

# Physics 6311: Statistical Mechanics - Homework 3

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due date: September 14, 2021

## Problem 1: Probability of a density fluctuation (10 points)

Consider two identical boxes, A and B.

- 10 particles are distributed over the two boxes at random. Calculate the probabilities  $P(4)$  and  $P(5)$  for finding exactly  $N_A = 4$  and  $N_A = 5$  particles in the box A, respectively. Calculate  $P(4)/P(5)$ .
- Repeat the calculations for  $10^{23}$  particles. Compare  $N_A = 0.4 * 10^{23}$  and  $N_A = 0.5 * 10^{23}$ . (For large  $n$  the factorial can be approximated by Stirling's formula  $n! = \sqrt{2\pi n} n^n e^{-n}$ )

## Problem 2: Transformation of random variables (10 points)

The random variables  $X$  and  $Y$  are independent and Gaussian distributed with averages  $\langle x \rangle = \langle y \rangle = 0$  and standard deviations  $\sigma_x = \sigma_y = 1$ .

- Calculate the characteristic functions of  $X$  and  $Y$ .
- Find the characteristic function for the random variable  $Z = X^2 + Y^2$ .
- Derive the probability density  $P_Z(z)$  of the random variable  $Z$
- Compute the moments  $\langle z \rangle$ ,  $\langle z^2 \rangle$ , and  $\langle z^3 \rangle$ . Find the first three cumulants.

## Problem 3: Power-law distribution (10 point)

The probability density of a random variable  $J$  has the power-law form

$$P_J(j) = \begin{cases} A(1-j)^y & (0 \leq j \leq 1) \\ 0 & \text{otherwise} \end{cases} .$$

where  $A$  and  $y$  are constants.

- Find the normalization constant  $A$  as a function of the exponent  $y$ . For which  $y$  is  $P_J$  a valid probability density?
- Calculate the average of  $J$  as well as the geometric mean. How do they compare for  $y \rightarrow -1$ ?

## Problem 4: Diode (10 points)

The current  $I$  across a diode is related to the applied voltage  $V$  via

$$I = I_0[e^{eV/(k_B T)} - 1] .$$

The diode is subject to a random voltage  $V$  which is Gaussian distributed with zero mean and variance  $\sigma^2$ . Calculate the probability density  $P(I)$  of the current. Find the most probable current and the average current.