38. Find the Wronskian of the given pair of functions:
   (a) e^{-2t} and te^{-2t}; (b) e^{-2t} and $\frac{3}{5}e^{-2t}$; (c) $\cos t$ and $\sin t$;
   (d) $\cosh t$ and $\sinh t$; (e) t^n and t^m; (f) t^n and mt^n;
   (g) t and te^t; (h) $\cos^2 t$ and $1 + \cos(2t)$.
39. If the Wronskian of y_1 and y_2 is $3e^{4t}$ and if $y_1(t) = e^{2t}$, find y_2.
40. If $b^2 - 4ac > 0$, calculate the (nonzero) Wronskian of two solutions of $ay'' + by' + cy = 0$.
41. Consider the equation $y'' + q(t)y = 0$.
   (a) If $q(t) \equiv -1$, find two solutions such that the Wronskian is always 1.
   (b) If $q(t) \equiv 1$, find two solutions such that the Wronskian is always 1.
   (c) If q is any continuous function, show that the Wronskian of any two solutions is independent of the time. Calculate the Wronskian.
42. For the equation $(p(t)y')' + q(t)y = 0$, where p is differentiable and never zero and q is continuous, calculate the Wronskian of any two solutions.
43. Show that $y_1(t) = t + 1$ and $y_2(t) = 2t + 4$ solve the equation $y = ty' + (y')^2$ but that $\alpha y_1 + \beta y_2$ in general is not a solution. Why does this not contradict Theorem 3.5 as presented in the lecture?
44. Find two solutions of the equation $t^2y'' - 2ty' + 2y = 0$ such that their Wronskian is not zero (hint: try t^α). Calculate this Wronskian and give the interval where the solution is valid. Finally, find the solution of the equation that satisfies $y(1) = 3$ and $y'(1) = 4$.
45. Consider the problem $t^2y'' + 3ty' + y = 0$.
   (a) For which interval can we ensure the existence of a solution?
   (b) Find a solution y_1 of the form $y_1(t) = t^\alpha$ for some real number α.
   (c) To find another solution, try $y_2(t) = v(t)y_1(t)$ for some function v.
   (d) Make sure that the Wronskian of y_1 and y_2 is not zero (if it is zero, try (a) and (b) again). Find this Wronskian.
   (e) Now find the solution that satisfies $y(e) = \frac{e+2}{e}$ and $y'(e) = \frac{e-2}{e^2}$.
46. Use steps similar as in the previous problem to solve $2t^2y'' + 3ty' - y = 0$, $y(1) = 3$, $y'(1) = 0$.
47. Here we consider the linear difference equation of second order $ay_{k+2} + by_{k+1} + cy_k = 0$.
   (a) Show that, if f and g both solve the equation, then so does $\alpha f + \beta g$.
   (b) If $a = 1$, $b = -7$, and $c = 6$, find the solution with $y_0 = -1$ and $y_1 = 4$ (hint: try α^k).
   (c) Find a, b, c for the Fibonacci sequence: $1, 1, 2, 3, 5, 8, 13, 21, \ldots$. Find the nth member y_n of this Fibonacci sequence. Use this formula to find y_{20}. Finally calculate $\lim_{n\to\infty} \frac{y_{n+1}}{y_n}$.
   (d) Find the solutions of the equation if $b^2 - 4ac > 0$.