Midterm #2, Math 203, Dr. M. Bohner. Nov 23, 98.

1. Find a basis and the dimension for each of the four fundamental subspaces of $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \end{bmatrix}$.

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2. Let $x = [1 \quad 2 \quad 3 \quad 4]^T$ and $y = [1 \quad 1 \quad 2 \quad 3]^T$. Find the angle between x and y. Also, find all vectors that are orthogonal to both x and y.

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3. A secret message (x, y) is linearly encoded and sent from A to B, where it is encoded again (also linearly, but maybe with a different code) and sent to C. Spies find out that the message $(1, 2)$ from A arrives as $(-1, 3)$ in B and as $(5, -4)$ in C. Also, they find that $(3, 5)$ from A arrives as $(15, -9)$ in C and $(4, 2)$ from B arrives as $(8, 2)$ in C. Now, if $(10, 4)$ arrives in C, which was the original message and which message arrived in B?

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4. Let $x, y \in \mathbb{R}^n$. Prove the inequality $\|x + y\| \le \|x\| + \|y\|$. (Hint: Start with calculating $\|x + y\|^2$ and use the Cauchy-Schwarz Inequality.)