1. Write the system of equations

$$x + 2y + 3z = 14, \quad 2x + 3y + z = 11, \quad 3x + y + 2z = 11$$

as $Av = b$, find the LDU decomposition of A, find c with $Lc = b$, and find v with $DUv = c$.

2. Determine which of the following sets are subspaces of $\mathbb{R}^3$ and give the dimension and basis for them:

$$\text{(a) } V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : \; x^2 + z^2 > 3 \right\} \qquad \text{(b) } V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : \; 2x + 6y = 4z \right\}.$$

3. Find the echelon form of

$$A = \begin{bmatrix} 1 & 3 & 0 & 2 & 1 \\ 1 & 3 & 0 & 4 & 2 \\ 2 & 6 & 0 & 6 & 3 \end{bmatrix},$$

the basic and free variables, rank$A$, all solutions to $Ax = 0$, and all solutions to $Ax = [7 \quad 10 \quad 17]^T$.

4. Given are the vectors

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \; x_2 = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}, \; x_3 = \begin{bmatrix} -4 \\ -3 \\ 2 \\ 1 \end{bmatrix}, \; x_4 = \begin{bmatrix} 0 \\ -\frac{4}{3} \\ 0 \\ 1 \end{bmatrix}, \; x_5 = \begin{bmatrix} 1 \\ 0 \\ 8 \\ 0 \end{bmatrix}.$$

Are x_1, x_2, and x_3 linearly independent? Are x_1, x_2, x_3, x_4, and x_5 linearly independent? Find three pairs (i, j) with $x_i \perp x_j$. Determine the angle between x_1 and x_5.

5. Find a basis and the dimension of each of the four fundamental subspaces of A from Problem 3.

6. Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

Find the least-square solution to $Ax = b$ and the projection of b onto the image of A.

7. Find all eigenvalues, eigenvectors, the trace, and the determinant of

$$A = \begin{bmatrix} 0 & 0.95 & 0.6 \\ 0.8 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix}.$$

8. If $S^{-1}AS = B$, prove that the characteristic polynomials of A and of B are the same.