Physics 6311: Statistical Mechanics - Homework 8

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due date: October 19, 2021

Problem 1: Particle number fluctuations in grand-canonical ensemble (10 points)

Consider a many-particle system in the grand-canonical ensemble characterized by a chemical potential μ and temperature T. Compute the variance of the particle number N and relate it to $(\partial N/\partial \mu)$. Use the result to discuss how the particle number fluctuations behave in the thermodynamic limit.

Problem 2: Identical particles in two-level system (10 points)

A quantum mechanical system has two single-particle states $|a\rangle$ and $|b\rangle$ with energies $\epsilon_a = -\epsilon_b = \epsilon$.

a) The system is occupied by two identical particles. Write down all possible states, the corresponding energies and the **canonical** probabilities for these states for bosons (S=0) and for fermions (S=1/2, but both particles being in the $\uparrow$ state). Using the canonical ensemble calculate the Helmholtz free energy, the entropy, the internal energy and the specific heat as functions of temperature.

b) Consider an additional term in the Hamiltonian, viz, an interaction between the particles of the form Un_an_b. where U is the interaction energy and n_a and n_b are the particle numbers of the two single-particle states. How do the canonical probabilities for the two-boson states from a) change as a result of U? Discuss the limits $U \to \infty$ and $U \to -\infty$.