

1. Solve the initial value problem  $ty' + 3y = t^2$ ,  $y(1) = 0$ .
2. Given are two tanks containing 3 gallons of water each. At time 0, the first tank contains 1 lb of dye thoroughly mixed, and there is no dye in the second tank. Now, water is entering the first tank at a rate of 3 gallons per minute. From the first tank water is flowing into the second tank at a rate of 3 gallons per minute. And water is leaving the second tank, also at a rate of 3 gallons per minute. Determine the amount of dye in the first tank at time  $t$  and determine the amount of dye in the second tank at time  $t$ .
3. Find all real solutions of  $y'' - 2y' + 2y = e^t$ .
4. A mass weighing 8 lb stretches a spring 6 inches. At time 0 the mass is released from a point 5 inches below the equilibrium position with upward velocity of 8 ft/sec. Determine the function which describes the subsequent free motion of the mass (ignoring any damped forces), find the period, frequency, and amplitude of the motion, and draw a picture.
5. Solve the initial value problem  $P_{n+2} = \frac{2}{3}P_{n+1} + \frac{2}{9}P_n$ ,  $P_0 = P_1 = 1$ .
6. Find a solution of the equation  $t^2y'' - ty' + y = 0$  (try  $y(t) = t^r$ ). Then, use the reduction of order method to find another linearly independent solution. Finally evaluate the Wronskian of these two solutions at 1.
7. Find the first five terms of a series solution about  $x_0 = -1$  for  $y'' - xy' - y = 0$ ,  $y(0) = y'(0) = 1$ .