Journal of Difference Equations and Applications,
Vol. 13, No. 6, June 2007, 561–562

Taylor & Francis
Taylor & Francis Group

Book Review

Discrete oscillation theory *by Ravi P. Agarwal*, *Martin Bohner*, *Said R. Grace and Donal O'Regan*, Contemporary Mathematics and Its Applications, Vol. 1, Hindawi Publishing Corporation, New York, 2005, xiv + 962 pages, $275.00 USD, ISBN: 977-5945-19-4.

Oscillation theory forms the largest part of the qualitative theory of difference equations. In the last few decades, the theory of difference equations has developed rapidly. There are at least three important reasons why difference equations should be investigated. First, they serve as discrete models describing various phenomena from real life. Second, they can be viewed as discrete counterparts of differential equations. Here, it should be noted that the development of a discrete theory is not generally close, and sometimes far from a simple imitation of arguments for the continuous theory. Third, difference equations or recurrence relations are important from a theoretical point of view—because they appear in other branches of science when solving various problems.

The aim of this book is to present a survey of oscillation theory of difference equations. The authors collect many of the important results in this area, classical ones as well as very recent ones, and treat them in a very orderly fashion. This is not an easy task, since there are hundreds of papers on this topic that have appeared in the last few years. Primarily, second-order difference equations are treated, ordinary as well as ones with deviating arguments or neutral ones. Results on first-order and higher-order equations and systems of difference equations are included as well. In addition to presenting a collection of known results, the authors present several new statements and observations.

There are nine chapters in this book. The first one is the longest one. Mainly, classical results concerning second-order linear difference equations are presented. Central concepts of oscillation theory are introduced, for example, a generalized zero, disconjugacy, and (non)oscillation of an equation. Discrete Sturmian theory is presented and two important tools are derived, namely, the variational principle and the Riccati technique. Dominant and recessive solutions, disfocality, and many (non)oscillation criteria are discussed.

Chapter 2 deals with a generalization of the results from the previous chapter to linear Hamiltonian difference systems, which contain Sturm–Liouville difference equations of even order as a special case. An alternate way of extending the theory of Chapter 1 is presented in Chapter 3, namely, a generalization to half-linear difference equations. Both Chapters 2 and 3 contain extensions of Reid's roundabout theorem, which is the central statement of oscillation theory.

Chapters 4 and 5 present oscillation theory for nonlinear difference equations, where the nonlinearities are natural generalizations of Emden–Fowler type nonlinearities. Superlinear and sublinear cases are distinguished. Oscillation criteria for linear and nonlinear difference equations with deviating arguments are presented in Chapter 6 while neutral difference

Journal of Difference Equations and Applications
ISSN 1023-6198 print/ISSN 1563-5120 online © 2007 Taylor & Francis
http://www.tandf.co.uk/journals
DOI: 10.1080/10236190701264982

equations are discussed in Chapter 7. Chapter 8 deals with stability and oscillation theory for differential equations with piecewise constant arguments.

The final chapter presents miscellaneous topics. Some of the topics include difference equations with periodic coefficients, oscillation of rational difference equations, and global attractivity and asymptotical stability for certain second-order equations.

The theory throughout the book is illustrated with many examples. Each chapter concludes with a section that is devoted to notes and bibliographical and historical remarks.

The text is very well organized and the reader is able to find a desired topic quite fast. The authors utilize their rich experiences working in the field of difference equations; they are well-known experts in this field (many important results are their own). This, together with the fact that the authors have carefully assembled results from other authors, make this book a very up-to-date and excellent reference. A majority of the statements in the book are proved. The proofs are presented with sufficient details so as to make the book easy to read.

I have no serious reprovals about this book. There are a few imperfections, which may have two origins. Some mistakes come from the original papers which were used as source texts. Others are due to the fact that practically the same result is presented in more than one place (e.g. Lemma 3.4.1 and Lemma 3.9.2—the results come from two different papers but their conclusions are the same). These imperfections are understandable, in view of the extent of this book.

Assembling the topics of this book was a very worthwhile project, for this text will surely serve as a good source of reference and a survey, useful to many scientists interested in the field of difference equations. For those who are experts in this field, this book is a must.

Pavel Řehák
Institute of Mathematics
Academy of Sciences of the Czech Republic
Žižkova 22, CZ-61662 Brno, Czech Republic
Email: rehak@math.muni.cz