81. A stock price follows geometric Brownian motion with an expected return of 16% and a volatility of 35%. The current price is \$40.
    (a) What is the probability that a European call option on the stock with an exercise price of \$40 and a maturity date in 6 months will be exercised?
    (b) Answer (a) if the call is replaced by a put.
82. Suppose that a stock price (in dollar) at the end of each of 15 consecutive weeks was 30.2, 32.0, 31.1, 30.1, 30.2, 30.3, 30.6, 33.0, 32.9, 33.0, 33.5, 33.5, 33.7, 33.5, 33.2 and estimate the stock price volatility.
83. Is S^{-2r/σ^2} the price of a tradeable security?
84. Calculate the price of a 3-month European put option on a non-dividend-paying stock with a strike price of \$50 when the current stock price is \$50, the risk-free interest rate is 10%, and the volatility is 30%.
85. What is the price of a European call option on a non-dividend-paying stock when the stock price is \$52, the strike price is \$50, the risk-free interest rate is 12%, the volatility is 30%, and the time to maturity is 3 months?
86. What is the price of a European put option on a non-dividend-paying stock when the stock price is \$69, the strike price is \$70, the risk-free interest rate is 5%, the volatility is 35%, and the time to maturity is 6 months?
87. Show that the Black–Scholes formulas for call and put options satisfy put–call parity.
88. Consider an option on a non-dividend-paying stock when the stock price is \$30, the exercise price is \$29, the risk-free interest rate is 5%, the volatility is 25%, and the time to maturity is 4 months. Find the price of the option if it is a
    (a) European put;
    (b) European call;
    (c) American call.
89. Consider a derivative that pays off S_T^n at time T, where S_T is the stock price at that time. Assume that the stock price follows geometric Brownian motion. Now assume that the price of the derivative can be written as $f(t, s) = h(t)s^n$, where h is an unknown function in one variable.
    (a) By substituting into the partial differential equation (BSM), derive an ordinary differential equation for h.
    (b) What is the boundary condition for the differential equation for h?
    (c) Solve the problem for h and hence find the price of the derivative.