8. Consider the linear first order equation with constant coefficients $y' = ry + k$.
   (a) Do steps I-III as in Example 2.1 to find all solutions.
   (b) Find all constant solutions.
   (c) Find the solution with $y(0) = 2$.
   (d) For a given point (t_0, y_0), find the solution that goes through this point.
   (e) Characterize all increasing solutions. Characterize all decreasing solutions.
   (f) Determine the behavior of the solutions as $t \to \infty$.
9. Find the solutions of the following initial value problems:
   (a) $y' = 5y - 1$, $y(0) = 2$;
   (b) $y' = -y + 4$, $y(1) = -1$;
   (c) $5y' = 2y - 3$, $y(-2) = 3$;
   (d) $3y' - 2y = 1$, $y(-1) = 0$;
   (e) $-2y' + 2y - 4 = 0$, $y(5) = 10$.
10. Let $N(t)$ be the number of atoms of a radioactive element at time t. We assume that the element disintegrates at a rate proportional to the amount present.
    (a) Find a differential equation for N;
    (b) Show that the half-time T, which is defined by $N(t_0 + T) = \frac{1}{2}N(t_0)$, is independent of t_0. Express this half-time as a function of only the constant of proportionality.
    (c) If the constant of proportionality is -0.03 (inverse of days), after what time will 100 mg of the radioactive material be reduced to 80 mg?
    (d) If 100 mg of the radioactive material are reduced to 80 mg in 6 days, determine the rate of proportionality and the amount of material left over after 8 days.
11. Consider a certain product on the market. Let a demand function $D(t)$ and a supply function $S(t)$ for this product be given. Also, let the function $P(t)$ describe the market price of the product (as a function of the time t). We assume that S and D depend linearly on the market price P: $D(t) = \alpha + aP(t)$, $S(t) = \beta + bP(t)$.
    (a) According to the model, should we assume $a < 0$ or $a > 0$?
    (b) According to the model, should we assume $b < 0$ or $b > 0$?
    (c) Now we assume that P is changing proportionally to the difference $D - S$, with constant of proportionality γ. According to the model, should we assume $\gamma < 0$ or $\gamma > 0$?
    (d) Derive a differential equation for P and solve it.
    (e) Calculate the so-called equilibrium price of the product, i.e., determine $\lim_{t\to\infty} P(t)$.