

21. Determine the types of the wave equation, heat equation, and Laplace equation.
22. Determine the type of the equation  $u_{xx} + 6u_{xy} - u_{yy} + 4u_x - 7u_y + u = 0$ .
23. Find the regions in the  $xy$ -plane where the equation  $xu_{xx} - 4(x+y)u_{xy} + 18yu_{yy} + u_x - u = 0$  is elliptic, hyperbolic, or parabolic.
24. Transform the equation  $u_{xx} + 2u_{xt} + u_{tt} = 2u$  into standard form. Solve the obtained standard PDE. Then use the transformation to obtain the solution of the original PDE.
25. Transform the following PDEs into standard form. For each step, write down exactly the transformation that is needed. Also, determine whether the equation is hyperbolic, elliptic, or parabolic.
  - (a)  $3u_{xx} + 4u_{tt} - u = 0$ ;
  - (b)  $4u_{xx} + u_{xt} + 4u_{tt} + u = 0$ ;
  - (c)  $u_{xx} + u_{tt} + 3u_x - 4u_t + 25u = 0$ ;
  - (d)  $u_{xx} - 3u_{xt} + u_{tt} + 2u = 0$ ;
  - (e)  $u_{xx} + 2u_{xt} + u_{tt} + u_x - 2u_t + u = 0$ .
26. Show that none of the three transformations introduced in Example 1.7 (i.e., rotation of axes, change of dependent variable, and change of scale) changes the type of the PDE when applied to a second order equation with constant coefficients.