Set	MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY	Founded 1870   Rolla, Missouri
	Section 7.2	

The Definition of the Laplace Transform

### Laplace Transforms: The Basic Goal

We wish to solve IVPs involving DEs of the form 
$$a_ny^{(n)}+a_{n-1}y^{(n-1)}+\cdots+a_1y'+a_0y=g(t)$$

with initial conditions

$$y(0)=y_0,y'(0)=y_1,\dots,y^{(n-1)}(0)=y_{n-1}$$

If g(t) is reasonably straightforward, we can use undetermined coefficients or variation of parameters.

If g(t) is more complicated, Laplace transforms often work.

### Solving IVPs Using Laplace: The Basic Idea

- 1. Convert the initial value problem into an algebraic equation using the Laplace transform
- 2. Solve the algebraic equation
- 3. Convert the solution of the algebraic equation into a solution of the initial value problem using the inverse Laplace transform

## Are Laplace Transforms Useful?

# Yes!

The Laplace transform allows us to solve initial value problems that cannot be solved using other techniques.

The Laplace transform is a fundamental tool in systems and in control theory.

### The Laplace Transform

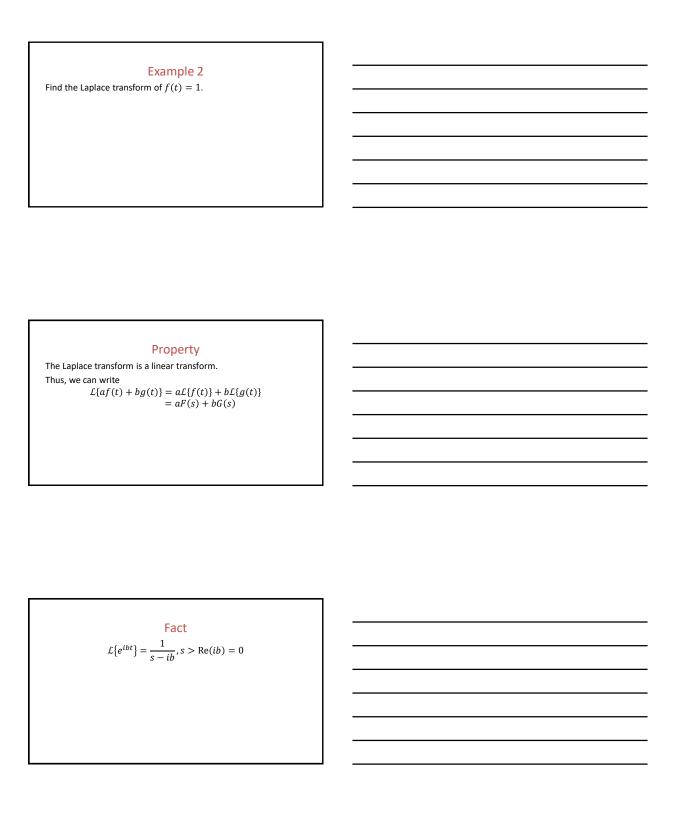
Let f be a function defined on  $[0,\infty)$ . Then, the Laplace transform of f at s is defined by

$$\mathcal{L}{f(t)} = F(s) = \int_{0}^{\infty} e^{-st} f(t) dt$$

for those values of  $\boldsymbol{s}$  for which the improper integral converges.

### Example 1

Find the Laplace transform of  $f(t) = e^{4t}$ .



### Example 3

Find the Laplace transforms of the functions  $f(t)=\cos(bt)$  and  $g(t)=\sin(bt)$  where b is a real constant.

### **Table of Laplace Transforms**

f(t)	$\mathcal{L}\{f(t)\} = F(s)$
1	$\frac{1}{s}$ , $s > 0$
$e^{at}$	$\frac{1}{s-a}, s > a$
$t^n$ , $n = 1, 2, \dots$	$\frac{n!}{s^{n+1}}, s > 0$
$\sin(bt)$	$\frac{b}{s^2 + b^2}, s > 0$
$\cos(bt)$	$\frac{s}{s^2 + b^2}, s > 0$

### Example 4

Find the Laplace transform of  $f(t) = 2e^{3t} + 3t^2 + 9\sin 4t$ 

Piecewise Continuity  If a function $f$ is continuous at every point of a finite interval $[a,b]$ except possibly at a finite number of points where $f$ has a jump discontinuity, then $f$ is said to be piecewise continuous on $[a,b]$ .  If $f$ is piecewise continuous on every possible interval $[0,N]$ for $N>0$ , then $f$ is said to be piecewise continuous on $[0,\infty)$ .	
Exponential Order  A function $f$ is said to be of exponential order $\alpha$ if there exist positive constants $T$ and $M$ such that $ f(t)  \leq Me^{\alpha t}$ for all $t \geq T$ .	
Theorem If $f$ is piecewise continuous on $[0,\infty)$ and of exponential order $\alpha$ , then $\mathcal{L}\{f(t)\}$ exists for $s>\alpha$ .	

# Find the Laplace transform of $f(t) = \begin{cases} e^t & 0 \le t < 1\\ 1 + e^{2t} & 1 < t \end{cases}$