Physics 6311: Test Prep Homework 7

due date: Tuesday, October 12, 2021

Problem 1: Exponential distribution (10 points)

A continuous random variable X is characterized by a probability density

$$P_X(x) = Ae^{-a|x-x_0|} \ .$$

Calculate the normalization constant A, the moments $\langle x \rangle$ and $\langle x^2 \rangle$, as well as the cumulants C_1 and C_2.

Problem 2: Quantum mechanical three-level system (10 points)

A quantum-mechanical system has three energy eigenstates $|0\rangle, |-1\rangle$ and $|1\rangle$ and energies $\epsilon_0 = 0$, $\epsilon_1 = \epsilon_{-1} = \epsilon$ $(\epsilon > 0)$.

a) Use the canonical ensemble to calculate the Helmholtz free energy, the entropy and the heat capacity as functions of temperature.

b) Calculate the occupation probabilities p_0, p_1 and p_{-1} of the three levels as functions of temperature. At what temperature is $p_1/p_0 = 2$? Discuss the sign of this temperature.

Problem 3: Ideal gas in rotating cylinder (10 points)

Consider a non-relativistic classical ideal gas of N particles (mass m) at temperature T in a cylindrical vessel of radius R and height H. The cylinder is rotating around its vertical axis with angular velocity ω.

a) Using the canonical ensemble, calculate the internal energy and the specific heat of the gas. [Hint: Work in a rotating reference frame and neglect the Coriolis force.]

b) Calculate how the particle density $n(r)$ changes with the distance r from the rotation axis. (Hint: the particle density $n(r)$ is a reduced probability density of the phase space density $\rho(\vec{r}, \vec{p})$.)

Problem 4: Broadening of spectral lines (10 points)

The atoms of a star emit light. The emission frequency of a particular element is ν_0 if the atom is a rest. Due to the thermal motion (temperature T) the observed frequency is shifted (Doppler

effect) to

$$\nu = \nu_0 \left(1 - \frac{v}{c} \cos\theta\right)$$

where v is the velocity of the atom and θ is the angle between the directions of motion and observation. Calculate the resulting intensity distribution $\rho(\nu)$. What is the width of the spectral line? (Assume the atoms to be noninteracting and to move non-relativistically!)