_\downarrow. \tag{L132}$$

$$Nn_\uparrow = Na\int_{-k_{F\uparrow}}^{k_{F\uparrow}} \frac{dk}{2\pi} \tag{L133}$$

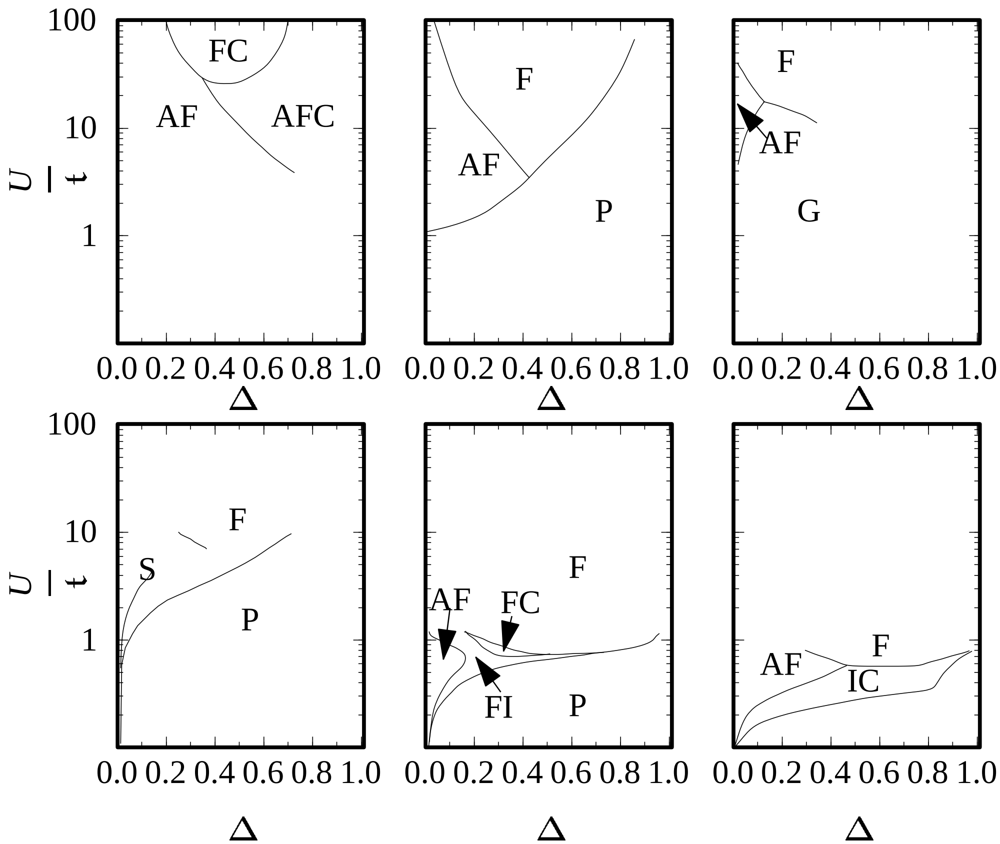
$$\Rightarrow \pi n_\uparrow = a k_{F\uparrow}. \tag{L134}$$

$$\mathcal{E}_0 = \frac{N}{\pi}\left[-2\mathfrak{t}\right]\left[\sin \pi n_\uparrow + \sin \pi n_\downarrow\right] + NU n_\uparrow n_\downarrow. \tag{L135}$$

$$\mathcal{E}_0 = \frac{-4\mathfrak{t}N}{\pi} \sin \pi n_\uparrow + NU n_\uparrow \left(1 - n_\uparrow\right). \tag{L136}$$

$$\frac{U}{\mathfrak{t}} > \frac{16}{\pi}, \tag{L137}$$

An Unsolved Problem...



F=Ferromagnetic

FC=Short-range ferromagnetic correlations

AF=Antiferromagnetic

AFC=Short-range antiferromagnetic correlations

P=Paramagnetic

FI=Ferrimagnetic

IC=Incommensurate, S=Spiral

G=Correlated state of Gutzwiller type

Figure 9: Six representative phase diagrams of the two-dimensional Hubbard model