1. Use the geometric method to find the general solution of $xu_x + u_y + (1+z^2)u_z = x + y$.

2. Transform $u_{xx} - 3u_{xt} + u_{tt} + 2u = 0$ into standard form and determine the type of the equation.

3. Solve $u_{xx} - u_{xt} - 2u_{tt} = 0$, $u(x,0) = x^2$, $u_t(x,0) = 0$ by "factoring the operator".

4. Solve (a) $u_{tt} = c^2 u_{xx}$, $u(x,0) = \tanh x$, $u_t(x,0) = 0$ and (b) $u_t = ku_{xx}$, $u(x,0) = \mathrm{e}^x$. Sketch the solutions at some times and describe the effect of the parameters.

5. If p and r are positive functions, prove that eigenfunctions to different eigenvalues of the problem

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[p(x)f'(x)\right] + q(x)f(x) + \lambda r(x)f(x) = 0, \quad f(0) = f(1) = 0$$

are orthogonal, using the scalar product $(f_1, f_2) = \int_0^1 r(x)f_1(x)f_2(x)\mathrm{d}x$.

6. By finding an appropriate Fourier series, determine the solution of

$$9u_{tt} = u_{xx}, \quad u(0,t) = u(\pi,t) = 0, \quad u(x,0) = 0, \quad u_t(x,0) = x(x-\pi).$$

7. Perform separation of variables to find the solution of the discrete problem

$$u(m+2,n) - 4u(m+1,n) - u(m,n+1) = 0, \quad u(0,1) = 0, \quad u(1,0) = 12, \quad u(1,1) = 60.$$

8. Use centered differences to approximate the harmonic function in $S = \{(x,y) : \ 0 < x < 1, \ 0 < y < 1\}$ that satisfies $u(x,0) = 54x^2(1-x)$ for $0 \le x \le 1$ and vanishes at all other points of the boundary of S (use step size $\frac{1}{3}$ for both x and y).