Physics 6311: Statistical Mechanics - Homework 10

---

due date: November 2, 2021

Problem 1: Radiation of Betelgeuse (8 points)

The luminosity (total amount of energy emitted per time) of the star Betelgeuse is about 10^4 times that of the sun. (The solar luminosity is approximately 3.828×10^{26} W.) The energy density $u(\epsilon)$ of Betelgeuse's radiation has its maximum at a photon energy $\epsilon \approx 0.8$ eV.

a) Find the surface temperature of Betelgeuse, assuming it emits blackbody radiation.

b) Estimate the radius of Betelgeuse.

c) Why is Betelgeuse called a red giant?

Problem 2: Background radiation (5 points)

In space there exists a background electromagnetic radiation corresponding to an equilibrium temperature of about 3K. At what frequency is the maximum of the energy density of this radiation? What is the corresponding wavelength?

Problem 3: Generalized Bose gas (15 points)

Consider a gas of noninteracting identical bosons of spin S in d dimensions. The single-particle energy-momentum relation is given by $\epsilon(\mathbf{p}) = A|\mathbf{p}|^z$ with positive prefactor A and exponent z.

a) Compute the density of states $g(\epsilon)$.

b) Calculate the maximum possible particle number in excited single-particle states as a function of temperature. For which values of d and z does the system show Bose-Einstein condensation?

c) If there is Bose-Einstein condensation, evaluate the critical temperature T_c

d) Find the specific heat for temperatures $T \leq T_c$.

e) Find the pressure for temperatures $T \leq T_c$.

Problem 4: Thermodynamics of Magnons (12 points)

Spin waves or magnons are elementary excitations of Bose type in ferromagnetic materials. Their dispersion relation is $\omega = D\ k^2$ for small frequencies $\omega \ll \omega_{max}$. Calculate the contribution of the magnons to the specific heat at low temperatures $k_B T \ll \hbar\omega_{max}$. (Hint: There is no conservation law for the magnon number, the rest mass is zero.)