12. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$. Find all matrices J for which $AJ = A$.

13. Prove that diagonal matrices of the same order commute.

14. Let D be an arbitrary diagonal matrix. In which case is D invertible? If it is invertible, what is D^{-1}?

15. Let A and D be square matrices of the same size. Assume that D is diagonal. Describe how AD looks like. How about DA?

16. Let A be a matrix of size $m \times n$. Find a matrix P such that P multiplied with A exchanges the ith row and the jth row of A. What needs to be done if the ith column and the jth column of A should be exchanged?

17. Suppose that $(I + A)^{-1}A = B$ holds for two matrices A and B.
    (a) Prove that A and B commute.
    (b) Prove that, if B is invertible and diagonal, then also A is invertible and diagonal.

18. Let $A_{11} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $A_{12} = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$, $B_{11} = \begin{bmatrix} 3 & -7 & -7 & 2 \end{bmatrix}$, and $B_{21} = \begin{bmatrix} -2 & 1 & 4 & 0 \\ 0 & 2 & 4 & 0 \end{bmatrix}$. Let $A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -7 & -7 & 2 \\ -2 & 1 & 4 & 0 \\ 0 & 2 & 4 & 0 \end{bmatrix}$. Show that $AB = A_{11}B_{11} + A_{12}B_{21}$. Also, state and prove a general theorem that can be used to solve such problems.

19. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. Find all vectors v that satisfy $Av = 0$.

20. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 0 \\ 5 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 1 \\ 6 \\ 10 \\ 14 \end{bmatrix}$. Find real numbers a, b, and c such that $av_1 + bv_2 + cv_3 = 0$.

21. We would like to find a 3×3-matrix that has each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 as its entries such that each row and each column and each of the two diagonals sums up to 15. Don't solve this problem, but just describe it as a system $Ax = b$ where $x \in \mathbb{R}^9$ is the vector that has the entries of the required matrix as its entries.

22. Find the inverse of the 5×5-matrix from Example 1.7 (b) (you may use Maple if you wish). Also, find the u_k, $1 \le k \le 5$ if $f(x) = 1$ and if $f(x) = x$. Compare them with the values $u(x)$ of the real solution of $u''(x) = f(x)$, $u(0) = u(1) = 0$.