

MISSOURI

ST

MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Founded 1827 | Rolla, Missouri

Section 9.2

Linear Algebraic Equations

Goals for Today

We need to look at using matrix methods to solve linear algebraic equations.

Recall: Elementary Row Operations

Replacement:
Replace one row by the sum of itself and a multiple of another row.

Interchange:
Interchange (swap) two rows.

Scaling:
Multiply all entries in a row by a nonzero constant.

Example 1

Apply elementary row operations to the augmented matrix of the system to solve the system of linear equations.

$$\begin{aligned}x_1 + 2x_2 + 3x_3 &= 4 \\4x_1 + 5x_2 + 6x_3 &= 7 \\7x_1 + 8x_2 + 10x_3 &= 10\end{aligned}$$

Row Equivalence

Two matrices are called row equivalent if there exists a sequence of elementary row operations which transform one matrix into another.

If the augmented matrices of two linear systems are row equivalent, then they have the same solution set.

Echelon Forms

A rectangular matrix is in (row) echelon form if it has the following three properties:

1. All nonzero rows are above any rows of all zeros.
2. Each leading entry of a row is in a column to the right of the leading entry of the row above it. (The leading entry of a row is the first nonzero entry.)
3. All entries in a column below a leading entry are zeros.

Echelon Forms

If a matrix in (row) echelon form also satisfies the following two conditions, then it is in reduced (row) echelon form:

4. The leading entry in each nonzero row is 1.
5. Each leading 1 is the only nonzero entry in its column.

Note that each matrix is row equivalent to one and only one reduced echelon matrix.

Pivot Positions

A pivot position in a matrix is a location in the matrix that corresponds to a leading 1 in the reduced echelon form of the matrix.

A pivot column of the matrix is a column which corresponds to a pivot position.

The Gauss-Jordan Row Reduction Algorithm

The Gauss-Jordan Row Reduction Algorithm provides an efficient procedure for transforming a matrix into either echelon or reduced echelon form.

Example 2

Describe the solutions of the following system in parametric vector form.

$$\begin{aligned}x_1 + 2x_2 - 3x_3 &= 5 \\ 2x_1 + x_2 - 3x_3 &= 13 \\ -x_1 + x_2 &= -8\end{aligned}$$

Then, give a geometric description of the solution set.

The Gauss-Jordan Row Reduction Algorithm

1. Begin with the leftmost nonzero column. This is a pivot column, and the pivot position is at the top.
2. Select your favorite nonzero entry in the pivot column as a pivot and, if necessary, interchange rows to place this entry in the pivot position.
3. Use row replacement operations to create zeros in all positions below the pivot.

The Gauss-Jordan Row Reduction Algorithm

4. Ignore the row containing the pivot and all rows (if any) above it. Then, repeat steps 1-3 for the remaining submatrix. Continue this process until there are no more nonzero rows to modify.

At this point, the matrix is in echelon form.
One more step will place the matrix in reduced echelon form.

The Gauss-Jordan Row Reduction Algorithm

5. Beginning with the rightmost pivot and working to the left, apply scaling operations to make each pivot a 1 and use replacement operations to create zeros above each pivot.

The matrix is now in reduced echelon form.
