bers. The basics of number theory are covered carefully in Chapter 6 and employed profitably in the final chapter, where notions of complexity and cryptography are introduced. The penultimate chapter has a good introduction to finite geometries and designs (dubbed "pretty creatures") and their connections with coding theory. A chapter on "Combinatorics in Geometry" rounds out the collection of atypical topics.

Exercises are included, sometimes inserted in the narrative; otherwise they appear at the ends of sections. Every problem has an entry in the "Answers" section at the back of the book, and often these are detailed explanations rather than just numerical final answers. The exercises are not as numerous as one would like to see in a textbook, and in some areas are lacking (e.g., only one problem in the section on inclusion-exclusion). Instructors considering using this text might contemplate whether they are prepared to supplement the exercises. That said, the exercises seem to be designed carefully and the student that works them diligently will benefit.

The authors intend this book to be an engaging introduction to the principal topics covered by the umbrella term of "discrete mathematics," and on this score they have succeeded admirably. The text is readable and entertaining, without sacrificing any rigor or cutting any corners. In the preface the authors say that "the aim of this book is not to cover discrete mathematics in depth." Consistent with this statement, certain decisions have been made about what to leave in and what to leave out. The title of the book contains the subtitle *Elementary and Beyond*, and in this respect they have also succeeded. Topics such as complexity, cryptography, coding theory, and finite geometries give the interested reader a glimpse into more advanced topics that build on the basic material and that might be sufficient to whet one's appetite for more.

This text is a welcome addition to the collection of undergraduate texts that cover combinatorics and graph theory. Its style and the inclusion of nontrivial advanced topics distinguish it from many others. With the addition of more basic counting concepts and more high-quality exercises it could be a real standout. It is worthy of serious consideration by an instructor for use in an appropriate course, or by a curious individual for independent study.

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Advances in Dynamic Equations on Time Scales. *Edited by Martin Bohner and Allan Peterson.* Birkhäuser Boston, Boston, MA, 2003. $69.95. xi+348 pp., hardcover. ISBN 0-8176-4293-5.

The theory of time scales was introduced in 1988 by Stefan Hilger in his Ph.D. thesis [1] (under the supervision of Bernd Aulbach) as a means to both unify and generalize continuous and discrete analysis. Many results in differential equations have analogous or at least similar counterparts in difference equations, and the theory of time scales aims at providing a framework to describe the two classical dynamic systems simultaneously and offering a deeper understanding of the *raison d'être* for a particular type of method, independent of the particular case that spawned it. A second goal of this unified theory is to create a systematic approach to extend results beyond the classical cases and discover new settings and new results for dynamics of, e.g., q-difference equations or even dynamics induced by the Cantor middle third set.

A time scale is defined as any closed subset of the real numbers with the topology that it inherits from the reals, the latter set being equipped with the standard topology. For an arbitrary time scale, the so-called *delta derivative* was introduced by Hilger, which for the time scale $\mathbb{R}$ is just the usual derivative d/dt, while in the case of the time scale $\mathbb{Z}$, this derivative is the forward difference operator. He also introduced—via antiderivatives—*delta integration*, which for $\mathbb{R}$ is the usual integral for continuous functions, whereas for $\mathbb{Z}$ this corresponds to summation over a corresponding set of integers. These two operations allowed the initial development of time scales calculus and the introduction of *dynamic equations* on time scales.

This theory of dynamic equations, just emerging from its infancy, has established itself as a very active and vibrant research area. Bohner and Peterson in 2001 published *Dynamic Equations on Time Scales* [2], in which the first four chapters are a self-contained, complete, and well-written introduction to dynamic equations, covering time scales calculus, first- and second-order equations, and self-adjoint equations. This first part, featuring many examples and exercises, should be accessible to a reader with a background in calculus and linear algebra. Proofs are carefully explained in detail, and it would make an excellent choice as a textbook for a first course on time scales. The book also contains four more chapters on linear systems, dynamic inequalities, linear symplectic systems, and extensions. This second part is still in large portions accessible to undergraduates, but some sections are aimed at a somewhat higher level. For example, in the chapter on linear systems, standard topics like existence and uniqueness or the variation of constants formula are presented in detail and can easily be followed. On the other hand, one section presents time scales versions of results on the asymptotic integration of linear systems corresponding to Levinson and Hartman and Wintner, which, due to proof techniques like the contraction principle, are probably not suitable for the introductory level and should, as the authors suggest, be left for a beginning graduate-level class or a topics class. This earlier book is well written (the reviewer especially likes the sections called "Notes and References" at the end of every chapter giving a quick summary together with references to continuous and discrete theory) and has an extensive bibliography. It has been well received and it most likely has found a permanent place on the desk of every mathematician working in time scales. It is also highly recommended as the starting point for any mathematician or student of mathematics interested in this relatively new field. While the book does not contain applications, this new perspective that time scales offer should prove to be interesting for the more applied mathematician.

