10. Show that the PDE $(u_x)^2 + (u_t)^2 = 0$ is not linear. Find its general solution.

11. Consider $au_x + bu_t = 0$. Use the geometric method twice to find the general solution (one time by eliminating the first component in u and a second time by eliminating the second component). Show that both representations of the general solution are the same.

12. Find the solution of $u_x - u_t + 2u = 1$ that satisfies $u(x, 0) = x^2$.

13. Consider the PDE $u_x + u_t = u + e^{x-t}$.

    (a) Apply the geometric method to obtain an idea how the general solution looks like.

    (b) Find the general solution.

    (c) Find the solution u with $u(x, 0) = g(x)$, where g is an arbitrary differentiable function.

    (d) Find the solution u with $u(x, 1) = g(x)$, where g is an arbitrary differentiable function.

14. Find the solution of $u_x + u_t + u = e^{x+2t}$ that satisfies $u(x, 0) = 0$.

15. Find the solution of $2u_x + 3u_t = 4u + x$ that satisfies $u(x, 0) = 9x^2$.

16. Consider the problem $u_x + 3u_t - u = 1$, $u(x, 3x) = g(x)$.

    (a) For which functions g does this problem have a solution?

    (b) Find two different solutions of the problem if $g(x) = 2\mathrm{e}^x - 1$.

17. Find the general solutions of the following equations. Where are they defined? Sketch some of the characteristic curves.

    (a) $xu_x + tu_t = 0$;

    (b) $xu_x + tu_t = t$;

    (c) $xu_x + tu_t = t^2 + x^3$;

    (d) $(1 + x^2)u_x + u_t = 0$.

18. Find the solution of $\sqrt{1 - x^2}u_x + u_t = 0$ that satisfies $u(0, t) = t$.

19. Find the solution of $tu_x + xu_t = 0$ that satisfies $u(0, t) = e^{-t^2}$.

20. Consider the equation $xu_t = tu_x$.

    (a) Find the general solution.

    (b) Find the solution that satisfies $u(x, 0) = 3x$.

21. Find the solution of $(t + x)u_x + (t - x)u_t = 0$ that satisfies $u(\cos(s), \sin(s)) = 1$ for all $s \in [0, 2\pi]$.

22. Find the solution of $4xu_x + 2tu_t = xt$ that satisfies $u(x, 1) = \phi(x)$ for some given function ϕ.

23. Find the general solution of $xu_x + u_y + (1 + z^2)u_z = x + y$.