Mathematics 204

Fall 2008

Final Exam

[1] Your Printed Name: ______________________________

[1] Your Instructor's Name: ______________________________

Your Section (or Class Meeting Days and Time): ______________________

1. **Do not open this exam until you are instructed to begin.**
2. All cell phones and other electronic noisemaking devices must be **turned off or completely silenced** (i.e. not on vibrate) for the duration of the exam.
3. The final exam consists of this cover page, 9 pages of problems containing 11 numbered problems, and two pages of Laplace transform formulas.
4. Once the exam begins, you will have 120 minutes to complete your solutions.
5. **Show all relevant work. No credit** will be awarded for unsupported answers and partial credit depends upon the work you show.
6. You may use the back of any page for extra scratch paper, but if you would like it to be graded, clearly indicate in the space of the original problem where the work is to be found.
7. The symbol [18] at the beginning of a problem indicates the point value of that problem is 18. The maximum possible score on this exam is 200.

	0	1	2	3	4	5	6	7	8	9	10	11	**Sum**
points earned													
maximum points	2	13	18	19	19	18	18	18	18	19	19	19	200

1.[13] State the order of each of the following differential equations. Are they linear or nonlinear? For each nonlinear equation, **CIRCLE** a term that makes it nonlinear.

	Order?	Linear?
$y'' + \sin(x+y) = \sin(x)$		
$y''' + ty' + \cos^2(t)y = t^3$		
$x' - t\ln(t)x = e^{-t}$		
$(1+y)y'' + ty' + y = e^t$		
$\left(\frac{d^2y}{dx^2}\right)^3 + \frac{dy}{dx} + y = 0$		

2.[18] Find the general solution of $y' = x^2e^{-2x} - 2y$.

3.[19] Find an explicit solution to the initial value problem $y' = (x-3)(y+1)^{2/3}$, $y(0) = -1$.

4.[19] According to Newton's empirical law of cooling, the rate at which the temperature of a body changes is proportional to the difference between the body's temperature and the ambient temperature. A thermometer reading 70^0 F is taken outside, where the air temperature is 10^0 F. After one minute, the thermometer reads 40^0 F.

(a) Write a differential equation and initial conditions that model the thermometer's temperature after t minutes.

(b) How long will it take for the thermometer to reach 15^0 F?

5.[18] Solve the differential equation $y^{(4)} - y = 2t^4$.

6.[18] Solve the differential equation $x^2y'' - xy' + y = \dfrac{3x}{\ln(x)}$ on the interval $x > 1$.

7.[18] (Use 32 ft/sec^2 as the acceleration of gravity in this problem.) A body weighing 16 pounds when attached to the end of a vertical spring stretches it 2 feet. Initially, the body is released with a downward velocity of 2 feet per second from a point 18 inches above the equilibrium position, and the subsequent motion takes place in a medium that offers a damping force numerically equal to ½ the instantaneous velocity. Write, **BUT DO NOT SOLVE**, an initial value problem that models the motion of the body if an external downward force of $10\cos(3t)$ pounds is applied starting at time $t = 0$.

8.[18] Solve the initial value problem $y'(t) - \int_0^t \cos(t-\tau)y(\tau)d\tau = 1 - \sin(t), \quad y(0) = 0.$

9.[19] Solve the initial value problem $y'' - 2y' + 2y = \delta(t-2)$, $y(0) = 0$, $y'(0) = 1$. Then compute the values of $y(1)$ and $y(3)$. (Your answers for $y(1)$ and $y(3)$ should not contain Heaviside unit step functions.)

10.[19] Solve the initial value problem $\mathbf{X}' = \begin{pmatrix} 1 & -1 \\ 4 & 1 \end{pmatrix}\mathbf{X}$, $\mathbf{X}(0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$. Perform all matrix calculations by hand. **NO CREDIT** will be awarded for unsupported answers.

11.[19] Let $\mathbf{A}=\begin{pmatrix}0 & 2\\ -1 & 3\end{pmatrix}$ and $\mathbf{F}(t)=\begin{pmatrix}e^t\\ -e^t\end{pmatrix}$. Given that $\mathbf{\Phi}(t)=\begin{pmatrix}e^{2t} & 2e^t\\ e^{2t} & e^t\end{pmatrix}$ is a fundamental matrix of $\mathbf{X}'=\mathbf{AX}$, use variation of parameters to find the general solution of $\mathbf{X}'=\mathbf{AX}+\mathbf{F}(t)$.

