

Physics 6311: Statistical Mechanics - Homework 4

due date: Tuesday, Sep 21, 2021

Problem 1: Additivity of the Shannon entropy (5 points)

Consider the joint probability distribution of two random variables X and Y . Show that if the two variables are statistically independent then the Shannon entropy S_S of the joint distribution is the sum of the entropies of the distributions of X and Y .

Problem 2: Shannon entropy of a random N -letter word (8 points)

Your printer malfunctions and prints random words, each consisting of N lower-case (English) letters. Find the Shannon entropy for this distribution of words

- if all letters of the alphabet occur with equal probability as each of the N letters (positions) in the word;
- if every second letter in the word is one of the vowels (a, e, i, o, u) while every other letter in the word is not a vowel

Hint: Use the result of problem 1, generalized to N random variables.

Problem 3: Maxima of entropy (12 points)

Consider the entropy of a discrete probability distribution given in terms of the probabilities p_i ($i = 1 \dots N$). Determine which p_i lead to the maximum entropy under the following constraints (Hint: Use Lagrange multipliers to enforce the constraints.):

- Normalization $\sum_i p_i = 1$
- Normalization $\sum_i p_i = 1$ and fixed average $\langle a \rangle = \sum_i p_i a_i$ of a quantity A with values a_i .
- Normalization, fixed average $\langle a \rangle$ and fixed variance σ_A^2 .

Problem 4: System of three-level atoms in the microcanonical ensemble (15 points)

A system consists of N non-interacting, distinguishable three-level atoms. Each atom can be in one of three states with energies $E_1 = E_2 = 0$ and $E_3 = \epsilon$.

- Work out the number of available states Ω at fixed N_3 (number of atoms in level 3).
- Calculate the entropy as a function of the total energy E (use Stirling's formula).
- Calculate the temperature.
- Express the energy as a function of temperature.