Physics 6311: Statistical Mechanics - Homework 6

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due date: Tuesday, Oct 5, 2021

Problem 1: Model of DNA (10 points)

A simple model of the DNA double helix molecule is analogous to a zipper: a chain of N links each of which can be open or closed. A closed link has energy ϵ_0, and an open link has energy $\epsilon_1 > \epsilon_0$. Replication of the DNA starts with the opening of the "zipper". Assume that it can only open from one end (say the left), i.e., a link can only be open if all links left of it are also open.

a) Calculate the partition function for this DNA model.

b) Find the average number of open links n as a function of N and temperature T .

c) Discuss the behavior of n in the limits of high and low temperatures.

Problem 2: Generalized equipartition theorem (10 points)

Consider a classical Hamiltonian of the form

$$H = \sum_{i=1}^{3N} \frac{1}{2} A_i |q_i|^n + \sum_{i=1}^{3N} \frac{1}{2} B_i p_i^2$$

where $n > 0$ is an exponent that characterizes the potential energy and A_i and B_i are positive constants. Using the canonical ensemble, calculate the internal energy and the specific heat at constant volume as functions of temperature.