10. State and prove the analogue of Proposition 2.17 for a minimum.

11. State and prove the analogue of Theorem 2.18 for an infimum.

12. Let $T = [0, 1)$ in an ordered field. Find $\min T$, $\max T$, $\inf T$, and $\sup T$.

13. Prove Theorem 2.20.

14. Let $A, B \subset \mathbb{R}$ with $A \subset B$. If $\sup A$ and $\sup B$ exist, prove $\sup A \le \sup B$.

15. Find $\sup\{x \in \mathbb{R} : \ x^2 < x\}$. Prove your claim.

16. Let $a, b, c \in \mathbb{R}$ and find all $x \in \mathbb{R}$ such that $ax^2 + bx + c = 0$.

17. Show that there exists exactly one positive $x \in \mathbb{R}$ with $x^3 = 3$.

18. Prove the following statements using the PMI:

    (a) $\forall n \in \mathbb{N} : \ 1 + 3 + 5 + \ldots + (2n - 1) = n^2$;

    (b) $\forall n \in \mathbb{N} : \ 1 + 4 + 9 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$;

    (c) $\forall n \in \mathbb{N} : \ 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n(n + 1) = \frac{n(n+1)(n+2)}{3}$;

    (d) $\forall n \in \mathbb{N} : \ \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \ldots + \frac{n}{n+1} \le \frac{n^2}{n+1}$.