1. Rewrite

$$u + 2v + 3w = 7, \quad 2u + 5v + 6w = 1, \quad 3u + 6v + 7w = 1$$

as an equation $Ax = b$, find the LDU Decomposition of A, find c such that $Lc = b$, and find x such that $DUx = c$. Give the solution of the original problem and check your solution.

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2. Given are the two matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 6 & 7 \end{bmatrix}.$$

Find AA^T, B^TA, $I - A$, $2B$, A^{-1}, B^{-1}, $\mathcal{R}(A)$, $\mathcal{N}(A)$, $\mathcal{R}(B^T)$, and $\mathcal{N}(B)$.

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3. Is the set of vectors in $\mathbb{R}^3$ that have zero as the second component a subspace of $\mathbb{R}^3$? How about the set of vectors in $\mathbb{R}^3$ that have a nonnegative number as the second component? (Prove your claims, of course).

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4. Let B, C, and X be real $n \times n$-matrices that satisfy

$$X^TX + B^TX + X^TB + C = 0$$

Show that under these assumptions C must be necessarily symmetric.
