Section 7.4

The Inverse Laplace Transform

Goals for Today

We have previously seen how to calculate the Laplace transform of a wide variety of functions and of derivatives of functions.

Today, we need to consider the inverse of this process.

In the next section, we will put all this together to solve initial value problems.

The Inverse Laplace Transform

If there exists a function $f(t)$ which is continuous on $[0, \infty)$ and which has Laplace transform $\mathcal{L}\{f(t)\} = F(s)$, then $f(t)$ is the inverse Laplace transform of $F(s)$.

Symbolically, we write

$$\mathcal{L}^{-1}\{F(s)\} = f(t)$$

Theorem

The inverse Laplace transform is linear.

Thus, if $\mathcal{L}\{f(t)\} = F(s)$ and $\mathcal{L}\{g(t)\} = G(s)$, we have

$$\mathcal{L}^{-1}\{aF(s) + bG(s)\} = af(t) + bg(t)$$

Example 1

Compute the inverse Laplace transform.

$$\mathcal{L}^{-1}\left\{\frac{5}{s} + \frac{9}{s+4}\right\}$$

Example 2

Compute the inverse Laplace transform.

$$\mathcal{L}^{-1}\left\{\frac{s-3}{s^2+4}\right\}$$

Example 3

Compute the inverse Laplace transform.

$$\mathcal{L}^{-1}\left\{\frac{2}{(s-1)^3}\right\}$$

Example 4

Compute the inverse Laplace transform.

$$\mathcal{L}^{-1}\left\{\frac{3}{s^2-4s+13}\right\}$$

Example 5

Compute the inverse Laplace transform.

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+3)(s^2+4)}\right\}$$

Example 6

Compute the inverse Laplace transform.

$$\mathcal{L}^{-1}\left\{\frac{s+2}{s^4+s^2}\right\}$$

Example 7

Compute the inverse Laplace transform.

$$\mathcal{L}^{-1}\left\{\frac{1-2s}{s^2+4s+5}\right\}$$