

Instructions: Each problem is worth 20 points. Only work on five problems. Designate one of the six problems as your extra credit problem. Write “EXTRA” on the page of the problem you designate as your extra credit problem. You may work on this designated problem to receive extra credit, but only if you have full credit on the other five problems. Only responses entered in the allocated space (no extra space allowed) for each problem will be graded. Present only the complete solution including all explanation (without scratch work, use your own extra paper for that purpose, do not turn in such scratch papers) neatly. You must support all of your answers in order to receive credit. Simplify your answers as much as possible. Do not remove the staples. Do not turn in the assignment sheet. Grades will be posted on the web this afternoon.

1. Verify that $x(t) = \begin{pmatrix} 6 \\ -8 \\ -4 \end{pmatrix} e^{-t} + 2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t}$ is a solution of $x' = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} x$.

2. Solve the IVP $x'_1 = 5x_1 - x_2$, $x'_2 = 3x_1 + x_2$, $x_1(0) = 2$, $x_2(0) = -1$.

3. Find the general solution of $x' = Ax$, where

$$(a) A = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \quad (b) A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \quad (c) A = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}.$$

4. Determine the critical value of α , where the qualitative nature of the phase portrait for $x' = \begin{pmatrix} \alpha & 1 \\ -1 & \alpha \end{pmatrix} x$ changes. Draw a phase portrait for the value of α equal to the critical value, slightly below the critical value, and slightly above the critical value.