85. Separate the variables to find solutions of the following problems:
   (a) $u(m+1,n) = u(m,n+1)$;
   (b) $u(m+1,n) = 4u(m,n+1)$;
   (c) $u(m+1,n) - 2u(m,n+1) - 3u(m,n) = 0$;
   (d) $u(m+2,n) = 4u(m,n+1)$.
   (e) $u(m+1,n) - u(m,n+1) + u(m,n) = 0$, $u(m,0) = 2^m$.
86. In Section 8.1 of the textbook, work on Problem 3.
87. Solve $u_t = u_{xx}$ in $[0,5]$ with $u = 0$ at both ends and $u(x,0) = x(5-x)$, using the forward difference scheme with $\Delta x = 1$ and $\Delta t = 0.25$ to find the approximate value of $u(2,1)$.
88. Solve $u_t = u_{xx}$ in $[0,5]$ with $u(0,t) = 0$ and $u(5,t) = 1$ for $t \geq 0$ and $u(x,0) = 0$ for $0 < x < 5$.
   (a) Compute $u(3,3)$ using the mesh sizes $\Delta x = 1$ and $\Delta t = 0.5$.
   (b) Write the exact solution as an infinite series. Calculuate $u(3,3)$ to three decimal places exactly and compare it with your answer in (a).
89. In Section 8.2 of the textbook, work on Problem 4.
90. In Section 8.3 of the textbook, work on Problems 1, 2, 4, 5, and 10.
91. Use centered differences to approximate the harmonic function in $S = \{(x,y):\ 0 < x < 1,\ 0 < y < 1\}$ that satisfies $u(x,0) = 54x^2(1-x)$ for $0 \leq x \leq 1$ and vanishes at all other points of the boundary of $S$ (use step size $\frac{1}{3}$ for both $x$ and $y$).
92. Work again on Problems 1–91 in order to get ready for the final.