Physics 6311: Statistical Mechanics - Homework 12

due date: Tuesday, November 16, 2021

Problem 1: Liquid ^{3}Helium (6 points)

Liquid ^{3}He is approximately a Fermi gas (spin 1/2). The density is 0.081 g/cm^3.

a) Calculate the Fermi energy (at zero temperature). Also calculate the Fermi velocity (the velocity corresponding to the Fermi energy).

b) At roughly what temperatures do you expect the fermionic character of ^{3}He to be important?

Problem 2: Fermions on a surface (10 points)

Consider an ideal gas of N spin-1/2 fermions of mass m on a planar surface of area A. Derive a closed form expression for the chemical potential as a function of temperature T (valid for all temperatures). Discuss the limits $T \to 0$ and $T \to \infty$.

Problem 3: Spin susceptibility of an ideal Fermi gas (12 points)

An ideal gas of N spin-1/2 fermions in a cube of of size L is under the influence of a weak magnetic field B. The field adds the term $\sigma \mu_B B$ to the single-particle energies where $\sigma = \pm 1$ for up and down spins, respectively. Neglect the effects of the field on the orbital motion of the fermions.

a) Find the Fermi momenta $k_{F\uparrow}$ and $k_{F\downarrow}$ for the up and down spins. (Because the field is weak, you can assume $\mu_B B \ll \epsilon_F$.)

b) Determine the magnetization $m = (\langle N_\uparrow \rangle - \langle N_\downarrow \rangle)/N$ at zero temperature. Here $\langle N_\uparrow \rangle$ and $\langle N_\downarrow \rangle$ are the numbers of spin-up and spin-down particles, respectively.

c) Determine the magnetic susceptibility (the so-called Pauli susceptibility) $\chi = (\partial m / \partial B)_T$.

Problem 4: Higher order corrections in Sommerfeld expansion (12 points)

Use the Sommerfeld expansion to $O(T^4)$ to calculate the chemical potential and the energy of a Fermi gas in a box.

a) Show that

$$\mu \quad = \quad \epsilon_F \left[1 - \frac{\pi^2}{12} \left(\frac{k_B T}{\epsilon_F} \right)^2 - \frac{\pi^4}{80} \left(\frac{k_B T}{\epsilon_F} \right)^4 \right] + O(T^6)$$

$$\frac{E}{N} = \frac{3}{5}\epsilon_F\left[1+\frac{5\pi^2}{12}\left(\frac{k_BT}{\epsilon_F}\right)^2-\frac{\pi^4}{16}\left(\frac{k_BT}{\epsilon_F}\right)^4\right]+O(T^6)$$

b) From a), determine the T^3 correction to the usual linear-in-T specific heat. Compare its magnitude in a typical metal (such as copper) with that of the Debye contribution of the phonons.