4. Show that the zero and the one in a field are unique.
5. Prove Proposition 2.4 from the lecture notes.
6. Assume K satisfies the field axioms. Let $a, b, c, d \in K$. Prove:
   (a) $(-a) \cdot (-b) = a \cdot b$;
   (b) $(a - b)c = ac - bc$;
   (c) $\frac{a}{b} : \frac{c}{d} = \frac{ad}{bc}$ provided $b, c, d \neq 0$.
7. Let $K = \{0, 1\}$ and define addition "+" by $0 + 0 = 1 + 1 = 0$, $0 + 1 = 1 + 0 = 1$ and multiplication "·" by $0 \cdot 0 = 0 \cdot 1 = 1 \cdot 0 = 0$, $1 \cdot 1 = 1$. Prove:
   (a) K satisfies the field axioms;
   (b) K does not satisfy the positivity axioms.
8. Prove the following rules in an ordered field:
   (a) $a^2 + b^2 \geq 0$, and $a^2 + b^2 = 0$ iff $a = b = 0$;
   (b) $0 \leq a < b$ and $0 \leq c < d$ imply $ac < bd$;
   (c) $0 \leq a < b$ implies $0 \leq a^2 < b^2$;
   (d) $0 < a < b$ implies $\frac{1}{a} > \frac{1}{b} > 0$;
   (e) $b, d > 0$ implies ($\frac{a}{b} < \frac{c}{d}$ iff $ad < bc$);
   (f) $b, d > 0$ and $\frac{a}{b} < \frac{c}{d}$ imply $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$;
   (g) $0 \leq a \leq \varepsilon$ for all $\varepsilon > 0$ implies $a = 0$.
9. Suppose K is an ordered field. Let $\alpha \in K \cap (0, 1)$ and $a < b$. Prove $a < \alpha a + (1 - \alpha)b < b$.