

40. Let $c \geq 1$. Discuss the continuity of f , where $f(x) = \begin{cases} \frac{6}{cx-1} & \text{if } x > 1 \\ 2x^2 + 1 & \text{if } x \leq 1. \end{cases}$
41. Prove that there exists a positive solution of the equation $\frac{1}{\sqrt{x+x^2}} = 2x - x^2$.
42. Prove or disprove that f is continuous at x_0 , using the “ ε/δ ” criterion:
- (a) $f(x) = x^3 + x$, $x_0 = 2$;
 - (b) $f(x) = \frac{x^3-2}{x+3}$, $x_0 = 1$;
 - (c) $f(x) = e(x)$, $x_0 = 1$ (see #39);
 - (d) $f(x) = \begin{cases} x & \text{if } x < 0 \\ x + 2 & \text{if } x \geq 0 \end{cases}$, $x_0 = 0$.
43. Determine whether the following functions are uniformly continuous:
- (a) $f(x) = 6x + 7$, $f : \mathbb{R} \rightarrow \mathbb{R}$;
 - (b) $f(x) = \frac{1}{1+x^2}$, $f : \mathbb{R} \rightarrow \mathbb{R}$;
 - (c) $f(x) = x^3$, $f : \mathbb{R} \rightarrow \mathbb{R}$;
 - (d) $f(x) = \frac{x}{x-1}$, $f : [2, \infty) \rightarrow \mathbb{R}$.
44. A function $f : D \rightarrow \mathbb{R}$ is called a Lipschitz function if there exists some $c \geq 0$ such that $|f(u) - f(v)| \leq c|u - v|$ for all $u, v \in D$. Find a Lipschitz function that is not uniformly continuous. Also find a uniformly continuous function that is not a Lipschitz function.