The book under review, *Advances in Dynamic Equations on Time Scales*, presents several important areas of recent research in time scales, particularly results obtained after the earlier book [2] was published, and is also concerned with more specialized topics. Edited by Bohner and Peterson and written by 21 researchers in time scales, it consists of nine chapters (plus one brief introductory chapter on time scale calculus). The individual chapters, each concerned with different, although loosely connected, research topics, are written by experts in their respective fields. The editors state that the book is a thorough introduction to areas of current research and that it provides an overview of the recent advances in the theory of time scales. It should not only prove valuable to graduate students and to researchers interested in the latest progress in this field, but it could also be used as a textbook for a second course in dynamic equations.

In the reviewer's opinion, this book is a valuable resource for researchers in this field as the first comprehensive presentation of several mainstream areas of recent or current research. It is also recommended for mathematicians or scientists with expertise in differential or difference equations and some background in time scales calculus who are curious to see how concepts from continuous and discrete analysis can be unified and generalized. However, caution needs to be used in the selection of material when used as a textbook for an advanced course.

Early chapters are on selected dynamic equations like logistic or Riccati equations and on nabla derivatives (here the derivative is taken backward in time, leading, for example, to the backward difference operator for difference equations). They are self-contained except for references to theorems and proofs in [2], which needs to be kept at hand, and the material is presented in detail. In the chapter on nabla derivatives, at times almost too much detail is provided given that the theory frequently follows the corresponding theory on delta derivatives. The book continues with a chapter on second order adjoint equations with derivatives both forward and backward in time. Here a motivation for this study as well as a comparison with previously established, corresponding results for second-order adjoint equations with delta derivatives would have been nice. Another chapter, concerned with Riemann and somewhat more briefly with Lebesgue integration on time scales, is interesting and accessible to a reader with a background in classical integration theory and some knowledge of basic time scales calculus. So far, the book provides examples and exercises and could be used as a textbook for a second course in time scales with [2] as a companion.

In the remaining chapters, a more specialized background knowledge in either differential or difference equations is helpful. For example, in the chapter on lower and upper solutions of boundary value problems, methods of quasi-linearization and generalized quasi-linearization are used to establish existence of solutions, upper and lower bounds of solutions, and results on convergence of these bounds. References to literature in the classical analysis as well as to research papers in the time scales setting are provided, but no explanation of the basic idea of quasi-linearization is offered, which makes it more challenging to follow for a reader without expertise in these techniques. However, exercises and examples are provided and helpful. A chapter on disconjugacy and higher order dynamic equations, assuming that the reader has some familiarity with disconjugacy of differential or difference equations, resembles a research paper more than a chapter in a textbook. In fact, large portions of this chapter were originally published as research papers and only minimally revised. Other indicators that this chapter is aimed more toward the expert than at a general audience are the lack of examples and exercises and a final section on open problems. The book concludes with a self-contained exploration of boundary value problems on infinite intervals and a study of symplectic dynamic systems.

In summary, this book is an important addition to the existing literature on time scales. Its comprehensive presentation of many central areas of current research and its extensive bibliography make it a welcome and valuable reference for researchers working in time scales. It might also be attractive for mathematicians with expertise in differential or difference equations who are interested in learning how at least some classical theories of continuous and discrete analysis can be unified and generalized. Anybody planning to use it as a textbook for a second course in time scales, however, should be prepared to provide more background material and additional exercises and examples for some of the chapters.

REFERENCES

[1] S. Hilger, *Ein Maßkettenkalkül mit Anwendung auf Zentrumsmannigfaltigkeiten*, Ph.D. thesis, Universität Würzburg, Würzburg, Germany, 1988.

[2] M. Bohner and A. Peterson, *Dynamic Equations on Time Scales*, Birkhäuser-Verlag, Basel, Switzerland, 2001.

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Two-Scale Stochastic Systems: Asymptotic Analysis and Control. *By Yuri Kabanov and Sergei Pergamenshchikov.* Springer-Verlag, New York, 2002. $69.95. xiv+266 pp., hardcover. ISBN 3-540-65332-5.

The perturbation theory of differential equations containing a small parameter is an important research area in applied mathematics. Singular perturbation theory, in particular, has played a prominent role in the historical development of the subject. For an excellent exposition, see O'Malley's book [3]. For stochastic differential equations, the asymptotic analysis was initially carried out by Khasminskii [1] in the 1960s and has been further developed by Papanicolaou [4], Kushner [2], and many others. Of particular interest is the randomly perturbed Tikhonov(–Levinson) system:

$$dx_t^\epsilon = f(t, x_t^\epsilon, y_t^\epsilon) + g(t, x_t^\epsilon, y_t^\epsilon)dw_t^x,\ x_0^\epsilon = x^0,$$

$$\epsilon dy_t^\epsilon = F(t, x_t^\epsilon, y_t^\epsilon) + \sigma(\epsilon)G(t, x_t^\epsilon, y_t^\epsilon)dw_t^y,\quad y_0^\epsilon = y^0,$$

where ϵ is a small parameter and w_t^x and w_t^y are independent Wiener processes. As pointed out by the authors of the book under review, extensive work has been done