Rec 7: Sept. 15, 2009

23.38 a) \( W_{\text{cons}} = -\Delta U = U_i - U_f \)

\[ U_i = 0 \ (\text{far}) \]

\[ V_f = q \cdot V_f \]
\[ \dot{q} = \lambda r \, d\theta \]
\[ \lambda = \frac{Q}{2\pi r} \]
\[ d\dot{q} = \frac{Q}{2\pi r} \, d\theta \]
\[ dV = \frac{kQ}{r} \]

\[ V = \frac{kQ}{2\pi r} \int_0^{2\pi} d\theta = \frac{kQ}{r} \]

\[ E_f = E_i \]
\[ K_f + V_f^2 = K_i + V_i \]
\[ K_f = \frac{1}{2} m V_f^2 = V_i \cdot \frac{1}{2} \frac{2V_i}{m} = \sqrt{2(1.69)} \]

\[ V_f = \frac{1.5 \times 10^{-3}}{1.5 \times 10^{-3}} = 67.4 \text{ m/s} \]

b) \( W_{\text{cons}} \) is independent of path so \( W \) is as well.

c) It will move to \( \infty \)

23.40 a) \[ E_{\text{sheet}} = \frac{1.61}{2 \varepsilon_0} \Rightarrow E = \frac{1.61}{\varepsilon_0} = \frac{47 \times 10^{-9}}{8.85 \times 10^{-12}} = 5.31 \times 10^3 \]

b) \( \Delta V = Ed = 0.33 \Rightarrow 116.8 \text{ V} \)

c) \( d \rightarrow 2d \); \( E \) stays same so \( V \) doubles
23.46 

\[ \frac{E}{\text{constant}} \] 

\[ \Delta V = -f \cdot E \cdot d \delta \]

Since \( d \delta \) and \( E \) are at \( 180^\circ \)

\[ \Delta V \] increases in moving away from sheet.

And this is independent of reference point.

b) Equipotentials are planes parallel to plastic sheet.

\[ \Delta V = Ed \Rightarrow d = \frac{\Delta V}{E} = \frac{4V}{107/\varepsilon_0} = \frac{2\varepsilon_0}{107} \]

\[ d = \frac{2(8.85 \times 10^{-12})}{4 \times 10^{-9}} = 2.95 \text{ mm} \]

23.86 

\[ V = A(x^2 - 3y^2 + z^2) \]

\[ E_x = -\frac{\partial V}{\partial x} = -2Ax \]

\[ E_y = -\frac{\partial V}{\partial y} = 0 \]

\[ E_z = -\frac{\partial V}{\partial z} = -2Az \]

b) \( W = q \int E \cdot \hat{k} \, dz \) (motion along \( z \)-axis)

\[ W = q \int_0^L (-2Az) \, dz = -qA^2 \int_0^L z = qA^2 z \Rightarrow A = \frac{W}{qA^2} \]

\[ A = (15 \times 10^{-5}) 25^2 = 640 \text{ V/m} \]

\[ \frac{E}{E} (z = 0.25) = -2 (640) (0.25)^2 = -320 \text{ V/m} \hat{k} \]

c) In planes \( y = \) constant.

So \( V = A(x^2 + z^2) + C \) where \( C = -3A^2 \)

Hence \( x^2 + z^2 = \frac{V-C}{A} = R^2 \) where \( R \) is constant

This is the equation for a circle of radius \( R \).

e) \[ R = \sqrt{\frac{V-C}{A}} \]

\[ C = -3 (1640) (0.25) = -7680 \]

\[ R = 5.74 \text{ m} \]
An insulating rod of length \( L \) has a total charge of \(+Q\) uniformly spread along its length. The rod lies along the x-axis with its left end located at \( x = a \).

(a) Find the electric potential at the origin.

\[
dV = \frac{kQ}{x} \Rightarrow \frac{kQ \, dx}{x} = k \left( \frac{Q}{L} \right) \frac{dx}{x} \\
V = \frac{kQ}{L} \ln \frac{L+q}{a}
\]

(b) A charge of \(-q\) is now moved from infinity to the origin. How much work was done by the external agent which moved the charge \(-q\)?

\[
(W_{\text{ext}})_{i \rightarrow f} = +\Delta V = +q \, \Delta V = +(-q) \left( V_f - V_i \right) \\
(W_{\text{ext}})_{i \rightarrow f} = -\frac{kQ}{L} \ln \frac{L+q}{a}
\]

(c) What is the direction of the electric force on the charge \(-q\) when it is at the origin?

to the right (along +x)
1. The electric potential in a certain region of space is: \( V = 12x^2 - 10x + 62 \). The \( x \)-component of the electric field at \( x = 2.0 \text{ m} \) is

\[
\frac{\Delta V}{\Delta x} = \frac{dV}{dx} = -24x + 10
\]

at \( x = 2 \), \( E_x = -48 + 10 = -38 \text{ V/m} \)

2. Two semicircular rods, one of radius \( R \) and the other of radius \( 2R \), and two short straight rods each of length \( R \) are located as shown. The rods have a uniform charge per unit length \( \lambda \). Determine the electric potential at the origin (you should assume that \( V = 0 \) at infinity).

**Do this piecewise.**

For segment 1

\[
dV = \frac{k\lambda dx}{2R} = \frac{k\lambda dx}{2R}
\]

\[
V_1 = k\lambda \int_0^\pi d\theta = k\lambda \pi R
\]

For segment 2

\[
dV = \frac{k\lambda dx}{R} = \frac{k\lambda dx}{R}
\]

\[
V_2 = k\lambda \int_0^\pi d\theta = k\lambda \pi R
\]

\[
V_2 = k\lambda \ln 2
\]

For segment 3

\[
dV = \frac{k\lambda dx}{x} = \frac{k\lambda dx}{x}
\]

\[
V_3 = k\lambda \int_0^2 d\theta = k\lambda \pi
\]

For segment 4

\[
dV = \frac{k\lambda dx}{x} = \frac{k\lambda dx}{x}
\]

\[
V_4 = k\lambda \int_x^{2R} dx = k\lambda \ln \frac{2R}{x}
\]

\[
V_4 = k\lambda \ln 2
\]

For segment 5

\[
dV = -\frac{k\lambda dx}{x} = -\frac{k\lambda dx}{x}
\]

\[
V_5 = k\lambda \int_0^{-x} d\theta = -k\lambda \int_{-x}^{2R} dx = -k\lambda \ln \frac{1}{x} = +k\lambda \ln 2
\]

\[
V_{total} = V_1 + V_2 + V_3 + V_4 = k\lambda \pi + k\lambda \ln 2 + k\lambda \pi + k\lambda \ln 2 = 2k\lambda [\pi + \ln 2]$