**Physics 6311: Statistical Mechanics - Homework 1**

due date: Tuesday, Aug 31, 2021

**Part A: Math warmup**

**Problem 1: Exact differentials** (14 points)

a) Test whether the following differentials are exact.

\[
\begin{align*}
\text{du}_a &= (y^2 - x^3) \, dx + (2xy + y^4) \, dy \\
\text{du}_b &= (2y^2 - x^2) \, dx - 2x \, dy
\end{align*}
\]

b) If the differential is exact, calculate the indefinite integral.

c) Check the dependence of the integral on the path of integration by explicitly integrating both differentials from point \((x_i, y_i) = (0, 0)\) to \((x_f, y_f) = (2, 2)\) on two different path, \((0, 0) \rightarrow (2, 0) \rightarrow (2, 2)\) and \((0, 0) \rightarrow (0, 2) \rightarrow (2, 2)\). Compare the results of the two path and that of a calculation using the indefinite integral (if it exists).

**Problem 2: Properties the \(\delta\) function** (10 points)

Compute the following integrals by manipulating the \(\delta\) function

\[
\begin{align*}
I_a &= \int_0^\infty dx \, x^2 \, \delta(e^x - 4) \\
I_b &= \int_{-\infty}^\infty dx \, \cos(\pi x) \, \delta(9 - x^2)
\end{align*}
\]

**Problem 3: Gaussian integrals** (10 points)

Compute the following integral in terms of \(A\) and \(B\).

\[
I = \int_{-\infty}^\infty \int_{-\infty}^\infty dx \, dy \, e^{-(x^2 - xy + y^2) + Ax + By}
\]

**Part B: Thermodynamics**

**Problem 4: Equilibrium states** (6 points)

Decide which of the following states is in an equilibrium state, a non-equilibrium steady state, or not a steady state. Explain your reasoning. In some cases, the state is not a true steady or equilibrium state but close to one. Discuss under what conditions it can be treated as a steady or equilibrium state.

a) a cup of hot tea, sitting on the table while cooling down
b) the wine in a bottle that is stored in a wine cellar
c) the sun
d) the atmosphere of the earth
e) electrons in the wiring of a flashlight switched off
f) electrons in the wiring of a flashlight switched on