# Physics 5413: Chaos, fractals, and nonlinear dynamics

### 1. What is a chaos? (Hilborn chapter 1)

- 1.1 Example: Population dynamics and the logistic map
- 1.2 Example: Driven damped pendulum
- 1.3 Example: Convecting fluids: Lorenz model
- 1.4 Determinism unpredictability, and the divergence of trajectories
- 1.5 The importance of being nonlinear

#### 2. Universality of Chaos (Hilborn chapter 2)

- 2.1 Feigenbaum numbers
- $2.2\,$  Renormalization of the logistic map

#### 3. Dynamics in state space (Hilborn chapter 3)

- 3.1 State space
- 3.2 Systems described by 1st order differential equations
- $3.3\,$  No-intersection theorem
- 3.4 Dissipative systems and attractors
- 3.5 One-dimensional state space
- 3.6 Structural stability of fixed points
- 3.7 State space volume
- 3.8 two-dimensional state space
- 3.9 Limit cycles
- 3.10 Poincaré sections and the stability of limit cycles
- 3.11 Bifurcations
- 4. Three-dimensional state space and chaos (Hilborn chapter 4)
  - 4.1 Heuristics
  - 4.2 Three-dimensional dynamical systems
  - 4.3 Quasi-periodic behavior
  - 4.4 Chaotic transients and homoclinic orbits
  - 4.5 Homoclinic tangles and horseshoes
  - 4.6 Lyapunov exponents and chaos

#### 5. Iterated maps and period doubling (Hilborn chapter 5)

- 5.1 Bifurcations, period doubling and chaos
- 5.2 Universality revisited
- 5.3 Tent map
- $5.4\,$  Gaussian map

## 6. Quasiperiodicity and chaos (Hilborn chapter 6)

- 6.1 Poincaré sections and winding numbers
- 6.2 Frequency locking
- 6.3 Sine-circle map
- $6.4\,$  Devil's staircase and Farey tree
- 6.5 Chaos

# 7. Intermittency and Crisis (Hilborn chapter 7)

- 7.1 The cause of intermittency
- 7.2 Quantitative theory of intermittency
- 7.3 Crises
- 8. Fractals and multifractals (Hilborn parts of chapters 9, 10; Strogatz chapter 11)
- 9. Pattern formation
- 10. Quantum chaos