TABLE OF LAPLACE TRANSFORMS

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1. 1	$\frac{1}{s}$
2. t	$\frac{1}{s^2}$
3. t^n	$\frac{n!}{s^{n+1}}, \quad n \ge 0$
4. $t^{-1/2}$	$\sqrt{\frac{\pi}{s}}$
5. $t^{1/2}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$
6. t^α	$\frac{\Gamma(\alpha+1)}{s^{\alpha+1}}, \quad \alpha > -1$
7. $\sin(kt)$	$\frac{k}{s^2+k^2}$
8. $\cos(kt)$	$\frac{s}{s^2+k^2}$
9. e^{at}	$\frac{1}{s-a}$
10. $\sinh(kt)$	$\frac{k}{s^2-k^2}$
11. $\cosh(kt)$	$\frac{s}{s^2-k^2}$
12. $\sinh^2(kt)$	$\frac{2k^2}{s\left(s^2-4k^2\right)}$
13. $\cosh^2(kt)$	$\frac{s^2-2k^2}{s\left(s^2-4k^2\right)}$
14. te^{at}	$\frac{1}{(s-a)^2}$
15. $t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}, \quad n \ge 0$
16. $e^{at}\sin(kt)$	$\frac{k}{(s-a)^2+k^2}$
17. $e^{at}\cos(kt)$	$\frac{s-a}{(s-a)^2+k^2}$

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
18. $e^{at}\sinh(kt)$	$\frac{k}{(s-a)^2-k^2}$
19. $e^{at}\cosh(kt)$	$\frac{s-a}{(s-a)^2-k^2}$
20. $t\sin(kt)$	$\frac{2ks}{\left(s^2+k^2\right)^2}$
21. $t\cos(kt)$	$\frac{s^2-k^2}{\left(s^2+k^2\right)^2}$
22. $\sin(kt)+kt\cos(kt)$	$\frac{2ks^2}{\left(s^2+k^2\right)^2}$
23. $\sin(kt)-kt\cos(kt)$	$\frac{2k^3}{\left(s^2+k^2\right)^2}$
24. $t\sinh(kt)$	$\frac{2ks}{\left(s^2-k^2\right)^2}$
25. $t\cosh(kt)$	$\frac{s^2+k^2}{\left(s^2-k^2\right)^2}$
26. $\frac{e^{at}-e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}$
27. $\frac{ae^{at}-be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}$
28. $\frac{a\sin(bt)-b\sin(at)}{ab\left(a^2-b^2\right)}$	$\frac{1}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$
29. $\frac{\cos(bt)-\cos(at)}{a^2-b^2}$	$\frac{s}{\left(s^2+a^2\right)\left(s^2+b^2\right)}$
30. $\sin(kt)\sinh(kt)$	$\frac{2k^2s}{s^4+4k^4}$
31. $\sin(kt)\cosh(kt)$	$\frac{k\left(s^2+2k^2\right)}{s^4+4k^4}$
32. $\cos(kt)\sinh(kt)$	$\frac{k\left(s^2-2k^2\right)}{s^4+4k^4}$

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
33. $\cos(kt)\cosh(kt)$	$\frac{s^3}{s^4+4k^4}$
34. $\sin^2(kt)$	$\frac{2k^2}{s\left(s^2+4k^2\right)}$
35. $\cos^2(kt)$	$\frac{s^2+2k^2}{s\left(s^2+4k^2\right)}$
36. $\frac{e^{bt}-e^{at}}{t}$	$\ln\left(\frac{s-a}{s-b}\right)$
37. $\frac{2(1-\cos(kt))}{t}$	$\ln\left(\frac{s^2+k^2}{s^2}\right)$
38. $\frac{2(1-\cosh(kt))}{t}$	$\ln\left(\frac{s^2-k^2}{s^2}\right)$
39. $\frac{\sin(at)}{t}$	$\arctan\left(\frac{a}{s}\right)$
40. $\frac{1}{\sqrt{\pi t}}e^{-a^2/4t}$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$
41. $1-\cos(kt)$	$\frac{k^2}{s\left(s^2+k^2\right)}$
42. $kt-\sin(kt)$	$\frac{k^3}{s^2\left(s^2+k^2\right)}$
43. $\delta(t)$	1
44. $\delta(t-t_0)$	e^{-st_0}
45. $e^{at}f(t)$	$F(s-a)$
46. $f(t-a)\mathcal{U}(t-a)$	$e^{-as}F(s)$
47. $\mathcal{U}(t-a)$	$\frac{e^{-as}}{s}$
48. $f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-\ldots-f^{(n-1)}(0)$
49. $t^nf(t)$	$(-1)^n\frac{d^n}{ds^n}F(s)$
50. $\int_0^t f(\tau)g(t-\tau)d\tau$	$F(s)G(s)$