due date: Tuesday, Oct 26, 2021

## Problem 1: Quantum corrections to classical ideal gas (20 points)

Calculate the lowest order quantum corrections to the energy and pressure of the classical ideal gas (as functions of particle number and temperature) both for fermions and for bosons. (Hint: Start from the Bose and Fermi occupation numbers. In the classical (Boltzmann) limit the

(Hint: Start from the Bose and Fermi occupation numbers. In the classical (Boltzmann) limit the average occupation numbers are small compared to 1. Expand the occupation numbers about this limit. Don't forget the corrections to  $\mu$ .)

## Problem 2: Adsorbed atoms in equilibrium with ideal gas (20 points)

- a) Consider a classical ideal gas of  $N \gg 1$  atoms (mass m) in a volume V. Use the canonical ensemble to calculate its chemical potential  $\mu_{\text{gas}}$  as a function of temperature T and particle number density N/V.
- b) Rewrite  $\mu_{\rm gas}$  as a function of pressure and temperature.
- c) Now consider a single adsorption site on a solid surface. It can either be empty (energy 0) or occupied by a gas atom (energy  $-\epsilon$  with  $\epsilon > 0$ ). Use the grand-canonical ensemble to calculate the grand partition function of the adsorption site as a function of temperature T and chemical potential  $\mu_{\rm sf}$ .
- d) Calculate the average number of atoms on the absorption site as a function of temperature T, and chemical potential  $\mu_{sf}$ .
- e) The gas and surface are brought into thermal and chemical equilibrium. Find the average number of atoms on the absorption site as a function of the pressure of the ideal gas and the temperature. Discuss the limits of vanishing and infinite pressure. (Hint: In chemical equilibrium the chemical potentials of the adsorbed atoms and the gas atoms are equal.)