19. Find a solution y_1 of the equation $t^2y'' - ty' + y = 0$ by trying $y(t) = t^r$. Then, proceeding as in Example 3.8 (iii) from the lecture notes, let y be the solution of the equation that satisfies $y(1) = 3$ and $y'(1) = -1$, find the Wronskian of y_1 and y by applying Abel's theorem, and finally find y by solving a first order linear equation.

20. Consider the problem $t^2y'' + 3ty' + y = 0$.

    (a) Find a solution y_1 of the form $y_1(t) = t^\alpha$ for some real number α.

    (b) To find another solution, try $y_2(t) = v(t)y_1(t)$ for some function v.

    (c) Make sure that the Wronskian of y_1 and y_2 is not zero (if it is zero, try (a) and (b) again). Find this Wronskian.

    (d) Now find the solution that satisfies $y(\mathrm{e}) = \frac{\mathrm{e}+2}{\mathrm{e}}$ and $y'(\mathrm{e}) = \frac{\mathrm{e}-2}{\mathrm{e}^2}$.

21. Use steps similar as in the previous problem to solve $2t^2y'' + 3ty' - y = 0$, $y(1) = 3$, $y'(1) = 0$.

22. For the following equations, find one particular solution (hint: Try $a\mathrm{e}^{bt}$ or $a\sin(bt) + c\cos(dt)$).

    (a) $y'' - 2y' - 3y = 3\mathrm{e}^{2t}$;

    (b) $y'' + 2y' + 4y = 2\mathrm{e}^{-t}$;

    (c) $y'' + 2y' + 5y = 3\sin(2t)$;

    (d) $y'' + y = 3\sin(3t) + 4\cos(3t)$.

23. Use the variation of parameters technique to find one particular solution:

    (a) $y'' + y' - 2y = 2t$;

    (b) $y'' + 4y = 3\sin(2t)$;

    (c) $y'' + 2y' + y = 3\mathrm{e}^{-t}$;

    (d) $y'' + y = \tan(t)$;

    (e) $y'' + 4y' + 4y = t^{-2}\mathrm{e}^{-2t}$.