

Physics 6311: Statistical Mechanics - Homework 1

due date: Tuesday, Aug 31, 2021

Part A: Math warmup

Problem 1: Exact differentials (14 points)

a) Test whether the following differentials are exact.

$$\begin{aligned} du_a &= (y^2 - x^3) dx + (2xy + y^4) dy \\ du_b &= (2y^2 - x^2) dx - 2x dy \end{aligned}$$

b) If the differential is exact, calculate the indefinite integral.

c) Check the dependence of the integral on the path of integration by explicitly integrating both differentials from point $(x_i, y_i) = (0, 0)$ to $(x_f, y_f) = (2, 2)$ on two different paths, $(0, 0) \rightarrow (2, 0) \rightarrow (2, 2)$ and $(0, 0) \rightarrow (0, 2) \rightarrow (2, 2)$. Compare the results of the two paths and that of a calculation using the indefinite integral (if it exists).

Problem 2: Properties the δ function (10 points)

Compute the following integrals by manipulating the δ function

$$\begin{aligned} I_a &= \int_0^\infty dx x^2 \delta(e^x - 4) \\ I_b &= \int_{-\infty}^\infty dx \cos(\pi x) \delta(9 - x^2) \end{aligned}$$

Problem 3: Gaussian integrals (10 points)

Compute the following integral in terms of A and B .

$$I = \int_{-\infty}^\infty \int_{-\infty}^\infty dx dy e^{-(x^2 - xy + y^2) + Ax + By}$$

Part B: Thermodynamics

Problem 4: Equilibrium states (6 points)

Decide which of the following states is in an equilibrium state, a non-equilibrium steady state, or not a steady state. Explain your reasoning. In some cases, the state is not a true steady or equilibrium state but close to one. Discuss under what conditions it can be treated as a steady or equilibrium state.

- a cup of hot tea, sitting on the table while cooling down
- the wine in a bottle that is stored in a wine cellar
- the sun

- d) the atmosphere of the earth
- e) electrons in the wiring of a flashlight switched off
- f) electrons in the wiring of a flashlight switched on