62. If S follows geometric Brownian motion, find the distribution of $\ln S$.

63. Solve $\mathrm{d}X = W\mathrm{d}t + t\mathrm{d}W$, $X(0) = 0$.

64. Suppose $X(t) = tW(t)$ and let Y be the solution of

$$\mathrm{d}Y = \frac{1}{2}Y\mathrm{d}t + Y\mathrm{d}W, \quad Y(0) = 1.$$

Find $\mathrm{d}(X(t)Y(t))$.

65. Find a stochastic differential equation solved by the process $X(t) = \frac{W(t)}{1+t}$.

66. Solve $\mathrm{d}X(t) = (\alpha(t) + \beta(t)X(t))\mathrm{d}t + \gamma(t)\mathrm{d}W(t)$, where α, β, γ are deterministic.

67. If R solves the CIR model, calculate the third moment of $R(t)$.

68. If R solves the Vasicek model, define the price of a zero-coupon bond with maturity T at time $t \in [0, T]$ by

$$B(t) = a(t)e^{-R(t)b(t)},$$

where

$$b(t) = \frac{1 - e^{-\beta(T-t)}}{\beta}$$

and

$$a(t) = \exp\left\{\left(\frac{\alpha}{\beta} - \frac{\sigma^2}{2\beta^2}\right)(b(t) - T + t) - \frac{\sigma^2}{4\beta}b^2(t)\right\}.$$

Show

$$\mathrm{d}B(t) = R(t)B(t)\mathrm{d}t - \sigma b(t)B(t)\mathrm{d}W(t)$$

and

$$\mathrm{d}\frac{1}{B(t)} = \frac{\sigma^2 b^2(t) - R(t)}{B(t)}\mathrm{d}t + \frac{\sigma b(t)}{B(t)}\mathrm{d}W(t).$$